



# MAILAM ENGINEERING COLLEGE

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Department of Electrical & Electronics Engineering

SUB CODE/NAME: EE8301 / Electrical Machines - I

YEAR / SEC : II / A & B

## SYLLABUS

### UNIT I MAGNETIC CIRCUITS AND MAGNETIC MATERIALS

Magnetic circuits - Laws governing magnetic circuits - Flux linkage, Inductance and energy - Statically and dynamically induced EMF - Torque - Properties of magnetic materials, Hysteresis and Eddy current losses - AC excitation, introduction to permanent magnets- Transformer as a magnetically coupled circuit.

### UNIT II TRANSFORMERS

Construction - principle of operation - equivalent circuit parameters - phasor diagrams, losses - testing - efficiency and voltage regulation-all day efficiency-Sumpner's test, per unit representation - inrush current - three phase transformers-connections - Scott Connection - Phasing of transformer-parallel operation of three phase transformers-auto transformer - tap changing transformers- tertiary winding.

### UNIT III ELECTROMECHANICAL ENERGY CONVERSION AND CONCEPTS IN ROTATING MACHINES

Energy in magnetic system - Field energy and co energy-force and torque equations - singly and multiply excited magnetic field systems-mmf of distributed windings - Winding Inductances, magnetic fields in rotating machines - rotating mmf waves - magnetic saturation and leakage fluxes.

### UNIT IV DC GENERATORS

Construction and components of DC Machine - Principle of operation - Lap and wave windings-EMF equations- circuit model - armature reaction -methods of excitation commutation - interpoles compensating winding -characteristics of DC generators.

### UNIT V DC MOTORS

Principle and operations - types of DC Motors - Speed Torque Characteristics of DC Motors starting and speed control of DC motors -Plugging, dynamic and regenerative braking testing and efficiency - Retardation test- Swinburne's test and Hopkinson's test - Permanent Magnet DC (PMDC) motors-applications of DC Motor

### TEXT BOOKS:

1. Nagrath, I.J. and Kothari.D.P., Electric Machines', McGraw-Hill Education, 2004

### REFERENCES

1. B.R. Gupta, 'Fundamental of Electric Machines' New age International Publishers, 3rd Edition, Reprint 2015.

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## UNIT – I

### MAGNETIC CIRCUIT AND MAGNETIC MATERIALS

**1. Write Lorentz force equation?(May-2011)**

If a particle carrying charge +Q and moving with velocity  $\vec{V}$  is present in a region where both electric and magnetic fields are present, then the particle in a motion experience forces  $\vec{F}_e$  and  $\vec{F}_m$  due to the electric and magnetic fields are present respectively and resultant force

$$\vec{F} = \vec{F}_e + \vec{F}_m = Q (E + \vec{V} \times \vec{B}) \text{ Newton}$$

**2. Give Analogy between electric circuit and Magnetic circuit (Dec-2010)(May 2013)**

S.No	Electric Circuit	Magnetic circuit
1	Current = $\frac{\text{Emf}}{\text{Resistance}}$	Flux = $\frac{\text{MMF}}{\text{Reluctance}}$
2.	Resistance = $\frac{\rho \ell}{A}$	Reluctance = $\frac{\ell}{\mu_o \mu_r A}$
3.	Conductance = $\frac{1}{\text{Resistance}}$	Permeance = $\frac{1}{\text{Reluctance}}$
4.	$E = \frac{V}{d}$	$H = \frac{NI}{l}$

**3. Define torque? (May-2010)**

A turning or a twisting force about an axis defined as torque.

**4. Distinguish between statically induced emf and dynamically induced emf.(Dec 2014)(May-Jun 2015)**

S.No	Statically Induced emf	Dynamically Induced emf
1	Magnetic field or flux changes its position but conductor remains constant	Conductor changes its position but magnetic field remains constant
2.	Eg: Transformer	Eg: Generator

**5. A conductor 80cm long moves at right angle to its length at a constant speed of 30 m/s in a uniform magnetic field of flux density 1.2T. Find the emf induced when the conductor motion is normal to the field flux?(May-2011)**

$$\begin{aligned} E &= B \ell v \sin \theta \\ &= 1.2 \times 80 \times 10^{-2} \times 30 \times \sin 90^\circ \\ E &= 28.8 \text{ V} \end{aligned}$$

**6. What are the core losses and how can this loss be minimized? (May-2012)**

When a magnetic material undergoes cyclic magnetization, two kinds of power losses occur on it. Hysteresis and eddy current losses are called as core loss

**Hysteresis loss**– This loss can be reduced by selecting good quality magnetic material.

**Eddy current loss** – This loss can be reduced by using thin laminations for the core.

**7. Clearly define MMF and EMF. (May-2012& Dec 2014)**

**MMF:** It is the cause for producing flux in a magnetic circuit. The amount of flux setup in the core depends upon the current (I) and number of turns (N).

$$\text{MMF} = NI$$

**EMF:** It is the driving force in the electric circuit

$$\text{EMF} = \text{Current} \times \text{Resistance}, \text{EMF} = I \times R \text{ (Volts)}$$

**8. What is hysteresis loss and how can this loss be minimized? (Dec-2011, 2013, 2016)**

When the magnetic material is subjected to **repeated cycles of magnetization and demagnetization**, it results into disturbance in the alignment of the various domain. **All the energy is never returned** through field completely collapses. **This loss of energy appears as heat** in the magnetic material. This can be minimized by selecting good quality magnetic material like Silicon.

**9. Define the term MMF and Reluctance (Dec 2009, 2003, 2007)**

$$\text{MMF} = NI \text{ (Ampere Turns)}$$

$$\text{Reluctance} = \text{MMF} / \text{flux} = NI / \phi \text{ (AT/Wb)}$$

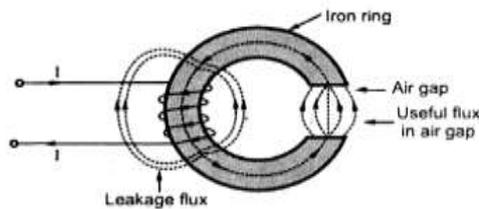
**10. A coil of 1000 turns in linking a flux of 0.01 Wb, The flux is reversed in an interval of 0.1 S. Calculate the average value of emf induced in the coil? (May-2007)**

$$e = -N \frac{d\phi}{dt} \text{ [Flux changing from 0.01 to -0.01]}$$

$$= -1000 \times \frac{-0.01 - 0.01}{0.1} = -1000 \times \frac{-0.02}{0.1} = 200V$$

**11. What is meant by leakage flux? (May-2004) (May 2016)**

The flux which is passing through unwanted path is called leakage flux.

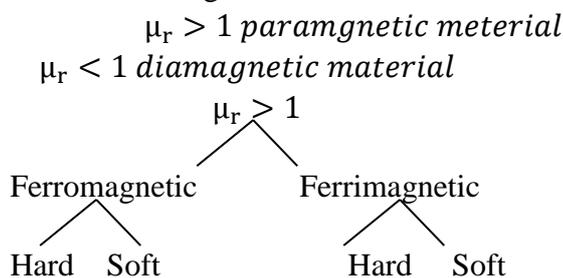


**12. State ohms law of magnetic circuit? (Apr-2009)**

$$\text{MMF} = \phi \times s$$

**13. Classify magnetic material? (Dec 2008)**

Magnetic material classified according to the nature of its relative permeability



For hard Ferro magnetic material- Alnico

Self Ferro magnetic material-Iron with its alloy with Ni, CO

Super magnetic material-Nichol, Zinc, Ferrites

**14. Mention the factors on which hysteresis loss depends?**

- The hysteresis loss is directly  $\propto$  to the area under the hysteresis curve.
- It is directly  $\propto$  to f (ie) number of cycle of magnetization
- It is directly  $\propto$  to volume of the material.

**15. Name the main magnetic quantities with their symbols having the following Units. Webers, Telsa, AT/wb, H/m. (Dec 2013)**

Weber-flux

Telsa-Magnetic flux density

AT/wb-Reluctance

H/m –permeability

**16. What is Permanent magnets?**

The PM is an important excitation source commonly employed for imparting energy to magnetic circuits used in rotating machines and other types of electromechanical devices. There are three classes of PM materials (or hard magnetic material) used for PMDC Motors. Alnicos, ceramics (ferrites) and rare earth material. Alnico material are used in motors up to 200kw while ceramic materials most economic in fractional KW motors. For very high temperature applications Alnico or rare earth cobalt magnets must be used.

**17. Define coactivity.**

It is the measure of MMF which, when applied to the magnetic circuit would reduce its flux density to zero, i.e., it demagnetizes the magnetic circuit.

**18. Mention some magnetic materials (May-Jun 2015)**

Alnicos, chromium steels, copper–nickel alloy, nickel, cobalt, tungsten and aluminum.

**19. What is magnetostriction?**

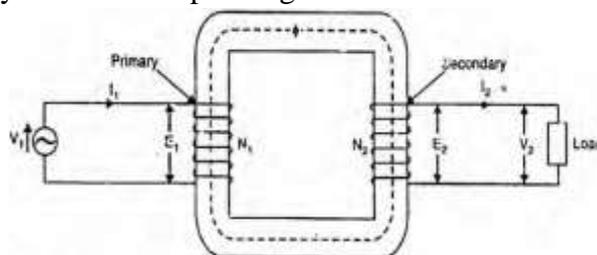
When ferromagnetic materials are subjected to magnetizing MMF, these may undergo small changes in dimension; this phenomenon is known as magnetostriction.

**20. Define magnetic reluctance (May -2014)**

The opposition offered by the magnetic circuit for the magnetic flux path is known as magnetic reluctance. It is analogous to electric resistance.

**21. What is magnetically circuit Transformer?**

A transformer is a static piece of equipment used either for raising or lowering the voltage of an A.C. supply with a corresponding decrease or increase in current.



It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in Fig. When the two circuits are placed very close to each other such that a magnetic flux produced by one circuit links with both the circuits, then the two circuits are said to be magnetically circuit.

**22. What are quasi-static fields? (May 2014)(Nov-Dec 2015)**

The field pattern in space is fixed but the field intensity at every point varies as a replica of the time variation of current.

**23. What is Stacking factor? (Nov-Dec 2015)**

The laminated construction helps to keep eddy current losses to low value. Due to stacks of lamination, the net cross sectional area occupied by the magnetic material less than its gross cross sectional area.

$$\text{Stacking factor} = \frac{\text{Net cross sectional area occupied by magnetic material}}{\text{gross cross – sectional area}}$$

Value always <1

**24. State Ampere’s Law. (May 2016)**

The integral around a closed path of the component of the magnetic field Tangent to the direction of the path equals  $\mu_0$  times the current intercepted by the area within the path

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$

**25. Define flux linkage. Dec - 2016**

It is known that a current carrying conductor is surrounded by concentric circles of magnetic. Flux linkage = Turns  $\times$  Flux =  $N\Phi$

**26. What is eddy current loss?**

When a magnetic core carries a time varying flux, voltages are induced in all possible path enclosing flux. Resulting is the production of circulating flux in core. These circulating current do no useful work are known as eddy current and have power loss known as eddy current loss.

**27. Define relative permeability. May 2017**

The ratio of flux density produced in that material to the flux density produced in air by the same magnetizing force.

$$\mu_r = \frac{\mu}{\mu_0} \text{ where, } \mu - \text{Absolute permeability of the material}$$

$\mu_0$ - Absolute permeability of air/vacuum

$\mu_r$  -Relative permeability of the material

**28. Give the expression for hysteresis losses and eddy current losses. May 2017**

**Hysteresis loss**= $K_h (B_m)^{1.6} f \times \text{Volume}$

$K_h$ -Characteristic constant of the material,  $B_m$ -Maximum flux density

$f$ -frequency in cycles per second

**Eddy current loss**= $K_e [B_{\max}]^2 f^2 t^2 v$  watt

$K_e$  - eddy current coefficient it depends on type of the material used

$f$ -numbers of complete magnetization cycle per second

$B_{\max}$  – maximum flux density,  $t$ - thickness of the lamination

$v$ - volume of the material.

**29. Define Magnetic flux density (Nov 2017)**

Magnetic flux density is defined as the amount of magnetic flux in an area taken perpendicular to the magnetic flux's direction.

**30. Define Self-inductance (Nov 2017)**

Self-inductance is defined as the induction of a voltage in a current-carrying wire when the current in the wire itself is changing.

**31. what are the types of magnetic losses?(May2018)**

1. Iron loss- hysteresis and eddy current losses
2. Copper loss-  $I^2R$  Losses

**32. How are hysteresis and eddy current losses minimized? (May 2018)**

Core losses majorly include Hysteresis loss and eddy current loss. Eddy Current loss can be reduced by increasing the number of laminations. The laminations provide small gaps between the plates. As it is easier for magnetic flux to flow through iron than air or oil, stray flux that can cause core losses is minimized.

**PART-B**

**1. Explain clearly the statically induced emf and dynamically induced emf?(May-2012) (Dec-2013, 2014, 2015, 2016)**

**Or**

**Derive the expression for self-inductance and mutual inductance and also define coefficient of coupling. (May 2017)**

EMF gets induced in a conductor, whenever their exits change in flux with that conductor, according to Faraday's Law. Such change in flux can be brought about by different methods. Namely,

1. Dynamically Induced emf
2. Statically Induced emf

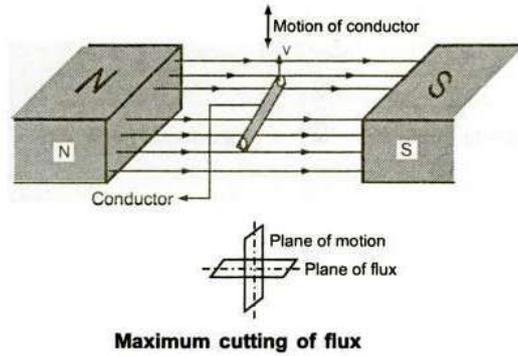
**1. Dynamically induced emf:**

The change in the flux linking with a conductor (or) circuit can be brought about by its motion relative to magnetic field. This is possible by moving flux with respect to coil conductor or circuit or it is possible by moving conductor, coil, and the circuit with respect to stationary magnetic flux.

**Magnitude of dynamically induced emf:**

Consider a conductor moves in the air gap between poles of magnet. If plane of the motion of the conductor is parallel to the plane of the magnetic field then there is no cutting of flux lines, no induced emf.

In second case, the motion of the conductor is perpendicular to the flux. Hence whole length of conductor is cutting the flux line hence there is maximum possible induced emf in the conductor. Under such condition plane of flux and plane of motion are perpendicular to each other.

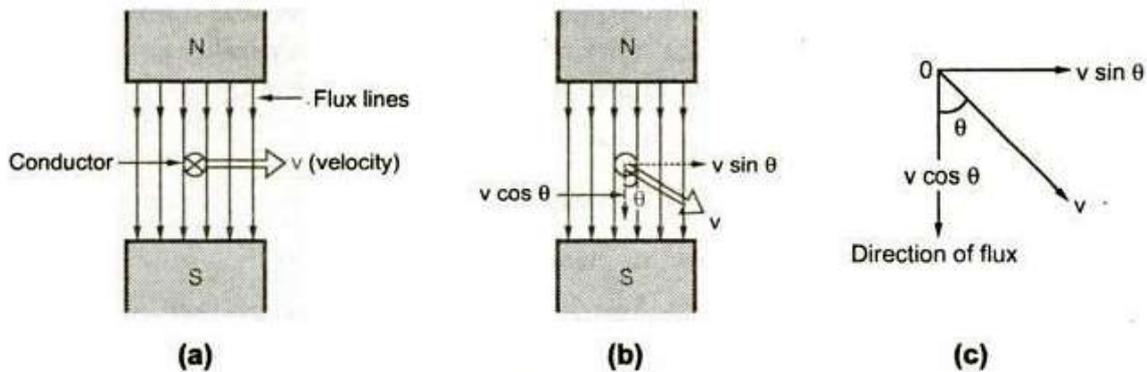


Consider a conductor moving with velocity  $v$  m/s such that its plane of motion or direction of velocity is perpendicular to the direction of flux lines as shown below.

$B$ -Flux density  $\text{wb/m}^2$

$\ell$ -Active length of conductor in meters

$V$ -Velocity in m/s



Let the conductor is moved through distance  $dx$  in a small time interval  $dt$ , then

$$\text{Area swept by conductor} = \ell \times dx \text{ m}^2$$

Flux cut by conductor = Flux density  $\times$  Area swept

$$d\phi = B \times \ell \times dx \quad \omega b$$

According to Faraday's law the magnitude of induced emf is perpendicular to rate of change of flux

$$e = \frac{\text{Flux cut}}{\text{Time}}$$

$$= \frac{d\phi}{dt} = \frac{B \ell dx}{dt}$$

$$e = Blv \text{ volts}$$

This is (maximum) induced emf when plane of motion is exactly perpendicular to the plane of flux. But if the conductor is moving with a velocity  $v$  at a certain angle  $\theta$  measured with respect to direction of field.

Under this condition,

Magnitude of Induced emf

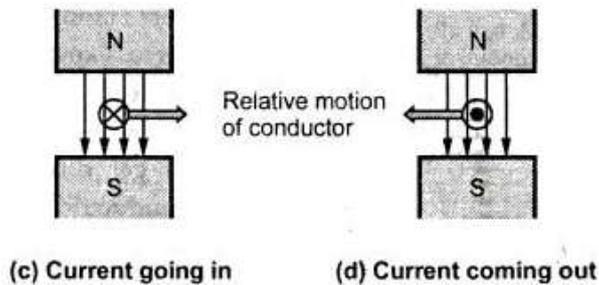
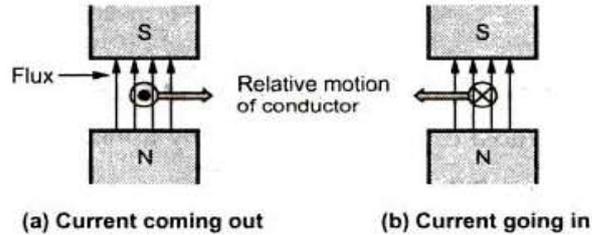
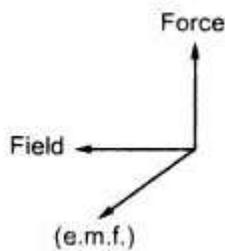
$$e = Blv \sin\theta \text{ volts}$$

$\theta$  – is measured with respect to plane of the flux.

**Direction of Induced emf**

1. Fleming's Right hand rule:

The thumb, fore finger, middle finger of right hand at right angle to one another. If fore finger points in the direction of magnetic field, thumb in the direction of motion of the conductor, then the middle finger will point in the direction of induced emf.



2. Fleming's Left hand Rule;

The fore finger, middle finger and thumb of the left hand at right angle to one another. If the fore finger points in the direction of magnetic field, the middle finger points towards the direction of current, then the thumb will points in the direction of motion of the conductor.

3. Lenz's law:

The direction of an Induced emf produced by the electromagnetic induction is such that it sets up a current which always opposes the cause that is responsible for inducing the emf.

$$e = -N \frac{d\phi}{dt}$$

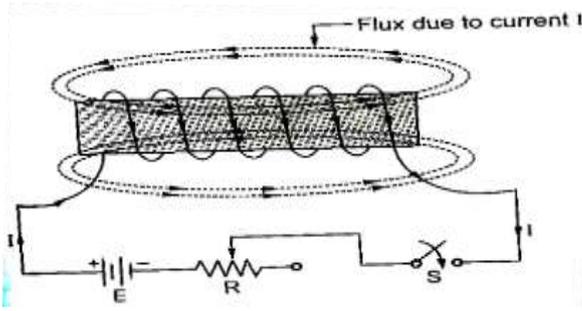
**2. Statically Induced EMF:**

The change in flux lines with respect to coil can be achieved without physically moving the coil or magnet is known as statically induced emf..

It is classified into

- ✓ Self induced e.m.f.
- ✓ Mutually induced e.m.f.

Self induced e.m.f.



Consider a coil having 'N' turns and carrying current 'I' when switch 'S' is in closed position. The current magnitude can be varied with the help of variable resistance connected in series with battery.

The flux produced by the coil links with the coil itself. The total flux linkages of coil will be  $N\Phi$  wb turns.

If 'I' changed flux also changed. Flux linkage also be changed. According to Faraday's law, due to rate of change of flux linkages there will be induced e.m.f in the coil. So without physically moving coil or flux there is induced e.m.f. in the coil. The phenomenon is called self induced.

**Expression or Magnitude of self induced e.m.f.**

According to Faraday's Law of electromagnetic induction, self induced e.m.f. can be expressed as,

$$e = -N \frac{d\Phi}{dt}$$

Negative sign indicates that direction of the e.m.f. is opposing change in current due to which it exists.

$$\Phi = \frac{\phi}{I} \times I$$

$$\text{Rate of change of flux} = \frac{\phi}{I} \times \text{Rate of change of current}$$

$$\frac{d\Phi}{dt} = \frac{\phi}{I} \cdot \frac{dI}{dt}$$

$$e = -N \frac{\phi}{I} \cdot \frac{dI}{dt}$$

$$= -\left(\frac{N\phi}{I}\right) \frac{dI}{dt}$$

$\frac{N\phi}{I}$  is called as coefficient of self inductance it is denoted by L

$$L = \frac{N\phi}{I}$$

$$e = -L \frac{dI}{dt} \text{ Volts}$$

**Expression for Coefficient of self-Inductance:**

$$L = \frac{N\phi}{I}$$

$$\Phi = \frac{\text{mmf}}{\text{Reluctance}} = \frac{NI}{S}$$

$$L = \frac{N\Phi}{I} = \frac{N^2}{S} = \frac{N^2}{\ell/\mu a}$$

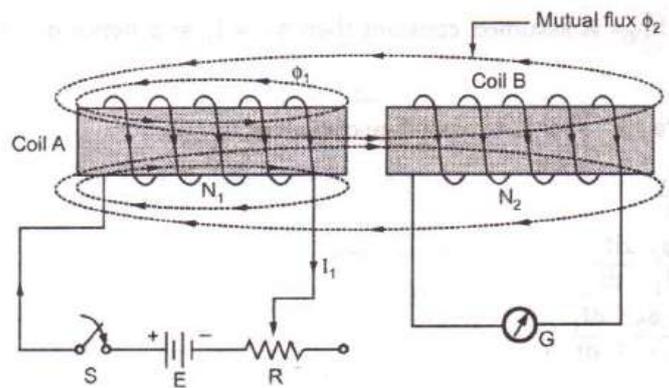
$$L = \frac{N^2 \mu_0 \mu_r a}{\ell}$$

$\ell$  - length of magnetic circuit

a- Area of cross section of magnetic circuit through which flux is passing.

### Expression for mutual Inductance or Mutually induced emf

If the flux produced by one coil is getting linked with another coil and due to change in this flux produced by first coil, there is induced e.m.f. in the second coil, then such an e.m.f. is called mutually induced e.m.f.



Consider two coils which are placed adjacent to each other as shown in figure. The coil A has  $N_1$  turns while coil B has  $N_2$  number of turns. The coil A has switch S, variable resistance R and battery of 'E' in series with it. A galvanometer is connected across coil B to sense induced e.m.f. and current because of it.

Current through coil A is  $I_1$  producing flux  $\Phi_1$ . Part of this flux will link with coil B, will complete its path through coil B as shown in figure (mutual flux  $\Phi_2$ ).

Now if current through coil A is changed by means of variable resistance R, then flux  $\Phi_1$  changes. Due to this, flux associated with coil B, which is mutual flux  $\Phi_2$  also changes. Due to Faraday's law there will be induced e.m.f. in coil B which will set up a current through coil B, which will be detected by galvanometer G.

### Magnitude of Mutually induced emf

$N_1$ -Number of turns of coil A

$N_2$ - Number of turns of coil B

$I_1$ -Current flowing through coil A

$\Phi_1$ -Flux produced due to current  $I_1$  in Weber's

$\Phi_2$ - Flux linking with coil B

According to Faraday's law, the induced e.m.f in coil B is

$$e_2 = - N_2 \frac{d\Phi_2}{dt}$$

-ve sign indicates that this emf will setup a current which will oppose the change of flux linking with it.

$$\Phi_2 = \frac{\phi_2}{I_1} \times I_1$$

$$\text{Rate of change of } \Phi_2 = \frac{\phi_2}{I_1} \times \text{Rate of change of current } I_1$$

$$\frac{d\phi_2}{dt} = \frac{\phi_2}{I_1} \cdot \frac{dI_1}{dt}$$

$$e_2 = -N_2 \frac{\phi_2}{I_1} \cdot \frac{dI_1}{dt}$$

$$e_2 = -\left(\frac{N_2 \phi_2}{I_1}\right) \frac{dI_1}{dt}$$

Here  $\frac{N_2 \phi_2}{I_1}$  is called coefficient of mutual Inductance

$$e_2 = -M \frac{dI_1}{dt}$$

### Expression for Coefficient of mutual Inductance

$$M = \frac{N_2 k_1 \phi_1}{I_1} \quad \& \quad M = \frac{N_1 k_2 \phi_2}{I_2}$$

Multiply both M

$$M \times M = \frac{N_2 k_1 \phi_1 N_1 k_2 \phi_2}{I_1 I_2}$$

$$M^2 = K_1 K_2 \frac{N_1 \phi_1}{I_1} \frac{N_2 \phi_2}{I_2}$$

$$M^2 = K_1 K_2 L_1 L_2$$

$$M = \sqrt{K_1 K_2 L_1 L_2} = \sqrt{K_1 K_2} \sqrt{L_1 L_2}$$

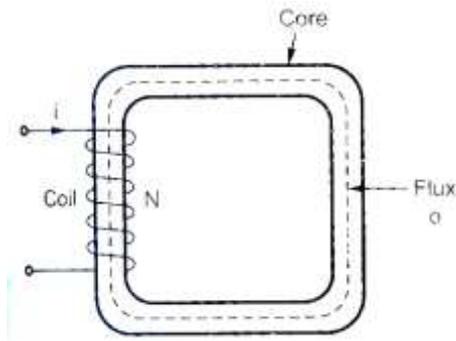
$$M = K \sqrt{L_1 L_2}$$

Where,  $K = \sqrt{K_1 K_2}$  is called coefficient of coupling.

## **2. Discuss the AC operation of Magnetic circuit (May-2012& Dec 2014, 2016)**

In many applications and machines such as transformers and A.C machines, the magnetic circuits are excited by A.C. supply. In such an operation, inductance plays vital role even in steady state operation through in D.C., it acts as a short circuit. In such cases the flux is determined by the A.C. voltage applied and the frequency. Thus the exciting current has to adjust itself according to the flux so that every time B-H relationship is satisfied.

Consider a coil having N turns wound on iron core. The coil carries an alternating current I varying sinusoidally, thus the flux  $\Phi$  produced by the exciting current 'I' is also sinusoidally varying with time.



$$\Phi = \phi_m \sin \omega t$$

Where,  $\phi_m$  = maximum value of flux in core

$$\omega = 2\pi f \quad \text{where, } f \text{ is frequency in Hz}$$

According to Faraday's law, the flux changes with respect to coil, the emf gets induced in the coil given by,

$$e = N \frac{d\phi}{dt} = N \frac{d}{dt} [\phi_m \sin \omega t]$$

$$e = N\phi_m \omega \cos \omega t \text{ volts}$$

$$E_m = \text{maximum value} = N\phi_m \omega$$

$$E = \text{r.m.s value} = \frac{E_m}{\sqrt{2}} = \frac{N\phi_m \omega}{\sqrt{2}}$$

$$E = \frac{N\phi_m 2\pi f}{\sqrt{2}} = 4.44 f N \phi_m$$

$$\phi_m = A_c B_m$$

$$E = 4.44 f N A_c B_m$$

### Energy stored under A.C operation.

Electric power input into the magnetic circuit through the coil terminals is given by

$$P = e \cdot i \quad \text{but } e = \frac{d\lambda}{dt}$$

$$P = \frac{d\lambda}{dt} \cdot i$$

Power is the rate of change of energy hence energy input which gets stored in the magnetic field during interval  $t_1$  to  $t_2$  is,

$$\omega f = \int_{t_1}^{t_2} p \, dt = \int_{t_1}^{t_2} i \frac{d\lambda}{dt} \, dt$$

$$\omega f = \int_{\lambda_1}^{\lambda_2} i \, d\lambda$$

Thus  $\omega f$  is the increase in the field energy as the flux linkages of the coil changes from  $\lambda_1$  and  $\lambda_2$  during the interval  $t_1$  and  $t_2$

$$H_c \ell_c = \text{m.m.f} = NI$$

$$I = \frac{Hc \ell c}{N}$$

Where  $\lambda = N\Phi$  and  $\Phi = B_c a_c$

$$\text{i.e. } \lambda = N a_c B_c$$

$$d\lambda = N a_c dB_c$$

The flux density of the core is changing from  $B_1$  to  $B_2$  as the flux linkages change from  $\lambda_1$  to  $\lambda_2$

$$\omega f = \int_{\lambda_1}^{\lambda_2} \frac{Hc \ell c}{N} d\lambda$$

$$\omega f = \int_{B_1}^{B_2} \left( \frac{Hc \ell c}{N} \right) N a_c dB_c$$

$$\omega f = \int_{B_1}^{B_2} Hc \ell c a_c dB_c$$

This is the field energy in terms of field quantities.

$$\omega f = a_c \ell c \int_{B_1}^{B_2} Hc dB_c$$

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### 3. Discuss in detail the magnetic circuits and electrical analogy of magnetic circuit (May-2011)

Magnetic circuit is defined as the closed path traced or followed by the magnetic lines of force, i.e. flux. Such a magnetic circuit is associated with different magnetic quantities as m.m.f, flux, reluctance, permeability etc,

Consider simple magnetic circuit which consists of an iron core with cross sectional area of 'a' m<sup>2</sup> with a mean length of 'l' m. A coil of N turns is wound on one of the sides of the square core which is excited by a supply. This supply drives a current I through the coil. This current carrying coil produces the flux which completes its path through the core fig (a).

This is analogous to simple electric circuit in which a supply i.e. m.f. of E volts drives a current I which completes its path through a cored conductor having resistance R. This analogous electrical circuit shown in fig (b).

- I=current flowing through coil
- N=Number of turns
- $\Phi$ =Flux in Weber's; B =Flux density in the core
- $\mu$ =Absolute permeability of the magnetic material
- $\mu_r$ =Relative permeability of the magnetic material

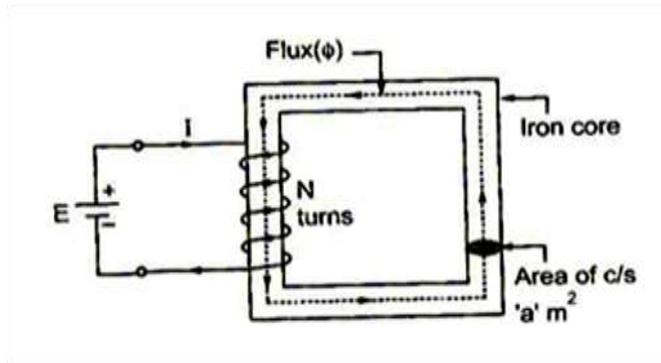


fig.a

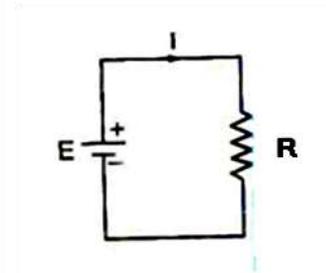


fig.b

$$H = \frac{NI}{\ell} \text{ AT/m}$$

Now flux density is,  $B = \mu H$

$$B = \frac{\mu_0 \mu_r NI}{\ell} \text{ wb/m}$$

$$\Phi = \beta a = \frac{\mu_0 \mu_r NI a}{\ell} \text{ wb/m}^2$$

$$\Phi = \frac{NI}{\frac{\ell}{\mu_0 \mu_r a}}$$

$$\Phi = \frac{\text{m. m. f}}{\text{Reuctance}} = \frac{F}{S}$$

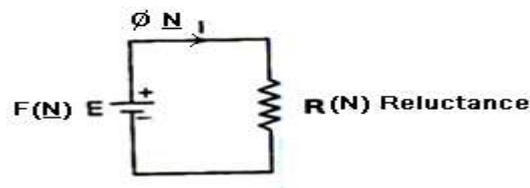
$NI =$  magneto motive force m.m.f in AT

$$S = \frac{1}{\mu_0 \mu_r a} \text{ (Reluctance)}$$

Flux is similar to in electric circuit

$$I = \frac{\text{e. m. f}}{\text{Resistance}}$$

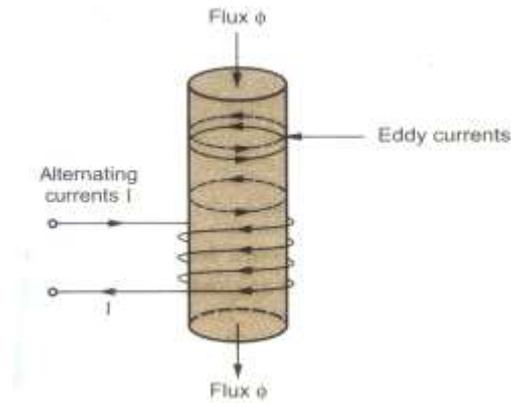
Electrical Analogy of simple magnetic circuit diagram



#### 4. What is eddy current? Explain in detail eddy current loss (May-2011,2015, 2016) (DEC 2015)

Consider a coil wound on the core and carries an alternating current, i.e current magnitude varies with respect to time, and then flux produced by current is also of alternating nature. So core is under the influence of the changing flux and under such condition, according to Faraday's law of electromagnetic induction emf gets induced in the core Now if the core is

solid, then the induced emf circulates currents through the core. Such currents in the core which are due to induced emf in the core are called as eddy currents. Due to such currents there is power loss in the core. Such loss is called eddy current loss.



Eddy current loss depend on the various factors

- i) Nature of the material
- ii) Maximum flux density
- iii) Frequency
- iv) Thickness of laminations used to constant to core soul
- v) Volume of magnetic material

$$\text{Eddy current loss} = K_e [B_{\max}]^2 f^2 t^2 v \quad \text{watt}$$

$K_e$  - eddy current coefficient it depends on type of the material used

$f$  - numbers of complete magnetization cycle per second

$B_{\max}$  - maximum flux density

$t$  - thickness of the lamination

$v$  - volume of the material.

**Reduce the eddy current loss;**

- 1) Higher resistivity of the magnetic material and longer length
- 2) Insurance the effective resistance resulting in reduction of eddy current loss
- 3) Dividing of the material into thin lamination with highly insulated material such as varnish is used for reduce eddy current loss.

Lamination thickness usually varies from 0.3 to 5mm for electromagnetic devices used for power station and 0.01 to 0.5 mm for devices used in electronic application.

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**5. Explain power loss that occurs in magnetic material when it undergoes a cyclic magnetization?(May-2011,2015, 2016)**

Two type of power loss appears in magnetic material when it undergoes a cyclic magnetization.

1. Hysteresis loss
2. Eddy current loss.

## 1. Hysteresis loss

When a magnetic material is subjected to repeated cycle of magnetization and demagnetization, it results into disturbance in the alignment of the various domains. Energy gets stored when magnetic field established and energy is returned when the fields collapses. Due to hysteresis, all the energy is never returned through field completely collapses. The loss of energy as heat in the magnetic material. This is called hysteresis loss. Due to hysteresis loss heat is produced.

Hysteresis loss depend up the following factors

- 1) It is directly  $\propto$  to the area under the hysteresis curve
- 2) It is directly  $\propto$  to  $f$
- 3) It is directly  $\propto$  to volume of material

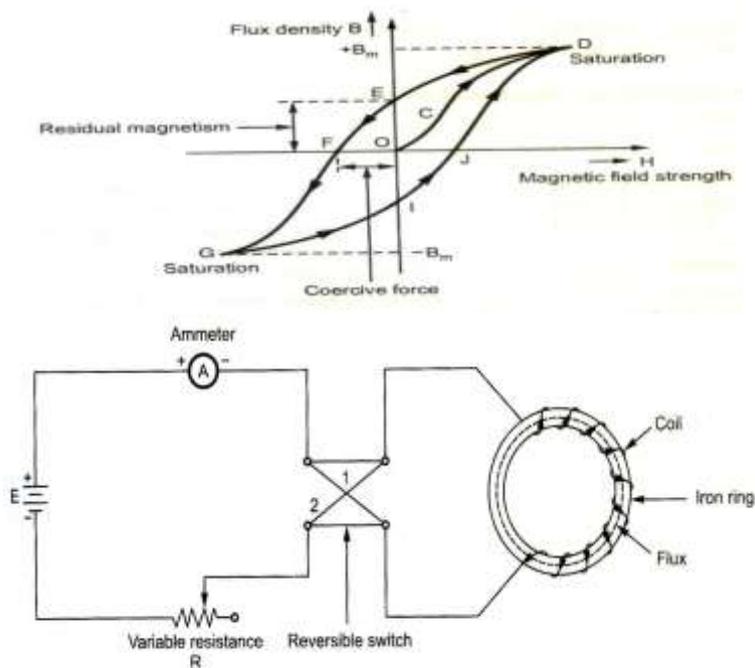
$$\text{Hysteresis loss} = K_h (B_m)^{1.6} f \times \text{Volume}$$

$K_h$ -Characteristic constant of the material

$B_m$ -Maximum flux density

$f$ -frequency in cycles per second

**Obtaining Hysteresis loop:**



- Initially variable resistance is kept maximum current through circuit low.  $H$  also low  $I$  is increased for low value of  $H$ ,  $B$  do not increase rapidly. But after knee point  $B$  increases rapidly upto certain point. This point is called **point of saturation**.
- After saturation point now current is again reduced to zero. Due to this field strength also reduces to zero. But  $B$  do not trace the same curve back but fall back as compared to previous magnetization curve. This phenomenon of falling back of flux density while demagnetization cycle is called **hysteresis**.

- iii) The value of flux density when exciting current through the coil and magnetic field strength is reduced to zero is called residual flux density. The magnitude of this residual flux or magnetism depends on the nature of the material of the core. And this property of material is called **retentivity**.
- iv) But now if it is required to demagnetize the core entirely then it is necessary to reverse the direction of current through the coil. This is possible with the help of the intermediate switch.
- v) If now this reversed current increased, core get saturated in opposite direction.
- vi) If I decreased to zero, core shows hysteresis properly, Hysteresis loss reduced by selecting good quality of magnetic material.

**Eddy current loss:**

**Ref. Question No :4**

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**6. Define Inductance of coil? (May-2010)**

According to Lenz law the direction of this induced e.m.f will be so as to oppose the cause producing it. The cause is the current I hence the self induced e.m.f. try to set up a current which is opposite direction to that of current I. When I increased self induced e.m.f. decreased the current tries to keep it to its original value. If current decreased self induced e.m.f. increased increase current to attain original value.

The property of the coil which opposes any change in the current passing through it is called self Inductance or Inductance

$$L = \frac{N\phi}{I} \text{ or } L = \frac{N^2}{S} \text{ or } L = \frac{N^2\mu_0\mu_r a}{l}$$

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**7. Define permeability of a magnetic material and the factors on which it depends (May-2010)**

It is related to the medium which magnet is placed. The force excited by one magnetic pole to other depends on the medium in which magnets are placed.

It is defined as the ability with which the magnetic material forces the magnetic flux through a given medium

**Types**

- i) Absolute permeability
- ii) Relative permeability

**i) Absolute permeability:**

It is the ratio of B in a particular medium to H producing that flux density.

$$\mu = \frac{B}{H}$$

Permeability of free space

$$\mu_0 = \frac{B}{H} \text{ in vacuum} = 4\pi \times 10^{-7} \text{ H/M}$$

If magnet placed in a free space or vacuum or in air then the ratio of B and H is called permeability of free space.

**ii) Relative permeability**

It is defined as the ratio of flux density produced in a medium to flux density produced in free space, under the influence of same H and under identical conditions

$$\mu_r = \frac{\mu}{\mu_0} \text{ where, H is same}$$

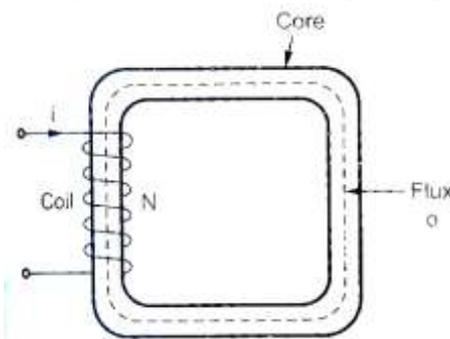
For free space  $\mu_r = 1$   $\mu = \mu_0 \mu_r$  H/m

$\mu_r$  - varies from 100 to 100,000 (metal like iron, steel)

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**8. Explain operation of magnetic circuit when A.C. current is applied to the coil wound on iron core. Draw the B-H curve and obtain an expression for hysteresis loss.**

Ref: Question No- 2 and 5 [Notes]



$$\Phi = \phi_m \sin \omega t$$

Where,  $\phi_m$  = maximum value of flux in core

$$\omega = 2\pi f \text{ where, } f \text{ is frequency in Hz}$$

According to Faraday's law, the flux changes with respect to coil; the emf gets induced in the coil given by,

$$e = N \frac{d\phi}{dt} = N \frac{d}{dt} [\phi_m \sin \omega t]$$

$$e = N\phi_m \cos \omega t \text{ volts}$$

$$E_m = \text{maximum value} = N\phi_m \omega$$

$$E = \text{r.m.s value} = \frac{E_m}{\sqrt{2}} = \frac{N\phi_m \omega}{\sqrt{2}}$$

$$E = \frac{N\phi_m 2\pi f}{\sqrt{2}} = 4.44 f N \phi_m$$

$$E = 4.44 f N A_c B_m$$

Energy stored under A.C operation.

Electric power i/p into the magnetic circuit through the coil terminals is given by

$$P = e \times i \text{ but } e = \frac{d\lambda}{dt}$$

$$P = \frac{d\lambda}{dt} \cdot i$$

Power is the rate of change of energy hence energy input which gets stored in the magnetic field during interval  $t_1$  to  $t_2$  is,

$$\omega_f = \int_{t_1}^{t_2} p \, dt = \int_{t_1}^{t_2} i \frac{d\lambda}{dt} \, dt$$

$$\omega_f = \int_{t_1}^{t_2} i \, d\lambda$$

Thus  $\omega_f$  is the increase in the field energy as the flux linkages of the coil changes from  $\lambda_1$  and  $\lambda_2$  during the interval  $t_1$  and  $t_2$

$$H_c \ell_c = m.m.f = Ni$$

$$I = \frac{H_c \ell_c}{N}$$

$$\omega_f = \int_{\lambda_1}^{\lambda_2} \frac{H_c \ell_c}{N} \, d\lambda$$

Where  $\lambda = N\phi$  and  $\phi = B_c a_c$ .

$$i.e. \lambda = N a_c B_c$$

The flux density of the core is changing from  $B_1$  to  $B_2$  as the flux linkages change from  $\lambda_1$  to  $\lambda_2$

$$\omega_f = \int_{B_1}^{B_2} \left( \frac{H_c \ell_c}{N} \right) N a_c \, dB_c$$

$$\omega_f = \int_{B_1}^{B_2} H_c \ell_c a_c \, dB_c$$

$$\omega_f = a_c \ell_c \int_{B_1}^{B_2} H_c \, dB_c$$

### 9. Compare the electric and magnetic circuit by their similarities and dissimilarities Similarities(Dec 2011& Dec 2014).

S.No	Electric circuit	Magnetic circuit
1	Path traced by current is called electric circuit	Path traced by magnetic flux is called magnetic circuit
2.	EMF is the driving force in electric circuit	MMF is the driving force in magnetic circuit
3.	There is current I in the electric circuit measured in amperes	There is flux $\phi$ in the magnetic circuit measured in Weber's
4	The flow of electrons decides the current in conductor	The number of magnetic lines of force decides flux
5.	Resistance oppose the flow of current	Reluctance opposed by magnetic path to the flux
6.	$R = \frac{\rho \ell}{A}$	$S = \frac{\ell}{\mu_0 \mu_r A}$

	$R \propto \ell$ $\frac{1}{\alpha} a \rightarrow$ depend on nature of material	$\frac{1}{\alpha} \mu$
7.	$I = \frac{\text{Emf}}{\text{Resistance}}$	$\phi = \frac{\text{MMF}}{\text{reluctance}}$
8.	$\delta = \frac{I}{a} \text{ A/m}^2$	$B = \frac{\phi}{a} \text{ wb/m}^2$
9.	Conductivity is Reciprocal of resistivity=1/R	Permeance is reciprocal of the reluctance=1/S
10.	Kirchhoff's current and voltage law is applicable to the electric circuit	Kirchhoff's m.m.f. law and flux law is applicable to the magnetic circuit

### Dissimilarities

Sl.No	Electric circuit	Magnetic circuit
1.	In the electric circuit the current actually flows i.e there is movement of electron	Due to m.m.f. flux gets established and does not flow in the sense in which current flows.
2.	There are many material which can be used as Insulator ie air, p.v.c., Synthetic resin etc, from which current cannot pass	There is no magnetic insulator as flux can pass through all the materials, even through the air as well.
3.	Energy must be supplied to the electric circuit to maintain the flow of current	Energy is required to create the magnetic flux, but it is not required to maintain it.
4.	The resistance and the conductivity are independent of current density ( $\delta$ ) under constant temp. But may change due to the temp	The reluctance, permeance and permeability are dependent on the flux density
5.	Electric lines of flux are not losed they start from positive charge and end on negative charge	Magnetic lines of flux are closed lines. They flow from N pole to S pole externally while S pole to N pole internally
6.	There is continuous consumption of electrical energy	Energy is required to create the magnetic flux and not to maintain it

### 10. Derive an expression for energy density in the magnetic field?(Dec-2010, 2015)

Induced emf in a coil

$$e = -L \frac{dI}{dt}$$

This opposes the supply voltage.

$$V = -e = -\left[-L \frac{dI}{dt}\right] = L \frac{dI}{dt}$$

$$\text{Power} = V \times I = L \frac{dI}{dt} \times I$$

$$E = \text{power} \times \text{time} = L \frac{dI}{dt} \times I \times dt$$

$$E = LI dI \text{ joules.}$$

The above expression is energy in current dI.

$$E = \int_0^I L dI$$

$$E = L \int_0^I I dI = \frac{LI^2}{2}$$

$$\text{Energy density} = \frac{\text{Energy}}{\text{Unit volume}}$$

We know that  $L = N \Phi / I$

$$E = \frac{1}{2} L I^2 = \frac{1}{2} \times \frac{N\Phi}{I} \times I^2$$

$$E = \frac{1}{2} N \Phi I$$

We know that  $NI = H\ell$

$$\Phi = Ba$$

$$E = \frac{1}{2} \times H\ell \times Ba$$

Where  $a \times \ell =$  volume of magnetic material per unit volume  $E = \frac{1}{2} B H$

Where  $B = \mu H$

$$\therefore \frac{E}{\text{Unit volume}} = \frac{1}{2} \mu H^2 \text{ Jolules/m}^3$$

$$\frac{E}{\text{Unit volume}} = \frac{1}{2} \frac{B^2}{\mu} \text{ Jolules/m}^3$$

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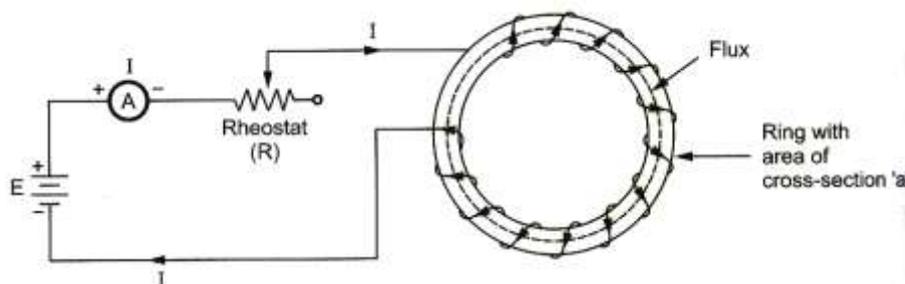
### 11. Explain in detail?(May-2010)

- 1) B-H relationship
- 2) Leakage flux
- 3) Fringing
- 4) Stacking factor

#### 1) B-H relationship:-

$H = \frac{NI}{l}$ , As current in coil changes magnetic field strength also changes. Due to this flux produced and hence the flux density also changes.

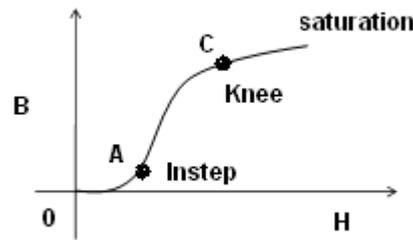
A graph b/w flux density (B) and the magnetic field strength (H) for magnetic material is called as its magnetization curve or B-H curve



The experiment need to find relationship b/w B & H.

A ring specimen has 'l' meter length, 'a' cross sectional area. Coil is wound for 'N' turns carrying a current I which can be varied by changing the variable resistance R connected in series, Ammeter connected series to measure current flux meter is introduced to measure flux.

'H' can be calculated as  $\frac{NI}{l}$  and b can be calculated by  $\frac{\Phi}{A}$ , with help of R. I is varied H also varied



**Initial portion:** - Low value H 'B' is not increase rapidly. It is denoted by OA

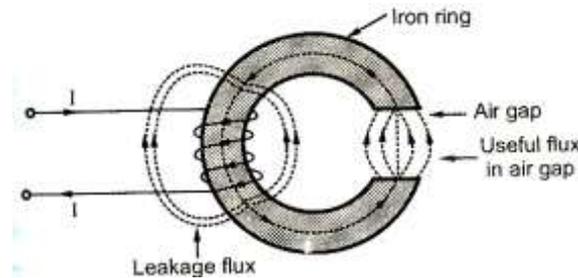
**Middle portion:** -H increases B also increases. The point at which portion of the curve bends is called knee point.

**Saturation portion:** - After knee point, rate of increase in B reduce drastically finally the curve parallel to 'X' axis. It is called saturation portion.

$$B = \mu H = \mu_0 \mu_r H; \quad \mu_r = \frac{B}{\mu_0 H}$$

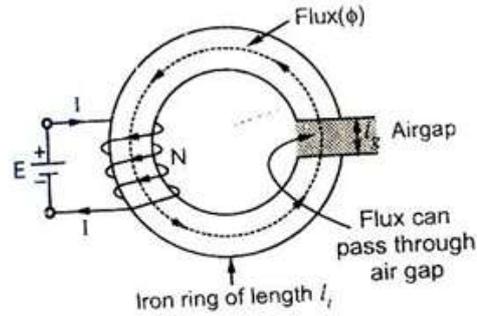
## 2) Leakage flux

Flux which does not follow the desired path is called leakage flux. The flux which is available in the air gap and utilized to produce the desired effect is called metal flux denoted by  $\phi_u$



## 3) Fringing

When flux enters into the air gap, it passing through the air gap in terms of parallel flux lines. There exists a force of repulsion between the magnetic lines of force which are parallel and having same direction. Due to this repulsive force there is tendency of the edge of the air gap. The tendency of **the flux to bulge out at the edges of the air gap** is called magnetic fringing.



**Effects:**

- 1) It increases the effective cross sectional area of air gape
- 2) It reduces the flux density in the air gap

**4) Stacking factor:-**

The laminated construction helps to keep eddy current losses to low value. Due to stacks of lamination, the net cross sectional area occupied by the magnetic material less than its gross cross sectional area.

$$\text{Stacking factor} = \frac{\text{Net cross sectional area occupied by magnetic material}}{\text{gross cross – sectional area}}$$

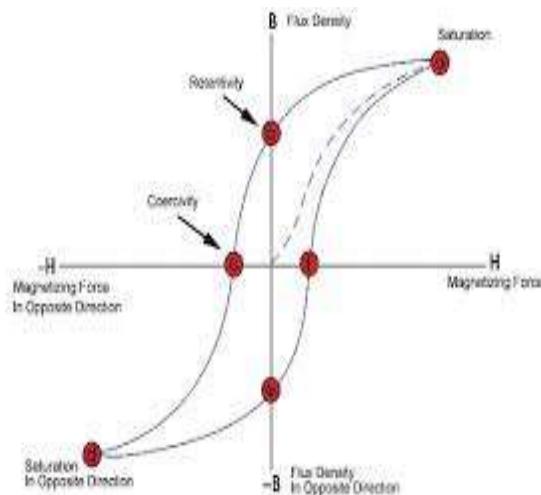
Value always <1

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**12. Explain the basic Properties of magnetic materials. Or State properties of magnetic material suitable for fabrication permanent magnet and electromagnet?(May 2015, 2016)**

**i)Non Liner B-H relationship:**

Magnetic materials are characterized by high permeability and nonlinear B-H relationship which exhibits both saturation and hysteresis. The B-H relationship H is the hysteresis loop shown in fig. for two values of maximum flux density. It is easily observed from this figure that B is a symmetrical two -valued function of H. at any given H, B is higher if H is reducing compared to when H is increasing.



This is the basic hysteresis property in which  $B$  lags behind. It can also be recognized as a memory type nonlinearity in which the materials remember its previous history. Further it is observed that the hysteresis loop becomes wider for increasing maximum flux densities. The dotted curve drawn through the positive and negative tips of the hysteresis loop with increasing maximum flux densities is the normal magnetization and is obtainable in virgin (Unmagnetized) material by increasing the DC magnetization in either direction. It is easily seen that because of hysteresis and saturation, the magnetic characteristic of a given material cannot be described by a few overall parameters but must be expressed in the form of a set of curves. It will be shown that the area of the hysteresis loop is the energy loss (it appears in the form of heat energy) per unit volume in one cycle of magnetization. This loss depends upon the quality of material and maximum flux density at which the material is operated.

### ii) Saturation:

Saturation is the state reached when an increase in applied external magnetic field  $H$  cannot increase the magnetization of the material further, so the total magnetic flux density  $B$  more or less levels off. (It continues to increase very slowly due to the vacuum permeability.) Saturation is a characteristic of ferromagnetic and ferromagnetic materials, such as iron, nickel, cobalt and their alloys.

### iii) Hysteresis:

Hysteresis is the lag in a variable property of a system with respect to the effect producing it as this effect varies, especially the phenomenon in which the magnetic flux density of a ferromagnetic material lags behind the changing external magnetic field strength.

### iv) Permeability:

Permeability is the measure of the ability of a material to support the formation of a magnetic field within itself. Hence, it is the degree of magnetization that a material obtains in response to an applied magnetic field.

**Or**

- The importance of magnetic materials is twofold:
- Magnetic materials are used to obtain large magnetic flux densities with relatively low levels of magnetizing force.
- Magnetic materials can be used to constrain and direct magnetic fields in well-defined paths.

Ferromagnetic materials, typically composed of iron and alloys of iron with cobalt, tungsten, nickel, aluminum, and other metals, are by far the most common magnetic materials.

- They are found to be composed of a large number of domains.
- When unmagnetized, the domain magnetic moments are randomly oriented.
- When an external magnetizing force is applied, the domain magnetic moments tend to align with the applied magnetic field until all the magnetic moments are aligned with the applied field, and the material is said to be fully saturated.
- When the applied field is reduced to zero, the magnetic dipole moments will no longer be totally random in their orientation and will retain a net magnetization component along the applied field direction.

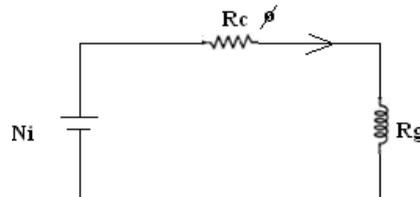
The relationship between  $B$  and  $H$  for a ferromagnetic material is both nonlinear and multivalued.

- In general, the characteristics of the material cannot be described analytically but are commonly presented in graphical form.

- The most common used curve is the *HB*-curve.
- DC or normal magnetization curve
- *Hysteresis loop*

**13. Draw and explain the typical magnetic circuit with air gap and its equivalent electric circuit. Hence derive the expression for air gap flux.(May 2013,Nov 2017)**

The series magnetic circuit can also have a short air gap. Such air gap is not possible in case of electric circuit.



Consider a ring having mean length of iron path as ' $l_i$ '

$$\text{Total m.m.f} = NI \quad \text{AT}$$

$$\text{Total reluctance} \quad S_T = S_i + S_g$$

Where  $S_i$  = reluctance of iron path

$$S_g = \text{Reluctance of air gap}$$

$$S_i = \frac{l_i}{\mu a_i} \quad \text{For } \mu = \mu_0 \mu_r$$

$$S_g = \frac{l_g}{\mu_0 \mu_r a_i} \quad \text{for air } \mu_r = 1$$

The cross sectional area of air gap is assumed to be equal to area of the iron ring

$$S_T = \frac{l_i}{\mu a_i} + \frac{l_g}{\mu_0 a_i}$$

$$\Phi = \frac{\text{m.m.f}}{\text{reluctance}} = \frac{NI}{S_T}$$

$$NI = S_i \Phi + S_g \Phi \quad \text{AT}$$

$$\Phi = \frac{NI}{S_i + S_g}$$

&&&&

**PROBLEMS**

**1. The total core loss of a specimen of silicon steel is found to be 1500 w at 50 Hz. Keeping the flux density constant the loss becomes 3000w when the frequency is raised to 75Hz. Calculate separately the hysteresis and eddy current loss at each of their frequencies?(May-2011, Dec- 2013, 2015)**

**Given:**

Core Loss=1500w, F=50, B=3000w, Frequency Is Raised=75hz

**Formula used:**

$$\omega_h = \eta B_m^{1.6} f$$

$$\omega_h = Af$$

$$\omega_e = K_e B_m^2 f^2 t^2 V$$

$$= Bf^2$$

**Solution:**

$$P = Af + Bf^2 \text{----- 1}$$

Divequ 1 by f

$$\frac{p}{f} = A + Bf$$

At 50Hz, the losses in 1500 watts

$$\frac{1500}{50} = A + 50B$$

$$30 = A + 50B \text{----- 2}$$

At 75 Hz

$$\frac{3000}{75} = A + 75B$$

$$40 = A + 75B \text{----- 3}$$

Solve 2, 3

$$A + 50B = 30$$

$$A + 75B = 40$$

$$B = 0.4$$

$$A = 10$$

**Result:**

At 50Hz

Hysteresis loss

$$Af = 10 \times 50$$

$$= 500 \text{ w}$$

Eddy current loss

$$Bf^2 = 0.4 \times 50^2$$

$$= 1000 \text{ watts}$$

At 75 Hz

$$\text{Hysteresis loss} = 10 \times 75 = 750 \text{w}$$

$$\text{Eddy current loss} = 0.4 \times 75^2 = 2250 \text{w}$$

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2. A ring composed of three sections. The cross section area is 0.001 m<sup>2</sup> for each section. The mean arc length area l<sub>a</sub>=0.3m, l<sub>b</sub>=0.2m, l<sub>c</sub>=0.1m, an air gap length of 0.1mm is cut in the ring, μ<sub>r</sub> for sections a, b,c, are 5000,1000, 10000 respectively. Flux in the air gap is 7.5×10<sup>-4</sup>wb. Find i)mmf ii) exciting current if the coil has 100 turns,iii) reluctance of the section(Dec-2011)

**Given:**

a=0.001 m<sup>2</sup>, l<sub>a</sub>=0.3m, l<sub>b</sub>=0.2m, l<sub>c</sub>=0.1m, ℓ<sub>g</sub> = of 0.1mm, μ<sub>r</sub>=5000,1000, 10000, ϕ = 7.5 × 10<sup>-4</sup>wb.

**Formula used:**

$$\text{Total reluctance} = R_g + R_a + R_b + R_c, R_g = \frac{\ell_g}{\mu_0 \mu_r g a}$$

Solution:

$$i) R_g = \frac{\ell_g}{\mu_0 \mu_{rg} a} = \frac{0.1 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 0.001} = 79554$$

$$R_a = \frac{\ell_a}{\mu_0 \mu_{ra} a} = \frac{0.3}{4\pi \times 10^{-7} \times 5000 \times 0.001} = 47748$$

$$R_b = \frac{\ell_b}{\mu_0 \mu_{rb} a} = \frac{0.2}{4\pi \times 10^{-7} \times 1000 \times 0.001} = 159109$$

$$R_c = \frac{\ell_c}{\mu_0 \mu_{rc} a} = \frac{0.1}{4\pi \times 10^{-7} \times 10000 \times 0.001} = 7957$$

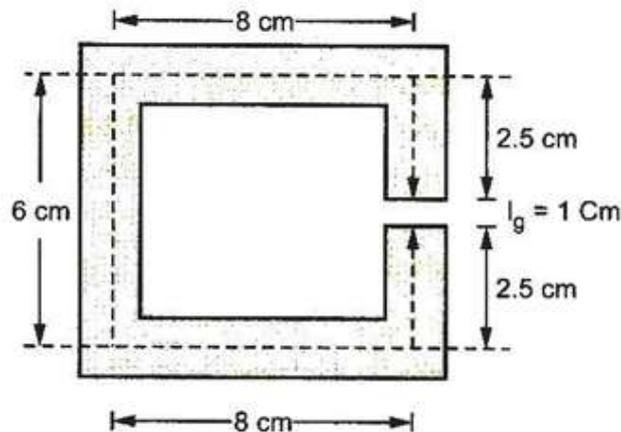
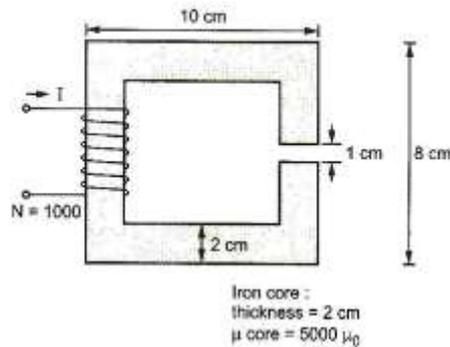
Total reluctance = **294368**

$$ii) \text{Mmf} = \text{flux} \times \text{reluctance} = 7.5 \times 10^{-4} \times 294585 \\ = 221 \text{ AT}$$

$$iii) \text{Current required} = \frac{\text{MMF}}{N} = \frac{221}{1000} \\ = 0.221 \text{ Amps.}$$

-----&&&&-----

**3. For the magnetic circuit shown in fig determine the current required to establish a flux density of 0.5T in the air gap.**



Given:

$$N=1000, B=0.5\text{T}, a=2 \times 2=4 \text{ cm}^2, l_g=1 \text{ cm}$$

$l_i = \text{length of iron path} = 8+8+6+5 = 27 \text{ cm}$

$$\mu_{\text{core}} = 5000\mu_0$$

$$\mu_r = 5000$$

Formula used:  $S_i = \frac{l_i}{\mu_0\mu_r a}$ ,  $S_g = \frac{l_g}{\mu_0 a}$ ,  $\Phi = \frac{\text{mmf}}{S_T}$

**Solution:**

$$\Phi = B \times a$$

$$= 0.5 \times 4 \times 10^{-4} = 0.2 \text{ mwb}$$

$$S_i = \frac{l_i}{\mu_0\mu_r a} = \frac{27 \times 10^{-2}}{5000 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}}$$

$$= 107.4295 \times 10^3 \text{ AT/wb}$$

$$S_g = \frac{l_g}{\mu_0 a} = \frac{1 \times 10^{-2}}{4\pi \times 10^{-7} \times 4 \times 10^{-4}}$$

$$= 19.8943 \times 10^6 \text{ AT/wb}$$

$$S_T = S_i + S_g = 20 \times 10^6 \text{ AT/wb}$$

$$\Phi = \frac{\text{mmf}}{S_T} = 0.2 \times 10^{-3} = \frac{NI}{20 \times 10^6}$$

$$I = 4 \text{ A}$$

-----&&&&-----

**4. The magnetic circuit has dimension:  $A_c = 4 \times 4 \text{ cm}^2$ ,  $bl_g = 0.06 \text{ cm}$ ,  $l_c = 40 \text{ cm}$  and  $N = 600$  turns. Assume the value of  $\mu_r = 6000$  for iron. Find the exciting current for  $B_c = 1.2 \text{ T}$  and corresponding flux and flux linkages. (May 2013)**

**Given:**  $A_c = 4 \times 4 \text{ cm}^2$ ,  $bl_g = 0.06 \text{ cm}$ ,  $l_c = 40 \text{ cm}$  and  $N = 600$ ,  $\mu_r = 6000$ ,  $B_c = 1.2 \text{ T}$

**Formula used:**

$$NI = \frac{B_c}{\mu_0\mu_r} l_c + \frac{B_g}{\mu_0} l_g, \quad \Phi = B_c A_c \quad \lambda = N\Phi$$

**Solution:**

Assume fringing neglected

$$A_c = A_g \text{ therefore } B_c = B_g$$

$$i = \frac{B_c}{\mu_0 N} \left( \frac{l_c}{\mu_r} + l_g \right)$$

$$= \frac{1.2}{4\pi \times 10^{-7} \times 600} \left( \frac{0.4}{6000} + 0.0006 \right)$$

$$= 1.06 \text{ A}$$

$$\Phi = B_c A_c = 1.2 \times 16 \times 10^{-4} = 19.2 \times 10^{-4} \text{ wb}$$

$$\lambda = N\Phi = 600 \times 19.2 \times 10^{-4} = 1.152 \text{ wb-turns}$$

Assuming fringing effect {If fringing is to be taken into account, one gap length is added to each dimension of the airgap constituting the area. then

$$A_g = (4 + 0.06) \times (4 + 0.06) = 16.484 \text{ cm}^2 = 16.484 \times 10^{-4} \text{ m}^2$$

$$B_g = \frac{\Phi}{A_g} = \frac{19.2 \times 10^{-4}}{16.484 \times 10^{-4}} = 1.165 \text{ T}$$

$$Ni = \frac{B_c}{\mu_0 \mu_r} l_c + \frac{B_g}{\mu_0} l_g$$

$$Ni = \frac{1.2 \times 0.4}{4\pi \times 10^{-7} \times 6000} + \frac{1.165 \times 0.0006}{4\pi \times 10^{-7}} = 619.90$$

$$I = \frac{619.90}{600} = 1.0345 \text{ A}$$

**5. The core loss (Hysteresis loss + eddy current loss ) for a given specimen of magnetic material is found to be 2000w at 50 hz .keeping the flux density constant, the frequency of the supply is raised to 75 hz resulting in a core loss of 3200W. Compute separately hysteresis and eddy current losses at both the frequencies. (Dec 2013).**

**Given:** Hysteresis loss + eddy current loss = 2000w, f = 50,

Hysteresis loss + eddy current loss = 3200, f = 75

**Formula used:**

$$\omega_h = \eta B_m^{1.6} f$$

$$\omega_h = Af$$

$$\omega_e = K_e B_m^2 f^2 t^2 V$$

$$= Bf^2$$

**Solution**

$$\omega_h = \eta B_m^{1.6} f; \quad \omega_h = Af;$$

$$\omega_e = K_e B_m^2 f^2 t^2 V;$$

$$= Bf^2$$

$$P = Af + Bf^2 \text{ ----- 1}$$

Divide equation 1 by f

$$\frac{P}{f} = A + Bf$$

**At 50Hz, the losses in 2000 watts**

$$\frac{2000}{50} = A + 50B$$

$$40 = A + 50B \text{ ----- 2}$$

**At 75 Hz the losses in 3200 watts**

$$\frac{3200}{75} = A + 75B$$

$$42.67 = A + 75B \text{ ----- 3}$$

Solve equation 2 & 3

$$A + 50B = 40$$

$$A + 75B = 42.67$$

$$B = 0.10668$$

$$A = 34.66$$

At 50Hz, the hysteresis and eddy current losses is calculated by

Hysteresis loss

$$Af = 34.66 \times 50 = 1733.3 \text{ w}$$

Eddy current loss

$$Bf^2 = 0.10668 \times 50^2 = 267 \text{ w}$$

At 75 Hz the hysteresis and eddy current losses is calculated by

$$\text{Hysteresis loss} = Af = 34.66 \times 75 = 2599.95 \text{ w}$$

$$\text{Eddy current loss} = Bf^2 = 0.10668 \times 75^2 = 600.75 \text{ w}$$

-----&&&-----

**6. A steel ring has a mean diameter of 20 cm, a cross section of 25 cm<sup>2</sup> and a radial air gap of 0.8 mm cut across it. When excited by a current of 1 A through a coil of 1000 turns wound on the ring core it produces an air gap flux 1 m wb. Neglecting leakage and fringing. Calculate**

**i) relative permeability of steel and**

**ii) total reluctance of the magnetic circuit. (Dec 2013)**

**Given:**  $d = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$ ,  $N = 1000$ ,  $a = 25 \text{ cm}^2 = 25 \times 10^{-4}$ ,  $\Phi_g = 1 \text{ mwb}$ ,  $l_g = 0.8 \times 10^{-3} \text{ m}$ ,  $I = 1 \text{ A}$

**Formula used:**  $s_g = \frac{l_g}{\mu_0 a}$ ,  $s_s = \frac{l_s}{\mu_0 \mu_r a}$

**Solution:**

$$d = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$$

$$N = 1000, a = 25 \text{ cm}^2 = 25 \times 10^{-4}, \Phi_g = 1 \text{ mwb}, l_g = 0.8 \times 10^{-3} \text{ m}, I = 1 \text{ A}$$

$$\text{Mean length} = l_t = d\pi = 0.6283 \text{ m (or)} 2\pi r$$

$$\text{Length of steel ring} = l_s = l_t - l_g = 0.6283 - 0.8 \times 10^{-3} = 0.6275 \text{ m.}$$

$$\text{i) } \mu = B/H$$

$$B = \text{flux/area} = 1 \times 10^{-3} / 25 \times 10^{-4} = 0.4 \text{ wb/m}^2$$

$$\text{MMF} = NI = 1000 \text{ AT,}$$

$$H = \text{MMF} / l_t = 1000 / 0.6283 = 1591.59 \text{ AT/m}$$

$$\mu = B/H = 0.4 / 1591.59 = 2.505 \times 10^{-4} \text{ H/m}$$

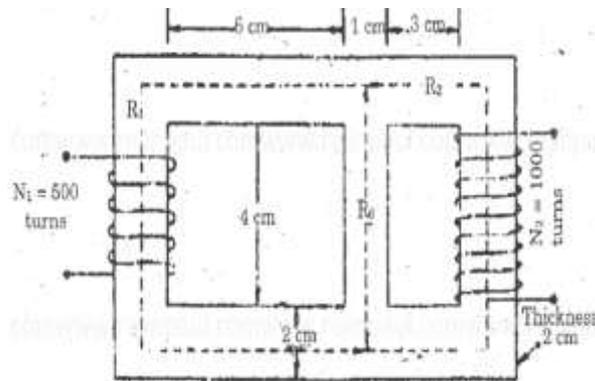
$$\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times \mu_r = 2.505 \times 10^{-4}, \mu_r = 199.34.$$

$$\begin{aligned} \text{Reluctance of the air gap} &= s_g = \ell_g / \mu_0 a = 0.8 \times 10^{-3} / (4\pi \times 10^{-7} \times 25 \times 10^{-4}) \\ &= 254647.9 \text{ AT/wb.} \end{aligned}$$

$$\begin{aligned} \text{Reluctance of the steel} &= s_s = \frac{\ell_s}{\mu_0 \mu_r a} = 0.6275 / (4\pi \times 10^{-7} \times 199.34 \times 25 \times 10^{-4}) \\ &= 2294379.03 \text{ AT/wb.} \end{aligned}$$

$$\begin{aligned} \text{Total reluctance of the magnetic circuit} &= s_g + s_s = 254647.9 + 2294379.03 \\ &= 2549026.93 \text{ AT/m.} \end{aligned}$$

8. For the magnetic circuit as shown below, find the self and mutual inductance between the two coils. Assume core permeability = 1600. (May 2014, Dec-2017)



Solution:

$$\ell_1 = (6 + 0.5 + 1) \times 2 + (4 + 2) = 21 \text{ cm}$$

$$\ell_2 = (3 + 0.5 + 1) \times 2 + (4 + 2) = 15 \text{ cm}$$

$$\ell_0 = 4 + 2 = 6 \text{ cm}$$

$$R_1 = \frac{\ell_1}{\mu_0 \mu_r A_1} = \frac{21 \times 10^{-2}}{4\pi \times 10^{-7} \times 1600 \times 2 \times 2 \times 10^{-4}} = 0.261 \times 10^6$$

$$R_2 = \frac{\ell_2}{\mu_0 \mu_r A_2} = \frac{15 \times 10^{-2}}{4\pi \times 10^{-7} \times 1600 \times 2 \times 2 \times 10^{-4}} = 0.187 \times 10^6$$

$$R_0 = \frac{\ell_0}{\mu_0 \mu_r A_0} = \frac{6 \times 10^{-2}}{4\pi \times 10^{-7} \times 1600 \times 1 \times 2 \times 10^{-4}} = 0.149 \times 10^6$$

### 1. Coil 1 excited with 1A

$$R = R_1 + R_0 \parallel R_2 = 0.261 \times 10^6 + 0.149 \parallel 0.187 = 0.344 \times 10^6$$

$$\phi_1 = \frac{NI}{R} = \frac{500 \times 1}{0.344 \times 10^6} = 1.453 \text{ mwb}$$

$$\phi_{21} = \frac{(1.453 \times 10^{-3}) \times (0.149 \times 10^6)}{(0.149 \times 10^6) + (0.187 \times 10^6)} = 0.649 \text{ mwb}$$

$$M_{21} = N_2 \phi_{21} = 1000 \times 0.649 \times 10^{-3} = 0.64 \text{ H}$$

### 2. Coil 2 excited with 1A

$$R = R_2 + R_0 \parallel R_1 = 0.284 \times 10^6$$

$$\phi_2 = \frac{N_2 I}{R} = \frac{1000 \times 1}{0.284 \times 10^6} = 3.52 \text{ mwb}$$

$$L_{22} = N_2 \phi_2 = 1000 \times 3.52 \times 10^{-3} = 3.52 \text{ H}$$

$$M_{12} = M_{21} = 0.64 \text{ H}$$

9. A single phase 50Hz 50KVA transformer for 6000/240V ratio has a maximum flux density of 1.4 Wb/m<sup>2</sup> and an effective core section of 150 cm<sup>2</sup>, the magnetizing current (RMS) is 0.1A. Estimate the inductance of each wire an open circuit. (Dec - 2014)

Given:  $E_1 = 6000 \text{ V}$ ,  $E_2 = 240 \text{ V}$   $f = 50 \text{ Hz}$ ,  $B_m = 1.4 \text{ wb/m}^2$   $A = 150 \text{ cm}^2$   $I = 0.1 \text{ A (rms)}$

Formula used:  $E_1 = 4.44 f \phi_m N_1$ ,  $L_1 = \frac{N_1 \phi_m}{I}$ ,  $L_2 = \frac{N_2 \phi_m}{I}$

**Solution:**

$E_1 = 6000 \text{ V}$ ,  $E_2 = 240 \text{ V}$   $f = 50 \text{ Hz}$ ,  $B_m = 1.4 \text{ wb/m}^2$   $A = 150 \text{ cm}^2$   $I = 0.1 \text{ A (rms)}$

$$B_m = \frac{\phi_m}{A}$$

$$1.4 = \frac{\phi_m}{150 \times 10^{-4}}$$

$$\phi_m = 0.021 \text{ wb}$$

$$E_1 = 4.44 f \phi_m N_1$$

$$6000 = 4.44 \times 50 \times 0.021 \times N_1$$

$$N_1 = 1287$$

$$E_2 = 4.44 f \phi_m N_2$$

$$240 = 4.44 \times 50 \times 0.021 \times N_2$$

$$N_2 = 51.48 = 52$$

$$L_1 = \frac{N_1 \phi_m}{I} = \frac{27.027}{0.1} = 270.3 \text{ H}$$

$$L_2 = \frac{N_2 \phi_m}{I} = \frac{1.092}{0.1} = 10.92 = 11 \text{ H}$$

10. The core of an electromagnet is made of an iron rod of 1 cm diameter, bent in to a circle of mean diameter 10 cm, a radial air gap of 1 mm being left between the ends of the rod. Calculate the direct current needed in coil of 2000 turns uniformly spaced around the core to produce a magnetic flux of 0.2 mwb in the air gap. Assume that the relative permeability of the iron is 150, that the magnetic leakage factor is 1.2 and that the air gap is parallel. (May 2017)

Given:

Mean diameter  $D = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$

$$A_a = \frac{\pi}{4} \times (1)^2 = \frac{\pi}{4} \text{cm}^2$$

$$\ell_g = 1\text{mm} = 1 \times 10^{-3} \text{m}$$

$$N = 2000, \quad \phi_g = 0.2 \text{ mwb}$$

$$\mu_r = 150, \quad \lambda = 1.2$$

$$\ell_i = \pi D = \pi \times 10 \times 10^{-2} = 0.3141 \text{m}$$

**Solution:**

AT for airgap:

$$B_g = \frac{\phi_g}{a} = \frac{0.2 \times 10^{-3}}{0.785 \times 10^{-4}} = 2.547 \text{ wb/m}^2$$

$$H_g = \frac{B_g}{\mu_0} = \frac{2.547}{4\pi \times 10^{-7}} = 2026838 \text{ AT/m}$$

$$AT_g = H_g \times \ell_g = 2026838 \times 1 \times 10^{-3} = 2026.83 \text{ AT}$$

**AT for iron path:**

$$\phi_i = \phi_g \times \lambda = 0.2 \times 10^{-3} \times 1.2 = 2.4 \times 10^{-4} \text{ wb}$$

$$B_i = \frac{\phi_i}{a} = \frac{2.4 \times 10^{-4}}{0.785 \times 10^{-4}} = 3.057 \text{ wb/m}^2$$

$$H_i = \frac{B_i}{\mu_r \times \mu_0} = \frac{3.057}{150 \times 4\pi \times 10^{-7}} = 16217.8 \text{ AT/m}$$

$$AT_i = H_i \times \ell_i = 16217.8 \times 0.3141 = 5094 \text{ AT}$$

$$\text{Total AT} = AT_g + AT_i = 2026.83 + 5094 = 7120.83 \text{ AT}$$

$$\text{Direct Current (I)} = \frac{\text{Total AT}}{N} = \frac{7120.83}{2000} = 3.56 \text{ A}$$


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**11. An iron rod 1.8 cm diameter is bent to form a ring of mean diameter 25cm and wound with 250 turns of wire. A gap of 1mm exists in between the end faces. Calculate the current required to produce a flux of 0.6mWb. Take relative permeability of iron as 1200. (May-18)**

Given Data:

$$\text{Mean Diameter} = 25 \times 10^{-2} \text{m}$$

$$N = 250$$

$$\ell_g = 1\text{mm} = 1 \times 10^{-3} \text{m}$$

$$\phi = 0.6 \text{ mwb} = 0.6 \times 10^{-3} \text{wb}$$

$$\mu_r = 1200 \quad \mu_0 = 4\pi \times 10^{-7}$$

$$d = 1.8 \text{cm}$$

Solution:

$$a = \frac{\pi d^2}{4} = \frac{\pi}{4} (1.8)^2 = 2.544 \text{cm}^2$$

$$a = 2.544 \times 10^{-4} \text{m}^2$$

$$\text{Flux density} = \frac{\phi}{a} = \frac{0.6 \times 10^{-3}}{2.544 \times 10^{-4}} = 2.358 \text{wb/m}^2$$

$$H_g = \frac{B}{\mu_0} = 1876436.8 \text{AT/m}$$

$$\begin{aligned}
\text{AT required for air gap} &= H_g \times l_g \\
&= 1876436.8 \times 1 \times 10^{-3} \\
AT_g &= 1876.43AT \\
H_i &= \frac{B}{\mu_0 \mu_r} = \frac{2.358}{4\pi \times 10^{-7} \times 1200} = 1563.7AT/m \\
l_i &= \pi D = \pi \times 25 \times 10^{-2} = 0.7854m \\
\text{AT required for iron path} &= H_i \times l_i = 1563.7 \times 0.7854 \\
AT_i &= 1228.12AT \\
\text{Total AT} &= AT_g + AT_i = 1876.43 + 1228.12 = 3104.55 \\
\text{Current } I &= \frac{AT}{T} = \frac{3104.55}{250} = 12.42A
\end{aligned}$$

**12. An electromagnetic relay has an exciting coil of 800 turns. The coil has a cross section of 5cm×5cm. find (a) coil inductance if the air gap length is 0.5cm. (b) field energy stored for a coil current of 1.25A (c) Permeance at air gap. (May-18)**

**Given:** N=800, l=0.5cm I=1.25A

$$L = \frac{N^2 a \mu_0 \mu_r}{l} = \frac{(800)^2 \times 25 \times 10^{-4} \times 4\pi \times 10^{-7} \times 1}{0.5 \times 10^{-2}} = 0.401H$$

$$\text{Field energy stored} = \frac{1}{2} LI^2$$

$$= \frac{1}{2} (0.401) \times (1.25)^2$$

$$\text{Field energy stored} = 0.313J$$

$$\text{Permeance} = \frac{1}{S} = \frac{1}{\frac{l}{\mu_0 \mu_r}}$$

$$\text{Permeance} = 6.28 \times 10^{-7}$$

**13. A toroidal core made of mild steel has a mean diameter of 16cm and a cross-sectional area of 3cm<sup>2</sup>. Calculate a) The MMF to produce a flux of 4 × 10<sup>-4</sup>Wb b) the corresponding values of the reluctance of the core and relative permeability.**

**Given:** d=16cm, a=3cm<sup>2</sup>  $\phi = 4 \times 10^{-4}Wb$

**Formula used:**  $B = \frac{\phi}{A}$ ,  $S = \frac{mmf}{\phi}$

**Solution:**

$$\text{The flux density} = B = \frac{\phi}{A} = \frac{4 \times 10^{-4}}{3 \times 10^{-4}} = 1.33Wb/m^2$$

**Magnetic field strength:**

$$H = 900AT/m$$

$$mmf = 950 \times \pi \times 16 \times 10^{-2} = 478AT$$

$$\text{Reluctance } S = \frac{mmf}{\phi} = \frac{478}{4 \times 10^{-4}} = 119.5 \times 10^{-4}AT/Wb$$

$$S = l/\mu A$$

$$119.5 \times 10^{-4} = \frac{\pi \times 16 \times 10^{-2}}{\mu \times 3 \times 10^{-4}}$$

$$\mu = \frac{\pi \times 16 \times 10^{-2}}{119.5 \times 10^{-4}} = \mathbf{0.1043} \times 10^{-2}$$

$$\mu_r = \frac{\mathbf{0.1043} \times 10^{-2}}{4\pi \times 10^{-7}} = 1116$$

**UNIT 2**  
**TWO MARKS**

**1. What is all day efficiency of Transformer?(Dec 2011) (May 2016)**

In distribution transformers, the energy output is calculated in kilo watt hours (Kwh) and energy spent in supplying the various losses is also determined in Kwh over its operation for a day.

$$\text{All day } \eta = \frac{\text{Output in kwh during a day}}{\text{Input in kwh during a day}}$$

**2. Why polarity test has to be done in a transformer?(Dec 2010)**

On primary side one end is '+ve' with respect to other at any instant. At the same instant, one end of secondary winding is '+ve' with respect to other. There relative polarities of primary and secondary windings must be known for operating transformer in parallel.

**3. Why short circuit test on a transformer performed on H.V side?(May 2010)**

The H.V. side is low current, so it is convenient to connected high voltage side to the supply and short the low voltage side. The voltage on high voltage side can be easily adjusted to such a value that the high voltage side carries the rated current. As this is low current side, no danger, safety measure be needed, with small ammeter we measure value.

**4. Explain why the wattmeter in O.C. test on transformer reads core loss and that in S.C. test reads copper loss at full load?May 2006, 2017**

When secondary side open  $I_2=0$ , therefore reflected current on primary side  $I_2=0$ .  $I_1=I_0$ .  $I_0$  is very small. When  $I_2=0$  Copper loss is zero &  $I_1=I_0$ . Therefore copper loss in Primary side is low. Total Cu loss in O.C. test negligible. But input voltage is high Therefore iron losses are at rated voltage will appear. While in S.C. test voltage is low hence no core losses are found out.

**5. Which equivalent circuit parameter can be determined from the open circuit test on a transformer?(May 2011)**

From the open circuit test the exciting branch parameters can be obtained which includes resistance  $R_o$  indicating core loss component and no load inductance  $X_o$  indicating magnetizing reactance.

**6. Why transformer rated in KVA?(Dec 2011, 2015)**

**Why transformer rating is expressed in KVA? May 2017**

Copper loss depends on current and iron loss depends upon voltage. Hence the total loss in a transformer depends upon volt-amp only and not on the phase angle between voltage and current i. e, it is independent of load power factor that's why we called transformer rating in KVA.

**7. Compare two winding transformer and auto transformer?(Dec 2011)**

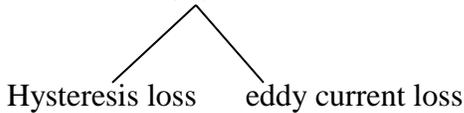
1. Higher. 2. Small size. 3. Smaller exciting current. 4. Lower cost. 5. Better voltage regulation compared to conventional two winding transformer.

6. Continuously varying voltage can be obtained

7. Requires less copper

**8. What are the no load losses in a two winding Transformer and state the reason for such losses?(Dec 2010)**

1. Core loss (or) Iron loss



2. Copper loss.

**9. Mention the condition to be satisfied for parallel operation of two winding Transformer?( May 2010)**

1. The voltage ratings of both Primary and Secondary must be same. i.e. transformation ratio are the same.
2. The transformer polarities must be connected properly, otherwise dead short circuit occurs.
3. The ratio of equivalent resistance to equivalent reactance of transformer should be same.
4. Equivalent impedances should be inversely proportional to the respective KVA ratings of the transformer.

**10. The emf per turn of 1 $\phi$  2200/220V 50Hz transformer is 11V. Calculate the number of primary and secondary turns. (May 2011)**

$$\begin{aligned} N_1=? \quad V_1=2200V \quad N_2=? \quad V_2=220V \\ K=\frac{V_2}{V_1} = \frac{220}{2200} = 0.1 \quad \frac{N_2}{N_1} = 0.1 \\ \text{emf per turn} = \frac{E_1}{N_1} = 11 \\ 11 = \frac{2200}{N_1}, \quad N_1 = 200, \quad N_2 = 0.1N_1, \\ N_2 = 20 \end{aligned}$$

**11. Define regulation of a transformer.(May 2010, Dec 2012, 2013, 2016)**

The regulation of a transformer is defined as reduction in magnitude of the terminal voltage due to load with respect to the no load terminal voltage

$$\% \text{ regulation} = \left( \frac{V_2(\text{on no-load}) - V_2(\text{when loaded})}{V_2(\text{on no-load})} \right) \times 100 \quad (\text{or}) \quad \left( \frac{E_2 - V_2}{V_2} \right) \times 100$$

**12. List the advantage and application of auto transformer (Dec 2012, 2013), (APR- 2015)  
(or) Mention the use of Auto transformer? (May 2006)**

**Advantages - Refer Q. No: 7**

**Applications**

1. for safely starting the m/c like induction motors, synchronous motors i.e as a starter
2. To give a small boost to a distribution cable to compensate for a voltage drop i.e, as a booster.
3. as a furnace transformer to supply power to the furnace at the required supply voltage.
4. for interconnecting the system which are operating roughly at same voltage level.
5. Used in laboratories, variable voltage regulators, variable voltage rectifiers etc.

**13. What happens if DC supply is applied to the transformer? (May 2012, Dec 2014, 2015)**

If D.C. Supply is given, the current will not change due to constant supply and transformer will not work. For D.C. supply  $f=0$ ,  $X_2=0$  total impedance is very low, therefore winding draw large current, this may cause burning of winding and heat generated and finally permanent damage to the transformer.

**14. Why all day  $\eta$  is lower than commercial efficiency? (May 2012)**

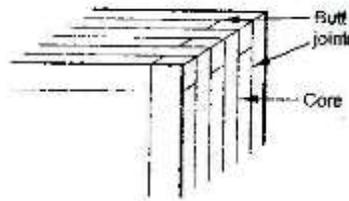
The load on distribution transformer varies continuously during the period of day. The primary of transformer always energized may be with or without energized may be with or without load continuously core loss appear and Cu loss depend upon the load condition. Due to these losses the all-day  $\eta$  is low than commercial  $\eta$ .

**15. Give the principle of transformer?(May 2010)**

The principle of mutual induction states that when two coils are inductively coupled and if current in one coil is changed uniformly then an emf gets induced in the other coil. This e.m.f can drive a current, when a closed path is provided to it. The transformer works on the same principle.

**16. What is staggering in the construction of Transformers? Explain with sketch?(Dec 2005)**

To avoid the high reluctance at the joint, the alternate layers are stacked differently to eliminate the joints. The butt joints are staggered in alternate layers.



**Advantages**

- 1) avoid continuous air gap
- 2) the reluctance of magnetic circuit gets reduced.
- 3) Continuous air gap reduces the mechanical strength of the core. It is need to increases the strength of the core.

**17. Define regulation up and down of the transformer? (Dec 2005)**

The net change in voltage when transformer is loaded is  $E_2 - V_2$  where  $E_2$  is no load voltage while  $V_2$  is load voltage. The change if divided by  $E_2$ , it is called down regulation. This change if divided by  $V_2$  it is called up regulation.

$$\% \text{ regulation down} = \frac{E_2 - V_2}{E_2} \times 100$$

$$\% \text{ regulation up} = \frac{E_2 - V_2}{V_2} \times 100$$

**18. How are sledding in transformer oil caused?(May 2006)**

While heating if the oil is exposed to oxygen then reaction takes place due to which large deposits of dark and heavy material are formed in the oil. This is called sledding of oil. Such deposits can log the cooling ducts. The sledding is possible due to decomposition of oil with long term and continuous use.

**19. What is the condition for max  $\eta$  in a single phase transformer?(May 2010)**

The max  $\eta$  ours when

$$P_i = I_2^2 R_{2e}$$

$$= P_{cu}$$

(i.e) Copper loss = Iron loss

**20. What is an ideal transformer?(May 2004)**

A Transformer said to be ideal if it satisfies following properties.

- i) It has no losses,
- ii) Its windings have zero resistance
- iii) Leakage flux is zero and
- iv) Permeability of core is high.

**21. Define tertiary winding.**

Another way of avoiding the trouble of oscillating neutral is to provide each of the transformers with a third or tertiary winding of relatively low kVA rating. This tertiary winding is connected in  $\Delta$  and provides a circuit in which the triple-frequency component of the magnetizing current can flow (with an isolated neutral, it could not). In that case, a sine wave of voltage applied to the primary will result in a sine wave of phase voltage in the secondary. As said above, the advantage of this connection is that insulation is stressed only to the extent of line to neutral voltage *i.e.* 58% of the line voltage.

**22. Why the efficiency of Transformer is more than rotating electrical Machine?(Dec 2009)**

There are no moving parts in transformer hence the friction and mechanical losses are absent in transformer. Hence  $\eta$  of transformer is more than rotating m/c.

**23. A 1000 KVA, 50 KV/40 KV, 1 $\phi$  Auto transformer is fully loaded find the current in the common section of the winding?(Dec 2009)**

$$\frac{V_2}{V_1} = \frac{40}{50} = \frac{4}{5} = K$$

$$\frac{I_1}{I_2} = K$$

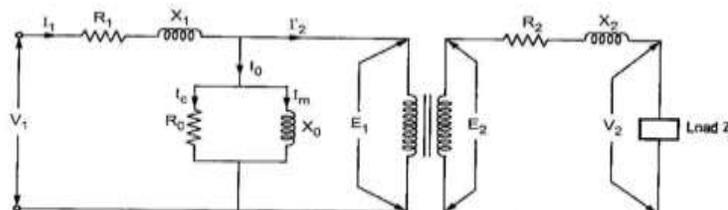
$$I_1 = \left(\frac{4}{5}\right) \times I_2$$

Current in 50 KV winding =  $\frac{1000}{50} = 20A$

$$I_1 = 20A$$

$$I_2 = \frac{20 \times 5}{4} = 25A$$

**24. Draw typical equivalent circuit of Transformer?(Dec 2007)**



**25. What are the losses in the transformer? And how those losses are minimized?(Dec 2014)**

1. **Hysteresis loss** - Hysteresis loss can be minimized by selecting materials for core such as silicon steel & steel alloys with low hysteresis coefficient and electrical resistivity.
2. **Eddy current loss** - Eddy current losses are minimized by laminating the core their moving parts.

**26. Differentiate between a core and shell type transformer.(May 2014,Dec-2017)**

S. No	Core Type	Shell Type
1	windings surround the core	core surround the winding
2	single magnetic circuit	double magnetic circuit
3	core has two limbs	core has three limbs
4	preferred for low voltage transformer	preferred for high voltage transformer
5	Easy maintenance	Difficult

**27. What is the basic purpose of tertiary winding? (May 2014)**

- 1.To supply the substation auxiliaries of aa voltage different from those of the primary and secondary windings.
2. Series capacitors or synchronous condenser may be connected to the tertiary winding for the reactive power injection into the system for voltage control.
2. Measuring the voltage of an HV testing transformer.

**28. List out any four three phase transformer connections.**

1. Star - Star connection ( $\lambda - \lambda$ )
2. Delta - Delta connection ( $\Delta - \Delta$ )
3. Star - Delta connection ( $\lambda - \Delta$ )
4. Delta - Star connection ( $\Delta - \lambda$ )
5. Open Delta (or 0 - V connection)
6. Scott connection (or) T-T connection

**29. What is inrush current? (May 2016)**

When a transformer is taken off-line, a certain amount of residual flux remains in the core due to the properties of the magnetic core material. The residual flux can be as much as 50 to 90% of the maximum operating flux, depending the type of core steel. When voltage is reapplied to the transformer the flux introduced by this source voltage builds upon that already existing in the core. In normal steady state operation condition the exciting current of a transformer is usually very low – less than 5% of the rated current. However at the moment when a transformer is connected to the power system, **a large inrush current will flow in the transformer during the transient period. This current may be high as 10 – 20 times the rated current.**

**30. What are the different types of testing of transformer?**

Polarity test, ratio test, Load test, Open and short circuit test and Sumpner's test

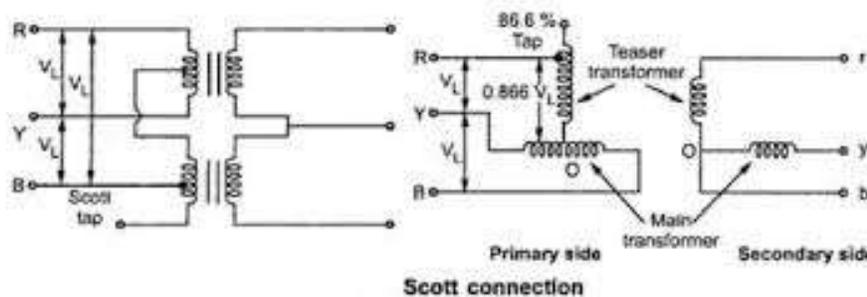
**31. What are the advantages of sumpner's test?**

- i) The power required to carry out the test is small.
- ii) The transformers are tested under full-load conditions.
- iii) The iron losses and full-load copper losses are measured simultaneously.
- iv) The temperature rise of the transformers can be noted

**32. Mention the Role of tertiary winding in transformer? APR/MAY 2015**

- To supply the substation auxiliaries at avoltage different from those of primary and secondary winding
- Static capacitors or synchronous condenser may be connected to the tertiary winding for reactive power injection for voltage control
- A delta connected tertiary reduces the impedances offered to the zero sequence current thereby allowing large earth fault current to flow for proper operations to productive equipment. Further, it limits voltage imbalance when load is unbalanced.
  - It also permits third harmonic currents to flow their by reducing third harmonics voltages. in this case, it is called as stabilizing winding.

**33. Draw Scott connection of a transformer. Dec - 2016**



**34. How to reduce leakage flux in transformer (Dec-2017)**

- 1. Reducing the magnetizing current to the minimum.
- 2. Reducing the reluctance of the iron core to the minimum.
- 3. Reducing the number of primary and secondary turn to the minimum.
- 4. Sectionalizing and interleaving the primary and secondary windings.

**35. Full load copper loss in a transformer is 1600W, what will be the losses at half load (May-2018)**

At half load the copper loss will be  $x^2 \times P_c = \left(\frac{1}{2}\right)^2 \times 1600 = 400W$

**36. Give the condition to be satisfied for parallel operation of transformer (May-2018)**

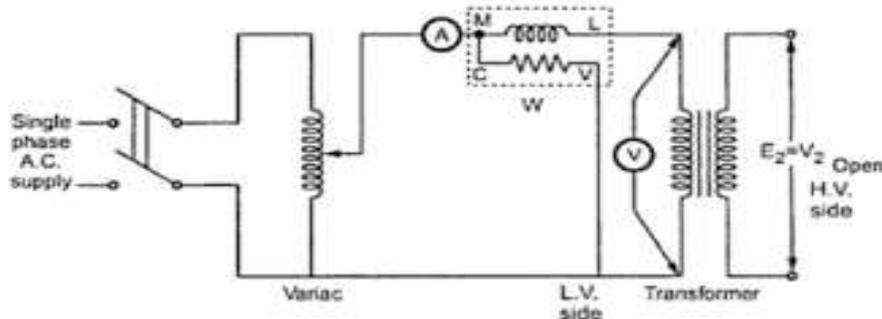
- 1. The secondary's of all transformer must have the same phase sequence.
- 2. Phase displacement between primary and secondary line voltages must be the same for all transformers which are to be operated in parallel.
- 3. Turns ratio and voltage ratio should be same.

## PART-B

1. Describe the method of calculating the regulation and efficiency of a 1 $\phi$  transformer by O.C. and S.C. tests?(May 2012, 2016)

### Open Circuit Test:

The transformer primary is connected to A.C. supply through ammeter, wattmeter and variac. The secondary is kept open. Low voltage side is used as primary and high voltage side as secondary to conduct O.C. test. Primary is excited by rated voltage with help of variac. The wattmeter measures input power ( $W_o$ ). The ammeter measures input current ( $I_o$ ). The voltmeter measures rated voltage ( $V_o$ ). If secondary is open means it works on no load. So the current drawn by the primary is no load current.



The components of no load O.C. test

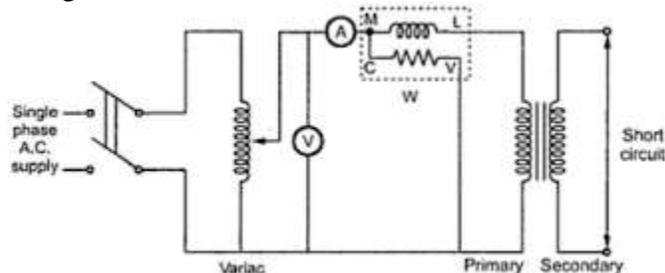
$$I_m = I_o \sin \phi_o, \quad I_c = I_o \cos \phi_o$$

$$W_o = V_o I_o \cos \phi_o, \quad \cos \phi_o = \frac{W_o}{V_o I_o}$$

$$R_o = \frac{V_o}{I_c} \Omega, \quad X_o = \frac{V_o}{I_m} \Omega$$

### Short Circuit Test:

The secondary is short circuited with the help of thick copper wire. As secondary is shorted, its resistance is very small and on rated voltage it may draw very large current. This short circuit current can cause overheating and burning of transformer. In order to avoid burning the primary is energized by low voltage and rated current. The current flowing through the winding are rated current hence total copper loss is full load copper loss. The wattmeter, Ammeter, voltmeter readings are note down



$$W_{SC} = \text{Full load copper loss}$$

$$W_{SC} = V_{SC} I_{SC} \cos \phi_{SC}$$

$$\cos \phi_{sc} = \frac{W_{sc}}{V_{sc} I_{sc}}$$

$$R_{ie} = \frac{W_{sc}}{I_{sc}^2}$$

$$Z_{ie} = \frac{V_{sc}}{I_{sc}}, X_{ie} = \sqrt{Z_{ie}^2 - R_{ie}^2}$$

$$\% \eta \text{ at full load} = \frac{V_2(I_2) \text{F.L.} \cos \phi}{V_2(I_2) \text{F.L.} \cos \phi + w_o + w_{sc}} \times 100$$

$$\% \eta \text{ at any load} = \frac{n \times (\text{VA rating}) \times \cos \phi}{n \times (\text{VA rating}) \times \cos \phi + w_o + n^2 w_{sc}} \times 100$$

Calculation of Regulation

$$\% R = \frac{I_2 R_{2\Omega} \cos \phi + I_2 X_{2\Omega} \sin \phi}{V_2} \times 100$$

$$= \frac{I_1 R_{1\Omega} \cos \phi + I_1 X_{1\Omega} \sin \phi}{V_1} \times 100$$

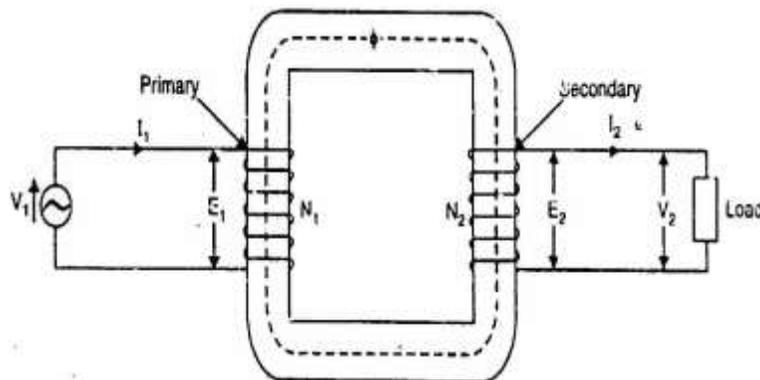
$I_1, I_2$  are rated currents for full load regulation.

For any other load, the currents  $I_1, I_2$  must be changed by fraction 'n'.

**2. Explain the principle of operation of a transformer. Derive its EMF equation. (MAY 2012, 2014, DEC-2017)**

**Working of Transformer**

A transformer is a static piece of equipment used either for raising or lowering the voltage of an a.c. supply with a corresponding decrease or increase in current. It essentially consists of two windings, the primary and secondary, wound on a common laminated magnetic core as shown in Fig. The winding connected to the a.c. source is called primary winding (or primary) and the one connected to load is called secondary winding (or secondary). The alternating voltage  $V_1$  whose magnitude is to be changed is applied to the primary.



Depending upon the number of turns of the primary ( $N_1$ ) and secondary ( $N_2$ ), an alternating e.m.f.  $E_1$  is induced in the primary. This induced e.m.f.  $E_2$  in the secondary causes a secondary

current  $I_2$ . Consequently, terminal voltage  $V_2$  will appear across the load. If  $V_2 > V_1$ , it is called a step up-transformer. On the other hand, if  $V_2 < V_1$ , it is called a step-down transformer

When an alternating voltage  $V_1$  is applied to the primary, an alternating flux  $\phi$  is set up in the core. This alternating flux links both the windings and induces e.m.f.s  $E_1$  and  $E_2$  in them according to Faraday's laws of electromagnetic induction. The E.M.F.  $E_1$  is termed as primary E.M.F. and E.M.F.  $E_2$  is termed as secondary e.m.f.

$$E_1 = -N_1 \frac{d\phi}{dt}$$

$$E_2 = -N_2 \frac{d\phi}{dt}$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Secondary and primary respectively. If  $N_2 > N_1$ , then  $E_2 > E_1$  (or  $V_2 > V_1$ ) and we get a step-up transformer. On the other hand, if  $N_2 < N_1$ , then  $E_2 < E_1$  (or  $V_2 < V_1$ ) and we get a step-down transformer. If load is connected across the secondary winding, the secondary e.m.f.  $E_2$  will cause a current  $I_2$  to flow through the load. Thus, a transformer enables us to transfer a.c. power from one circuit to another with a change in voltage level. The following points may be noted carefully:

- (i) The transformer action is based on the laws of electromagnetic induction.
- (ii) There is no electrical connection between the primary and secondary.

The a.c. power is transferred from primary to secondary through magnetic flux.

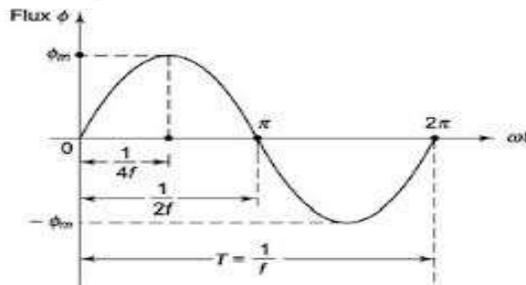
- (iii) There is no change in frequency i.e., O/P power has the same frequency as the input power.
- (iv) The losses that occur in a transformer are:

- (a) core losses—eddy current and hysteresis losses
- (b) copper losses—in the resistance of the windings

In practice, these losses are very small so that output power is nearly equal to the input primary power. In other words, a transformer has very high efficiency.

### EMF Equation

When the primary is excited by an alternating voltage  $V$ , it circulates alternating current, produced an alternating flux  $\phi$ . The primary winding has  $\alpha$ , number of turns. The alternating flux  $\phi$  linking with the primary winding itself produces an e.m.f.  $E_1$ . The secondary winding itself produces an e.m.f.  $E_2$



$\Phi$ -flux;  $\Phi_m$ -max value of flux

$N_1$ -Number of Primary turns;  $N_2$ -Number of Secondary turns

$f$ -frequency

$E_1$ -R.M.S value of Primary induced emf

$E_2$ - R.M.S value of Secondary induced emf

Average e.m.f per turn=Average rate of change of flux

Average emf per turn= $\frac{d\phi}{dt}$

$$\frac{d\phi}{dt} = \frac{\text{change in flux}}{\text{Time required to change in flux}}$$

Consider the  $\frac{1}{4}$ <sup>th</sup> cycle of the flux complete cycle gets completed in  $1/f$  seconds. In  $\frac{1}{4}$ <sup>th</sup> period the change in flux is from  $\theta$  to  $\phi_m$ .

$$\frac{d\phi}{dt} = \frac{\phi_m - 0}{1/4f} = 4f\phi_m \text{ volts.}$$

As  $\phi$  is sinusoidal, the induced emf in each turn of both the windings is also sinusoidal.

$$\text{Form factor} = \frac{\text{R.M.S value}}{\text{Average value}} = 1.11$$

R.M.S value =form factor  $\times$  Average value

$$= 1.11 \times 4f\phi_m = 4.444f\phi_m.$$

If  $N_1$  number turns in Primary

$$E_1 = N_1 \times 4.444f\phi_m \text{ volts}$$

If  $N_2$  number turns in Secondary

$$E_2 = N_2 \times 4.444f\phi_m \text{ volts}$$

Thus the e.m.f. equation is  $E = 4.444f T\phi_m$  volts

where T – total no. of turns

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### 3. Define the voltage Regulation of two winding Transformer and explain its significance? (Dec 2010)

Because of voltage drop across the primary and secondary impedance it is observed that the secondary terminal voltage drops from its no load value ( $E_2$ ) to load value ( $V_2$ ) as load and load current increases.

Regulation is defined as change in magnitude of the secondary terminal voltage, when full load, i.e rated load of specified power factor supplied at rated voltage is reduced to no load, with primary voltage maintained constant expressed as the percentage of the rated terminal voltage,

$E_2$ -Secondary terminal voltage on no load

$V_2$ - Secondary terminal voltage on given load.

$$\% \text{ voltage regulation} = \frac{E_2 - V_2}{V_2} \times 100$$

The ratio  $\frac{E_2 - V_2}{V_2}$  is called per unit regulation.

As load increases, the voltage drops tend to increase and  $V_2$  drops more.

In case of lagging power factor  $V_2 < E_2 \rightarrow$  + 've voltage regulation, while in leading p.f  $E_2 < V_2 \rightarrow$  - 've voltage regulation

$$\% R = \frac{E_2 - V_2}{V_2} \times 100 = \frac{\text{Total voltage drop}}{V_2} \times 100.$$

$$\% R = \frac{I_2 R_{2e} \cos \phi \pm I_2 X_{2e} \sin \phi}{V_2} \times 100.$$

Zero voltage regulation:

At certain leading power factor we get  $E_2 = V_2$  and regulation becomes zero.

$$\cos \phi = \cos \phi \left\{ \tan^{-1} \frac{V_R}{V_{\infty}} \right\}$$

**4. Explain the reason for tap changing in transformer? State on which winding the tapes are provided? Why?(Dec 2010)**

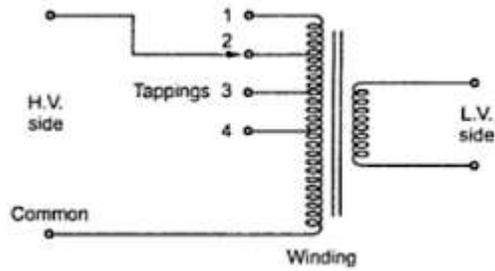
In power system network the voltage supplied by transformer can be varied by changing its transformation ratio. This is changing its transformation can be varied by changing its transformation ratio. This is achieved by tappings on transformers. It is a leads which are connected to various points on a transformer winding. The tappings are placed either on high voltage (or) low voltage (or) sometimes on both high and low voltage windings.

Tappings are needed in supply network because the voltage at consumer premises should remain in the permissible limits as declared by the electric supply company. It is also required to control active and reactive power with change in load, the voltage variations are to be adjusted. Depending on the constructional availability, the tapping may be in phase and (or) neutral point (or) in the middle of the winding.

1. If tapping are available in phase end, the number of bushing instructors are reduced.
2. When tapping available in neutral point then insulation between various parts is small which is economical especially in high voltage transformer.
3. Tapping are placed near the Centre of the winding if large voltage variation is required which also reduces magnetic asymmetry.

Normally tapping are provided on high voltage winding due to following reasons

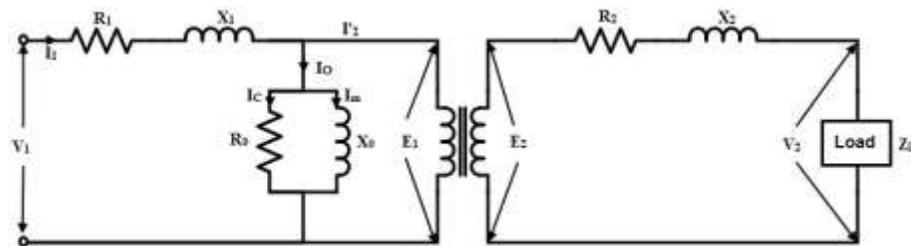
- i) A fine voltage regulation is possible with high voltage winding as it carries large number of turns.
- ii) The low voltage winding of the transformer carries large current. So if tappings are provided on low voltage side then there are difficulties encountered in the interruption of high currents which makes its impracticable.
- iii) For the reason of requirement of insulation, the low voltage winding is placed near the core while the high voltage winding is placed outside. Hence practically it is easier and simpler to provide tapping on the high voltage winding.
- iv) If tapping are provided on low voltage side then the exact voltage regulation may not be provided.
- v) In case of step down transformer, it is added advantage to provide tapping's on H.V. side. At light loads, the L.V side voltage increases. It is required to decrease this voltage by adjusting the tapping on H.V. side to a position number of turns are large.



Transformer with tapping on h.v side

5. Explain clearly the causes for voltage drop in a power transformer on load and develop the equivalent circuit for a single phase transformer?(Dec 2011, 2014 & May- 2014)

Consider a transformer supplying the load



$R_1, R_2$ -Primary and secondary resistances

$x_1, x_2$ -Primary and secondary leakage resistances

$Z_L$ -load impedance

$I_1, I_2$ -Primary and secondary current

$\bar{I}_1 - \bar{I}_0 + \bar{I}_2'$

$I_0$ -no load current.

$I_2'$  -load component of current in primary

Primary Voltage  $V_1 = -\bar{E}_1 + I_1 \bar{Z}_1$

Secondary Voltage  $V_2 = \bar{E}_2 - I_2 \bar{Z}_2$

**Unity p.f load**

$E_1$  lags  $\phi$  by  $90^\circ$

$E_1, E_2$  are in phase

$V_2$  in particular direction

$I_2$  is in phase with  $V_2$

Add  $I_2 R_2 + I_2 X_2$  to  $V_2$  to get  $E_2$

Reverse  $I_2$  is  $I_2'$

$I_0 + I_2' = I_1$

**Lagging p.f load**

$I_2$  lags  $V_2$  by  $\phi_2$

$I_2 R_2, I_2 X_2, I_2', I_1, I_1 R_1, I_1 X_1$  are changed.

$I_2 X_2$  leads  $I_2$  by  $90^\circ$

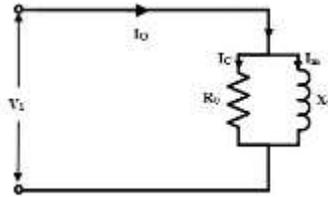
**Leading p.f load**

$I_2$  leads  $V_2$  by  $\phi_2$

**Equivalent circuit of transformer:-**

Equivalent circuit means the combination of fixed and variable resistance and reactance's, which exactly simulates performance and working of the machine.

No load equivalent circuit

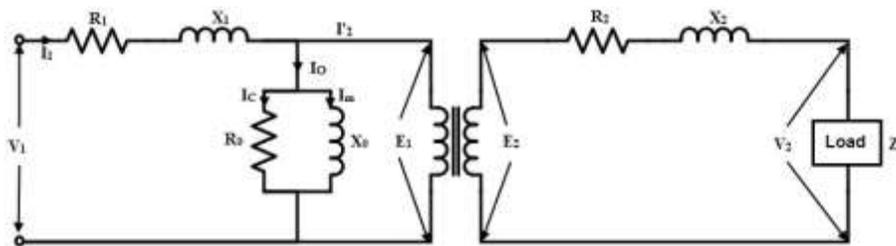


$$I_m = I_0 \sin \phi_0, \quad I_w = I_0 \cos \phi_0$$

$I_m$  produces flux and is assumed to flow through reactance  $X_0$  called no load reactance while  $I_c$  is active component representing core losses, hence is assumed to flow through the resistance  $R_0$ . The circuit consists of exciting circuit.

$$R_0 = \frac{V_1}{I_c}, \quad X_0 = \frac{V_1}{I_m}$$

When load is connected to the transformer then secondary current  $I_2$  flows. This cause voltage drop across  $R_2$  and  $X_2$ . Due to  $I_2$ , primary draws an additional current  $I_{21} = I_2/K$ , now  $I_1$  is the phasor addition of  $I_0$  and  $I_{21}$ . This  $I_1$  causes the voltage drop across primary resistance  $R_1$  and reactance  $X_1$



Transferring secondary parameters to primary we get,

$$R_2' = \frac{R_2}{K^2}, \quad X_2' = \frac{X_2}{K^2}, \quad Z_2' = \frac{Z_2}{K^2}$$

$$E_2' = \frac{E_2}{K} I_2' = K I_2 \text{ Where } K = \frac{N_2}{N_1}$$

While transferring the values remember the rule that

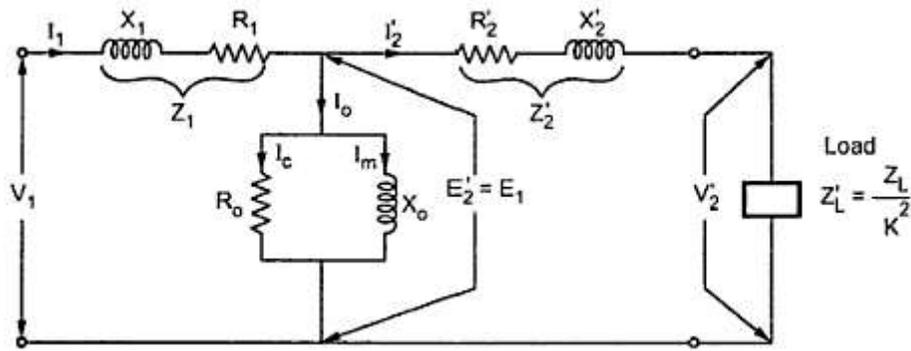
Low voltage winding → high current → low impedance

High voltage winding → low current → High impedance

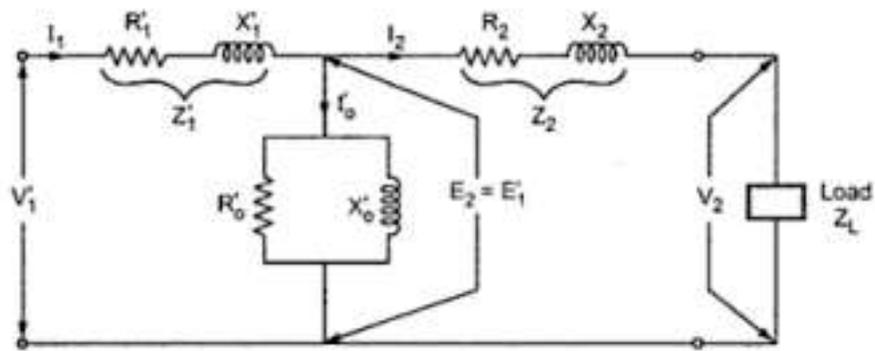
Equivalent circuit ref to primary,

$$R_1' = K^2 R_1, \quad X_1' = K^2 X_1, \quad Z_1' = K^2 Z_1$$

$$E_1' = K E_1, \quad I_1' = \frac{I_1}{K}, \quad I_0' = \frac{I_0}{K}$$



Similarly, Equivalent circuit Referred to secondary



**6. Derive an expression for saving of copper when an auto transformer is used (Dec 2011, 2014, 2015)**

For any winding the cross section of winding is proportional to the current I. The total length of winding is proportional to the number of turns N. Hence the weight of copper is proportional to the product N and I

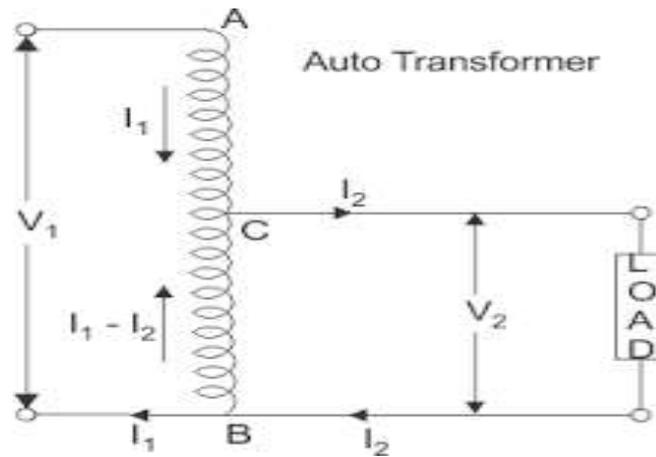
Weight of copper  $\propto N I$

I - Current in the winding, N - Number of turns of the winding

$$AB = N_1$$

$$BC = N_2$$

$$AC = N_1 - N_2$$



$W_{Tw}$  = Total weight of copper in two winding transformer

$W_{AT}$  = Weight of copper in auto transformer

**In the two winding transformer.**

Weight of copper of primary  $\propto N_1 I_1$

Weight of copper of secondary  $\propto N_2 I_2$

$$W_{Tw} \propto N_1 I_1 + N_2 I_2 \text{-----1}$$

**In Auto transformer**

Weight of copper in section AC  $\propto (N_1 - N_2) I_1$

Weight of copper in section BC  $\propto N_2 (I_2 - I_1)$

$$W_{AT} \propto (N_1 - N_2) I_1 + N_2 (I_2 - I_1) \text{-----2}$$

Divide 1 and 2

$$\frac{W_{Tw}}{W_{AT}} = \frac{N_1 I_1 + N_2 I_2}{(N_1 - N_2) I_1 + N_2 (I_2 - I_1)}$$

$$= \frac{N_1 I_1 + N_2 I_2}{N_1 I_1 - N_2 I_1 + N_2 I_2 - N_2 I_1}$$

$$= \frac{N_1 I_1 + N_2 I_2}{N_1 I_1 + N_2 I_2 - 2N_2 I_1}$$

$$K = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

$$\frac{W_{Tw}}{W_{AT}} = \frac{N_1 I_1 + KN_1 \cdot I_1 / K}{N_1 I_1 + KN_1 \left(\frac{I_1}{K}\right) - 2(KN_1) I_1}$$

$$= \frac{2N_1 I_1}{2N_1 I_1 - 2(KN_1) I_1}$$

$$= \frac{2N_1 I_1}{2N_1 I_1 (1-K)} = \frac{1}{1-K}$$

$$W_{AT} = (1 - k) W_{TW}$$

$$\text{Saving of copper} = W_{TW} - W_{AT}$$

$$= W_{TW} - [(1 - k)W_{TW}]$$

$$= W_{TW} - W_{TW} + kW_{TW}$$

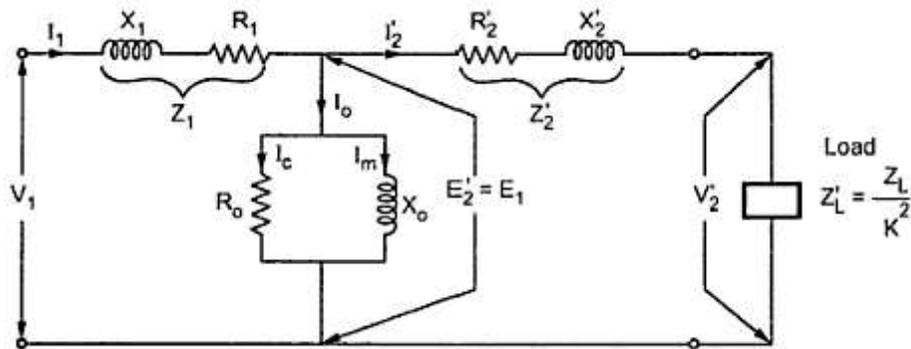
$$\text{Saving of copper} = kW_{TW}$$

Saving of copper is K times the total weight of copper in two winding transformer.

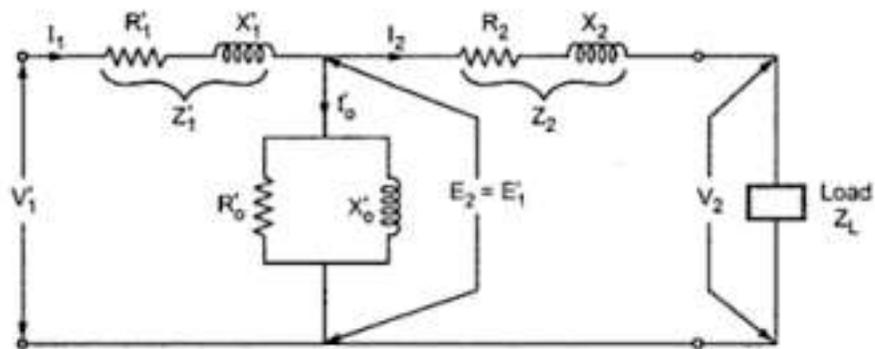
**7. Explain in detail the tests required to obtain the equivalent?(May 2010)**

**Test - Refer Q. No: 1 (Part – B)**

Equivalent circuit Ref to primary



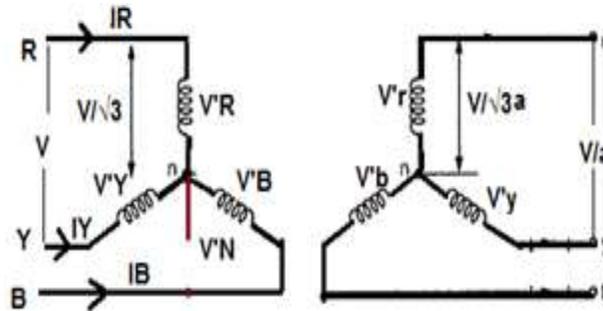
Equivalent circuit Ref to Secondary



## 8. Explain in details various types of 3 $\phi$ types of transformer connections?(May 2010, 2016)

### 1. Star-Star connection.

Here, both primary and secondary winding are connected in star. This connection is economical for small high voltage transformer as phase voltage is  $\frac{1}{\sqrt{3}}$  times that of line voltage, number of turns per phase and the quantity of Insulation required is minimum. The ratio of line voltages on primary and secondary sides is the same as the transformation ratio of ratio of each transformer. There is phase shift of 30° b/w the phase voltage and line voltage on both primary and secondary side.



$$V_{ph1} = \frac{V_{L1}}{\sqrt{3}} \quad V_{ph2} = K \left( \frac{V_{L1}}{\sqrt{3}} \right) V_{L2} = \sqrt{3} V_{ph2}$$

### Advantages

- 1) Phase voltage is  $\frac{1}{\sqrt{3}}$  times line voltage hence less number of turns are required. Stress on insulation is less.
- 2) Phase current is same as line current. Winding carry high currents. This makes cross section of the winding is high.  
∴ Winding are mechanically strong
- 3) There is no phase shift between primary and secondary voltage
- 4) Due to neutral, 3  $\phi$  4 wire system possible

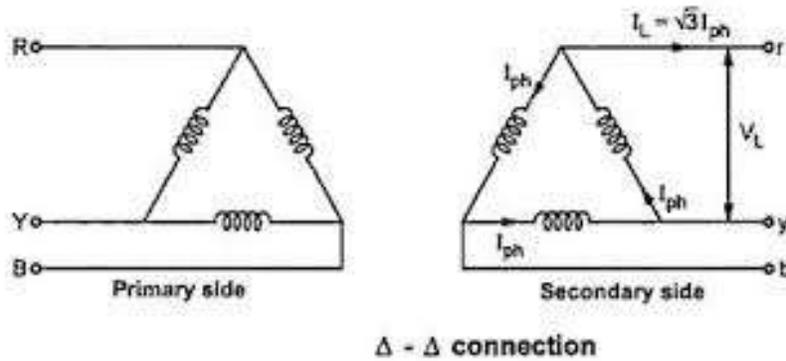
### Disadvantages

- 1) If load on secondary side is balanced then the performance of this connection is not satisfactory then shifting of neutral point is possible
- 2) It produce 3<sup>rd</sup> harmonics.

### 2. Delta-Delta connection

In this type of connection, both the three phase primary and secondary windings are connected in delta as shown in figure.

There is no phase shift between primary and secondary voltages.



$$V_{L1} = V_{ph1}$$

$$V_{ph2} = K V_{ph1} = K \cdot V_{L1}$$

$V_{L1}$  – Line voltage on Py.side  
 $V_{L2}$  - Line voltage on Sy.side  
 $V_{ph1}$ -Phase voltage on Py. Side  
 $V_{ph2}$ - Phase voltage on Sy. Side

### Advantages

- 1) No third harmonics present
- 2) If the load unbalanced 3  $\phi$  voltages are constant.
- 3) If one of the transformer disabled then supply can be continued with remaining two transformers of course of reduced  $\eta$ .
- 4) There is no distortion in the secondary side.
- 5) Phase voltage equal to line voltage but phase current  $\frac{1}{\sqrt{3}}$  time of line current

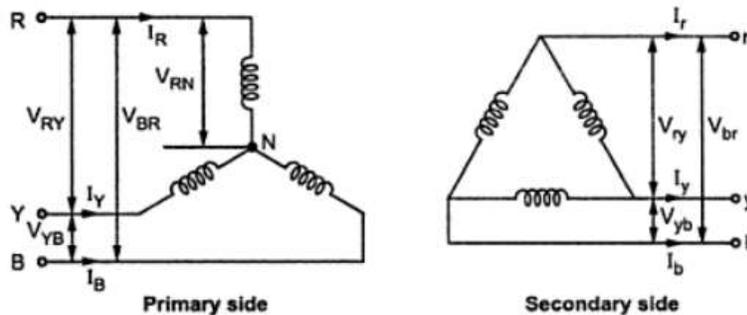
### Disadvantage

Not suitable for 3 $\phi$ four wire connection.

### 3.Star-Delta connection

In this type of connection, the primary is connected in star fashion while the secondary is connected in delta fashion as shown below.

The delta connections on secondary side allows third harmonic current to flow which provides a sinusoidal flux.



$$V_{ph1} = \frac{V_{L1}}{\sqrt{3}} ; \quad V_{L2} = K \frac{V_{L1}}{\sqrt{3}}$$

### Advantages

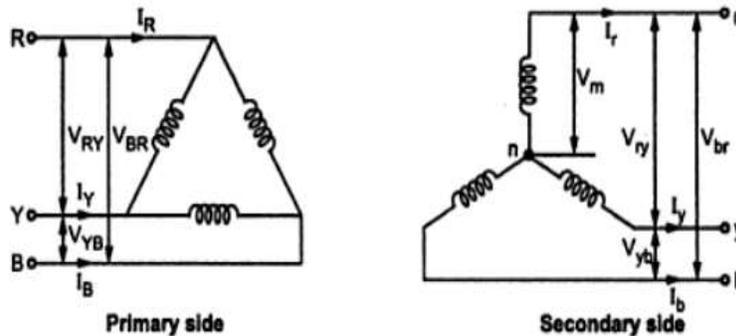
1. Primary side is star connected  $\therefore$  few number of turns required.
2. Avoid distortion due to Primary side neutral point.
3. Hence unbalanced loads can handle safely.

### Disadvantages

Secondary voltage is not in phase with the primary. Not possible to operate this connection parallel with star-star, Delta-Delta connected Transformer.

### 4. Delta-Star connection

In this type of connection, the primary is connected in delta fashion while the secondary is connected in star fashion as shown in figure.



$$V_{L1} = V_{ph1}$$

$$V_{L2} = (\sqrt{3}K)V_{L1}V_{ph2} = K V_{ph1}$$

### Advantages

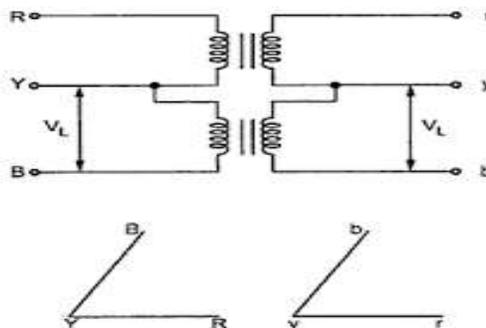
1. Cross section required on primary side less.
2.  $3\phi$  4 wire system is possible
3. No distortion
4. Due to star connection, economical saving of cost
5. Large unbalanced load can handle.

### Disadvantage

So Limitation is there.

### 5. Open delta (or) V-V connection

In delta – delta connected three phase transformer, if one of the transformer is unable to operate then the supply to the load can be continued with the remaining two transformer at the cost of reduced efficiency.



The connection thus obtained is called V – V connection or open delta connection. Consider the figure in which three phase supply is connected to the primaries. At the secondary side three equal three phase voltages will be available on no load.

$$\Delta\text{-}\Delta \text{ capacity} = \sqrt{3} V_L I_L = 3V_L I_{ph} \text{----- 1}$$

$$\text{V-V capacity} = \sqrt{3} V_L I_L = \sqrt{3} V_L I_{ph} (I_L = I_{ph} \text{ for V-V connection}) \text{-----2}$$

$$\frac{2}{1} = \frac{\sqrt{3} V_L I_{ph}}{3V_L I_{ph}} = \frac{1}{\sqrt{3}} = 0.577 = 57.7\%$$

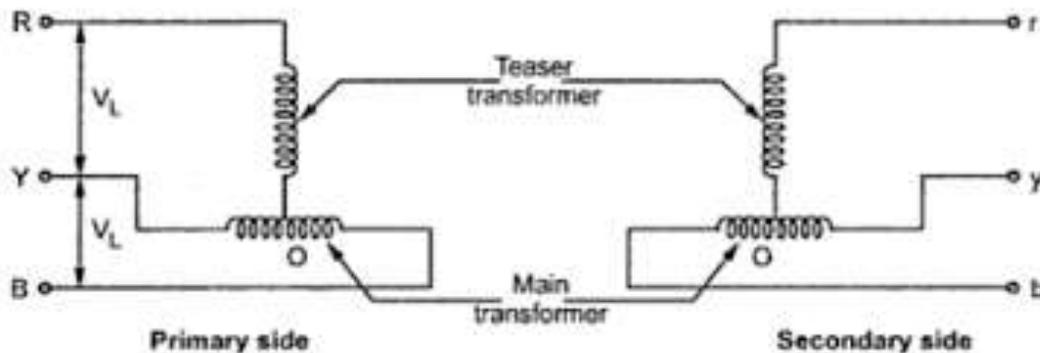
3phase load can be carried without exceeding the ratings of the transformer is 58% of original load.

The limitation with V-V connection.

1. Average p.f at which V-V bank is operating is less than that with the load.
2. The two transformers in V-V bank operate at different p.f except for balanced UPF load
3. The terminal voltage available on the secondary side become unbalanced.

### 6. Scott connection (or) T - T connection

This type of connection uses two transformer as that of open delta connection. With the help of this type of connection the conversion from three phase is possible which is shown in figure.



T.T connected primary & secondary side one of the transformer in the connection is called main transformer which is provided with a 50% tapping on both primary and secondary winding. The other transformer known as teaser transformer having 0.866 tap is connected to centre tap on main transformer on primary & secondary side. Horizontal winding is called as main winding. Vertical winding is called as teaser winding.

$$\text{The available KVA capacity} = V_L I_L + 0.866 V_L I_L = 1.866 V_L I_L$$

$$\text{Actual KVA capacity} = \sqrt{3} V_L I_L$$

$$\frac{\text{KVA actually used}}{\text{Available KVA}} = \frac{1.732 V_L I_L}{1.866 V_L I_L} = 92.8\%$$

∴ It is more economical than open Δ .

9. 1) Draw equivalent circuit of Tr. (Dec 2009)- Ref Question No: 7

2) Explain working principle of Auto Tr.

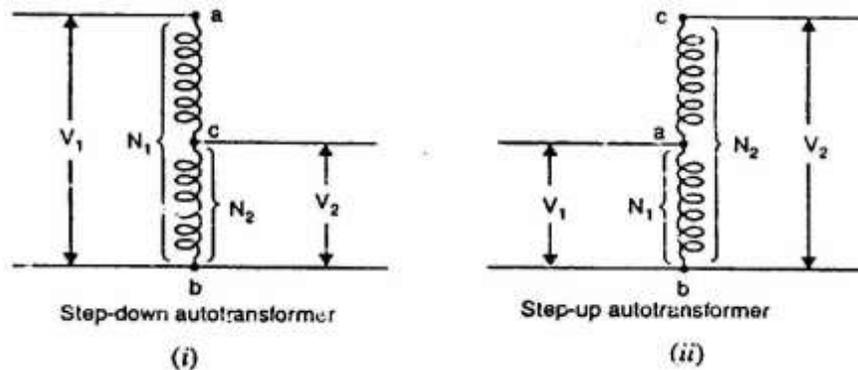
3) Different type of 3phase connection - Ref Question No:8

## 2) Working principle of Auto Transformer

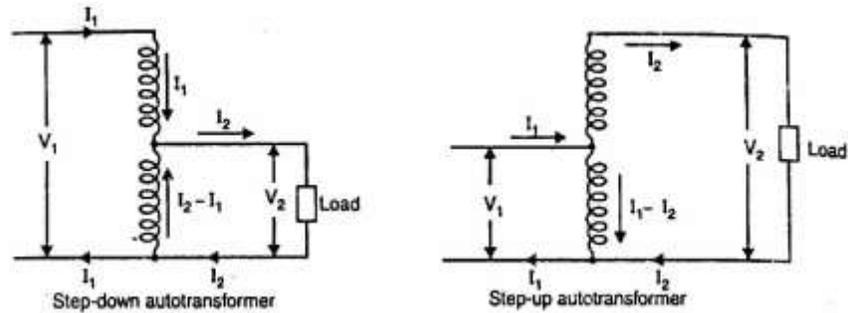
An autotransformer has a single winding on an iron core and a part of winding is common to both the primary and secondary circuits. Fig. (i) shows the connections of a step-down autotransformer whereas Fig. (ii) shows the connections of a step-up autotransformer. In either case, the winding ab having  $N_1$  turns is the primary winding and winding bc having  $N_2$  turns is the secondary winding. Note that the primary and secondary windings are connected electrically as well as magnetically. Therefore, power from the primary is transferred to the secondary conductively as well as inductively (transformer action). The voltage transformation ratio  $K$  of an ideal autotransformer is

$$K = \frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

Note that in an autotransformer, secondary and primary voltages are related in the same way as in a two winding transformer



Below figure shows the connections of a loaded step-down as well as step-up auto transformer. In each case,  $I_1$  is the input current and  $I_2$  is the output or load current. Regardless of autotransformer connection (step-up or step-down), the current in the portion of the winding that is common to both the primary and the secondary is the difference between these currents ( $I_1$  and  $I_2$ ). The relative direction of the current through the common portion of the winding depends upon the connections of the autotransformer. It is because the type of connection determines whether input current  $I_1$  or output current  $I_2$  is larger. For step-down autotransformer  $I_2 > I_1$  (as for 2-winding transformer) so that  $I_2 - I_1$  current flows through the common portion of the winding. For step-up autotransformer,  $I_2 < I_1$ . Therefore,  $I_1 - I_2$  current flows in the common portion of the winding



In an ideal autotransformer, exciting current and losses are neglected. For such an autotransformer, as  $K$  approaches 1, the value of current in the common portion ( $I_2 - I_1$  or  $I_1 - I_2$ ) of the winding approaches zero. Therefore, for value of  $K$  near unity, the common portion of the winding can be wound with wire of smaller cross-sectional area. For this reason, an autotransformer requires less copper.

**10. What are tap changing transformer? Explain on load tap changing transformer.(Dec 2009)**

**Ref Question No. 4 (Part – B)**

### On load tap changing transformer

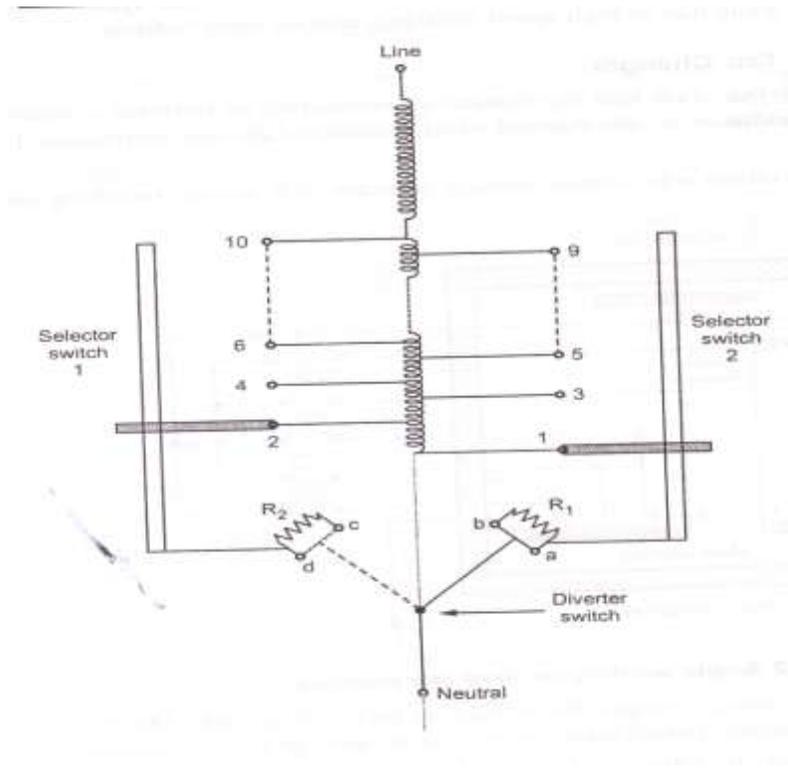
Under the load condition, it is required to maintain the voltage on secondary side of the transformer with the help of certain arrangement when transformer is connected to a system.

Tap changing is done by motor operated mechanism, remote control or with a help of handle

In tap changing mechanism, selector switch is used, it should not break current, additional separate oil filled compartment is used to mount diverter switch which breaks the load current by interrupted are which can form carbon. It should not mixed with oil in the main tank to decrease the dielectric strength. Make before break switch is used in transition period.

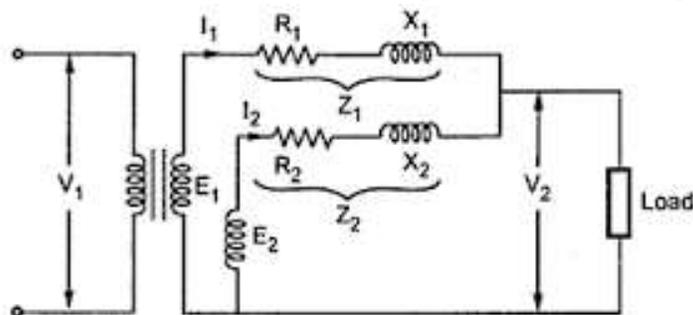
The selector switch 1 and 2 are provided on taps 1 and 2 respectively. The diverter switch is connecting tap 1 to the neutral terminal.

- ✓ Resistance  $R_1$  short circuited.
- ✓ External diverter switch is moved to open the constant a.
- ✓ The load current flow through resistance  $R_1$  and b.
- ✓ The contact c closes those to open the resistance  $R_1$  when the moving contact of diverter switch contains its movement to the left. Resistance  $R_1$ ,  $R_2$  connected across taps 1,2 now current flow b,c
- ✓ Further movement of diverter switch to left makes contact 'b' to open. Now current from tap 2 to flow through  $R_2$  & C.
- ✓ If switch closes 'd', short circuit  $R_2$ . Thus current flows through tap 2 and d.



**11. Explain parallel operation of single or three phase transformer?(Dec 2008) (May 2016)**

1. Condition to be obtained for parallel operation of transformer
2. The supply system voltage and frequency must suit the primary windings of the transformers
3. The transformers that are connected must have same polarity.
4. **In case of 3 $\phi$  Tr.** The Transformer should have same angular displacement and same phase sequence.
5. The voltage ratio of primaries and secondary's of the transformer must be same.
6. The % of impedance should be equal in magnitude and have same X/R ratio in order to avoid circulating currents and operating at different power factors.
7. If the transformer have the different KVA ratings, the equivalent impedances should be inversely proportional to individual KVA rating to avoid circulating currents.



Parallel operation of Tr with equal voltage

$$I_1 Z_1 = I_2 Z_2 = I Z_{equ}$$

$$I_1 = \frac{I Z_{equ}}{Z_1} = \frac{I Z_2}{Z_1 + Z_2} \dots\dots\dots 1$$

$$I_2 = \frac{IZ_{equ}}{Z_2} = \frac{IZ_1}{Z_1+Z_2} \dots\dots\dots 2$$

Multiply by  $V_2$  in equation 1

$$V_2 I_1 = V_2 I \frac{Z_2}{Z_1+Z_2}$$

Multiply by  $V_2$  in equation 2

$$V_2 I_2 = V_2 I \frac{Z_1}{Z_1+Z_2}$$

But  $V_2 I * 10^{-3}$  is Q i.e the combined load in KVA

$$Q_1 = Q \frac{Z_2}{Z_1+Z_2} = Q \frac{1}{1+\frac{Z_1}{Z_2}}$$

$$Q_2 = Q \frac{Z_1}{Z_1+Z_2} = Q \frac{1}{1+\frac{Z_2}{Z_1}}$$

-----&&&-----

**12. Explain the constructional details and working of core type and shell type transformer with neat sketches.(May 2013)(Dec 2013)**

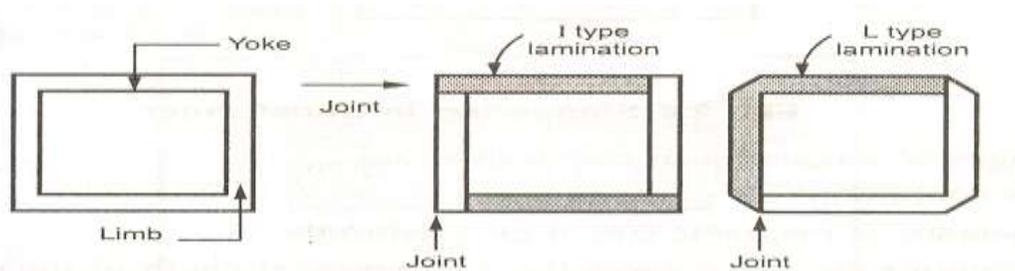
**Parts of transformer**

1. **Primary winding** : the winding , which is connected to supply is called primary winding
2. **Secondary winding** : the winding which is connected to load is called secondary winding
3. **Transformer core:** the two winding of the transformer are magnetically coupled through the core. The core is made up of silicon steel to reduce the hysteresis loss. It is also made up of limited stamping which are insulated from each other by core varnish or oxide layer to reduce the eddy current loss. The thickness of lamination varies from 0.35 mm to 0.5 mm. the two types of transformer cores are : A) core type b) shell type

**Construction**

**Transformer core:**

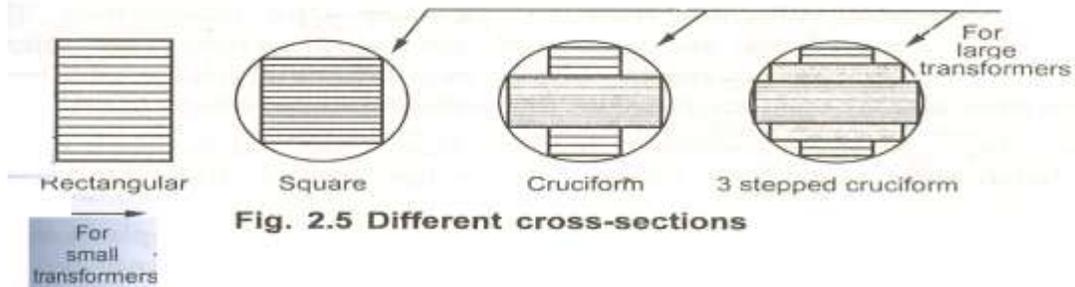
The core of the transformer is either square or rectangular in size, it is further divided into two parts. The vertical portion on which coils are wound is called limb while the top and bottom horizontal portion is called yoke of the core.



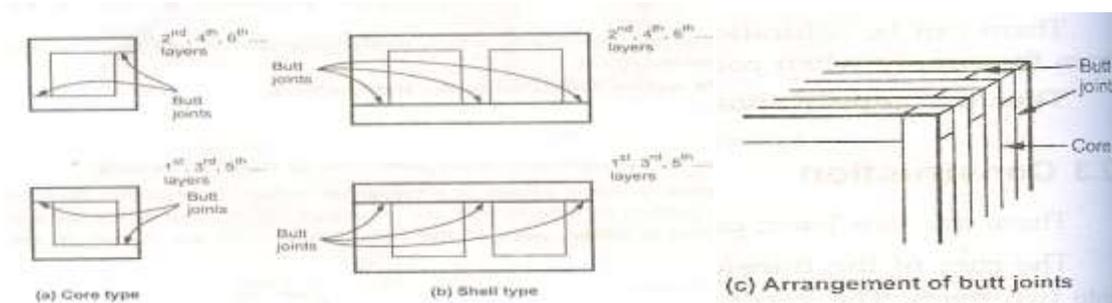
Core is made up of lamination. Because of laminated type of construction, eddy current losses gets minimized. Generally high grade steel laminations are used, these lamination are insulated from each other by using insulation like varnish.

All lamination are varnished, laminations are overlapped so that to avoid the air gap at the joints. For this generally L shaped or I shaped lamination are used which shown

The cross section of the limb depend on the type of coil to be used either circular or rectangular. The different cross section of limbs, practically used as shown



To avoid high reluctance at the joint, the alternate layers are stacked differently to eliminate the joints. This called staggering. The butt joints are staggered in alternate layers.

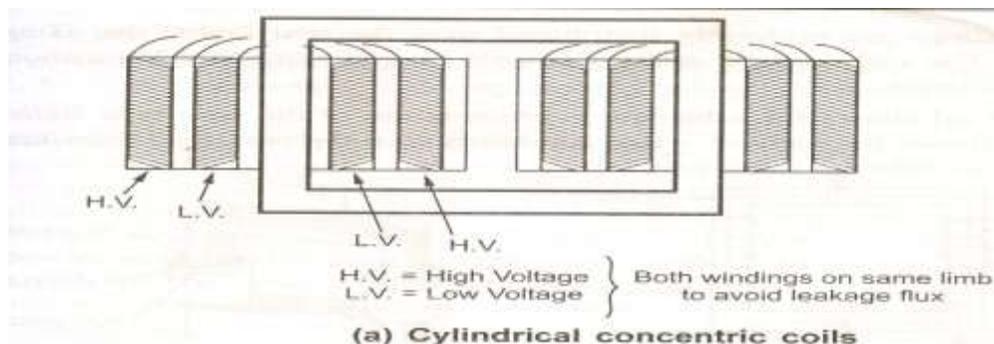


The advantages of staggering in transformer are,

1. It avoids continuous air gap
2. The reluctance of magnetic circuit get reduced
3. The continuous air gap reduces the mechanical strength of the core. The staggering helps to increase the mechanical strength of the core.

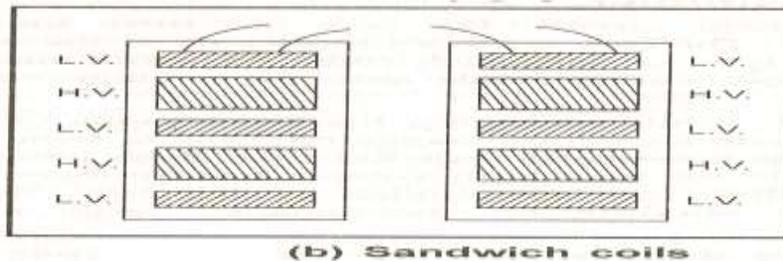
### Types of winding:

The coils are wound on the limbs and are insulated from each other. In the basic of transformer shown, the two winding wound are shown on two different limbs i.e. primary on one limb while secondary on other limb. But due to the leakage flux increases which affect the transformer performance badly. Similarly it is necessary that the winding must be very close to each other to have high mutual inductance. To achieve this, the two winding are split into number of coils and are wound adjacent to each other on the same limb. A very common arrangement is cylindrical concentric coils are shown



Such cylindrical coils are used in the core type transformer. These coils are mechanically strong, these are wound in the helical layer. The different layers are insulated from each other by paper, cloth or mica. The low voltage winding is placed near the core from ease of insulating it from core. The high voltage is placed after it.

The other type of coil which is very commonly used for the shell type of transformer is sandwich coils. Each high voltage portion lies between the two low voltage portions sandwiching the high voltage portion. Such subdivision of winding into small portion reduces the leakage flux. Higher the degree of sub-division, smaller in reactance .the sandwich coil is shown.



The top and bottom coils are low voltage coils. All the portions are insulated from each other by paper

### Construction of single phase transformer

The various construction used for the single phase transformer

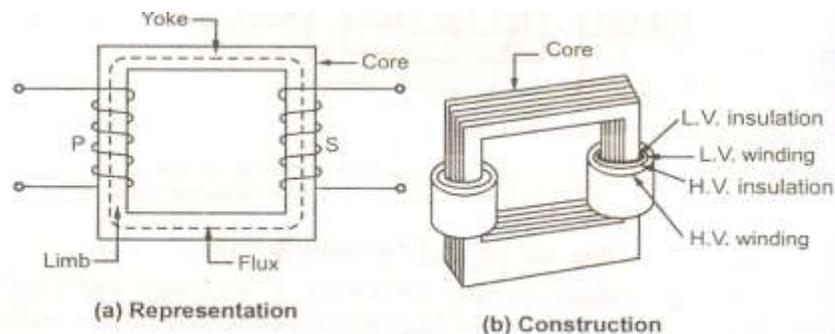
The various construction used for the single phase transformer are,

1. Core type
2. Shell type
3. Berry type

### Core type transformer

It has a single magnetic circuit. The core is rectangular having two limbs. The winding encircles the core. The coils used are of cylindrical type. As mentioned earlier, the coils are wound in helical layers with different layer insulated from each other by paper or mica. Both the coils are placed on both the limbs. The low voltage coil is placed inside near the core while high voltage coils surround the low voltage coils. Core is made up of large number of thin lamination.

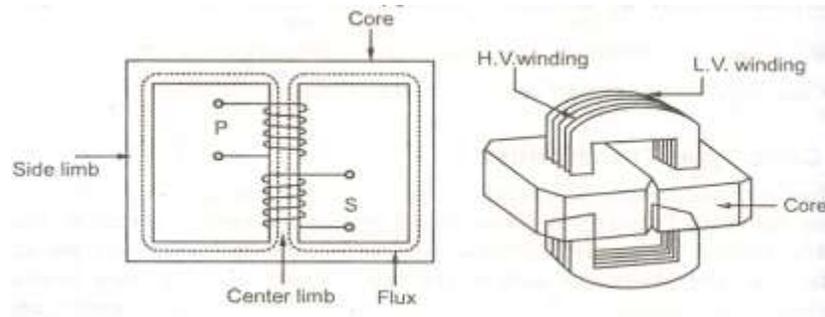
As the winding are uniformly distributed over the two limb,the natural cooling is more effective. The coils can be easily removed by removing the lamination of the top yoke, for maintenance.



## Shell type transformer

It has a double magnetic circuit. The core has three limbs. Both the winding are placed on the central limb. The core encircles most part of the winding. The coils used are generally multilayer disc type or sandwich coils, as mentioned earlier, each high voltage coil is in between two low voltage coils and low voltage coils are nearest to top and bottom of yoke.

The core is laminated. While arrange the lamination of the core, the care is taken that all the joints at alternate layer are staggered. This is done to avoid narrow air gap at the joint, Right through cross section of the core. Such joints are called over lapped or imbricated joints. Generally for very high voltage transformer, the shell type construction is preferred.



**Fig.Shell type transformer**

## Berry type transformer

This has distributed magnetic circuit. The number of independent magnetic circuits are more than 2. Its core construction is like spokes of a wheel. Otherwise it is symmetrical to that of shell type.

The transformer are generally kept in tightly fitted sheet metal tanks. The tanks are constructed of specified high quality steel plate cut, formed and welded into rigid structures. All the joints are painted with a solution of light blue chalk which turns dark in the presence of oil, disclosing even the minutest leaks. The tanks are filled with special insulating oil. i) keeps the coil cool by circulation and ii) provides the transformer an additional insulation .The oil should be absolutely free from alkalies, sulphur and specially from moisture. Presence of very small moisture lowers the dielectric strength of oil, affecting its performance badly. Hence the tanks are sealed air tight to avoid the contact of oil with atmospheric air and moisture. In large transformer, the chamber called breathers are provided. The breathers prevent the atmospheric moisture to pass on to the oil .the breathers contain the silica gel crystal which immediately absorb the atmospheric moisture. Due to long and continuous use, the sludge formed in the oil which can contaminate the oil. Hence to keep such sludge separate from the oil in main tank, an air tight metal drum is provided, which is placed on the top of tank. This is called conservator.

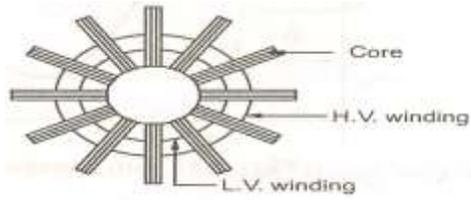


Fig- Berry type transformer

**13. Explain the condition for maximum efficiency?(Dec 2013)**

Due to the losses in a transformer, the output power of a transformer is less than the input power supplied.

$$\begin{aligned} \text{Power output} &= \text{power input} - \text{Total losses} \\ \text{Power input} &= \text{Power output} + \text{Total losses} \\ &= \text{power output} + P_i + P_{cu} \end{aligned}$$

The efficiency of any derive is defined as the ratio of the power output to power input. So for a transformer the efficiency can be expressed as,

$$\text{Transformer efficiency } \eta = \frac{\text{Output power}}{\text{Input power}}$$

$$\eta = \frac{\text{Output power}}{\text{Input power} + \text{Losses}} = \frac{\text{Output power}}{\text{Output power} + \text{iron Losses} + \text{copper Losses}}$$

$$\text{Output} = V_2 I_2 \cos \phi$$

Where,  $V_2$  –Secondary terminal voltage on load

$I_2$  –Secondary current at load ;  $\cos \phi$  -Power factor of the load

Iron loss,  $P_i = W_o$ , determined from O.C. Test

Copper loss  $P_{cu} = W_s$ , determined S.C. Test at full load

Copper losses at a load n times of full load =  $n^2 P_{cu}$

$$\text{So, Transformer efficiency } \eta = \frac{nV_2 I_2 \cos \phi}{nV_2 I_2 \cos \phi + P_i + n^2 P_{cu}}$$

Condition for maximum efficiency

$$\text{Output power} = V_2 I_2 \cos \phi_2$$

If  $R_{02}$  is the total resistance of the transformer referred to secondary, then

$$\text{Total copper loss } P_{cu} = I_2^2 R_{02}; \text{ Total loss} = P_i + P_{cu}$$

$$\eta = \frac{\text{Output power}}{\text{Input power}} = \frac{\text{Output power}}{\text{Input power} + \text{losses}} = \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + P_i + P_{cu}}$$

$$\eta = \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \phi_2 + P_i + I_2^2 R_{02}}$$

Divide both numerator and denominator by  $I_2$

$$\eta = \frac{V_2 \cos \phi_2}{V_2 \cos \phi_2 + P_i/I_2 + I_2 R_{02}} \quad \text{----- 1}$$

For maximum value of efficiency for given  $\cos \phi_2$  (p.f) the denominator must have the least value. The condition for maximum efficiency is obtained by differentiating the denominator and equating it to zero.

$$\frac{d}{dI_2} (\text{denominator}) = 0$$

$$\frac{d}{dI_2} \left( V_2 \cos \phi_2 + \frac{P_i}{I_2} + I_2 R_{02} \right) = 0$$

$$0 - \frac{P_i}{I_2^2} + R_{02} = 0 \quad \text{----- 2}$$

Iron loss = copper loss i.e., constant loss = Variable loss

Hence efficiency of a transformer will be maximum when copper losses are equal to iron losses.

From equation 2 the load current corresponding to maximum efficiency is given by,

$$I_2 = \sqrt{\frac{P_i}{R_{02}}}$$

#### 14. Explain operation of a transformer with necessary vector diagrams

i) on no load and

ii) on load with upf, lagging and leading power factor. (Dec 2014, Dec 2017)

##### i) On no load

Assume an ideal transformer *i.e.* one in which there were no core losses and copper losses. But practical conditions require that certain modifications be made in the foregoing theory. When an *actual* transformer is put on load, there is iron loss in the core and copper loss in the windings (both primary and secondary) and these losses are not entirely negligible.

Even when the transformer is on no-load, the primary input current is not wholly reactive. The primary input current under no-load conditions has to supply (i) iron losses in the core *i.e.* hysteresis loss and eddy current loss and (ii) a very small amount of copper loss in primary (there being no Cu loss in secondary as it is open). Hence, the no-load primary input current  $I_0$  is not at  $90^\circ$  behind  $V_1$  but lags it by an angle  $\phi_0 < 90^\circ$ . No-load input power

$$W_0 = V_1 I_0 \cos \phi_0$$

Where,

$\cos \phi_0$  is primary power factor under no-load conditions.

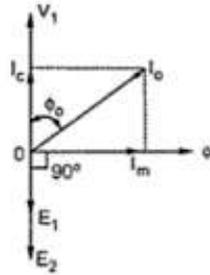
As seen from Fig., primary current  $I_0$  has two components

- i) One in phase with  $V_1$ . This is known as **active** or **working** or **iron** loss component  $I_w$  because it mainly supplies the iron loss plus small quantity of primary Cu loss

$$I_w = I_0 \cos \phi_0$$

- ii) The other component is in quadrature with  $V_1$  and is known as **magnetizing** component  $I_\mu$  because its function is to sustain the alternating flux in the core. It is wattless

$$I_\mu = I_0 \sin \phi_0$$



Obviously,  $I_0$  is the vector sum of  $I_w$  and  $I_\mu$ , hence  $I_0 = \sqrt{I_w^2 + I_\mu^2}$

The following points should be noted carefully:

- i) The no-load primary current  $I_0$  is very small as compared to the full-load primary current. It is about 1 per cent of the full-load current.
- ii) Owing to the fact that the permeability of the core varies with the instantaneous value of the exciting current, the wave of the exciting or magnetising current is not truly sinusoidal. As such it should not be represented by a vector because only sinusoidally varying quantities are represented by rotating vectors. But, in practice, it makes no appreciable difference.
- iii) As  $I_0$  is very small, the no-load primary Cu loss is negligibly small which means that no-load primary input is practically equal to the iron loss in the transformer.
- iv) As it is principally the core-loss which is responsible for shift in the current vector, angle  $\phi_0$  is known as hysteresis angle of advance.

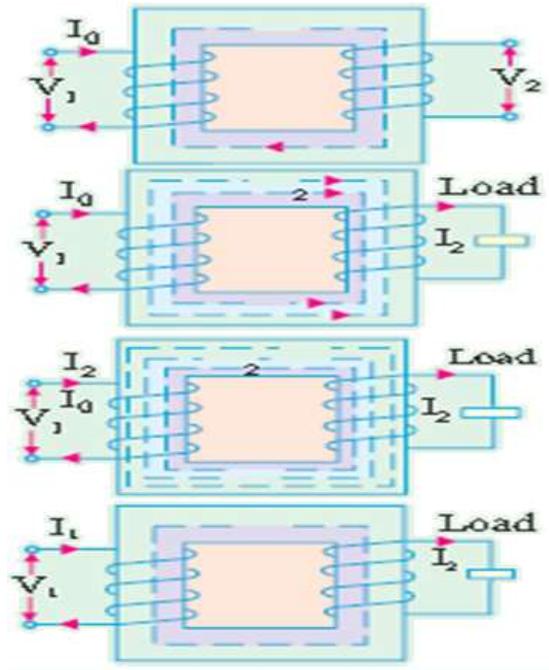
### **Transformers on Load**

When the secondary is loaded, the secondary current  $I_2$  is set up. The magnitude and phase of  $I_2$  with respect to  $V_2$  is determined by the characteristics of the load. Current  $I_2$  is in phase with  $V_2$  if load is non-inductive, it lags if load is inductive and it leads if load is capacitive.

The secondary current sets up its own m.m.f. ( $=N_2I_2$ ) and hence its own flux  $\Phi_2$  which is in opposition to the main primary flux  $\Phi$  which is due to  $I_0$ . The secondary ampere-turns  $N_2I_2$  are known as **demagnetising** amp-turns. The opposing secondary flux  $\Phi_2$  weakens the primary flux  $\Phi$  momentarily, hence primary back e.m.f.  $E_1$  tends to be reduced. For a moment  $V_1$  gains the upper hand over  $E_1$  and hence causes more current to flow in primary.

Let the additional primary current be  $I_2'$ . It is known as **load component of primary current**. This current is anti phase with  $I_2$ . The additional primary m.m.f.  $N_1 I_2'$  sets up its own flux  $\Phi_2'$  which is in opposition to  $\Phi_2$  (but is in the same direction as  $\Phi$ ) and is equal to it in magnitude. Hence, the two cancel each other out. So, we find that the magnetic effects of secondary current  $I_2$  are immediately neutralized by the additional primary current  $I_2'$  which is brought into existence exactly at the same instant as  $I_2$ .

Hence, whatever the load conditions, the net flux passing through the core is approximately the same as at no-load. An important deduction is that due to the constancy of core flux at all loads, the core loss is also practically the same under all load conditions

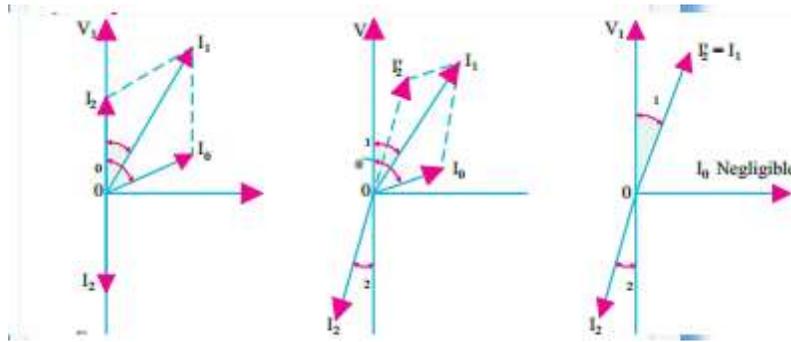


$$N_2 I_1 = N_1 I_2$$

$$I_2 = N_2 I_1 / N_1$$

$$= K I_1$$

Hence, when transformer is on load, the primary winding has two currents in it; one is  $I_0$  and the other is  $I_2'$  which is anti-phase with  $I_2$  and  $K$  times in magnitude. **The total primary current is the vector sum of  $I_0$  and  $I_2'$ .**



The vector diagrams for a load transformer when load is non-inductive and when it is inductive (a similar diagram could be drawn for capacitive load). Voltage transformation ratio of unity is assumed so that primary vectors are equal to the secondary vectors. With reference to Fig. (a),  $I_2$  is secondary current in phase with  $E_2$  (strictly speaking it should be  $V_2$ ). It causes primary current  $I_2'$  which is anti-phase with it and equal to it in magnitude ( $K = 1$ ). Total primary current  $I_1$  is the vector sum of  $I_0$  and  $I_2'$  and lags behind  $V_1$  by an angle  $\phi_1$ .

Fig. (b) vectors are drawn for an inductive load. Here  $I_2$  lags  $E_2$  (actually  $V_2$ ) by  $\phi_2$ . Current  $I_2'$  is again anti phase with  $I_2$  and equal to it in magnitude. As before,  $I_1$  is the vector sum of  $I_2'$  and  $I_0$  and lags behind  $V_1$  by  $\phi_1$ .

It will be observed that  $\phi_1$  is slightly greater than  $\phi_2$ . But if we neglect  $I_0$  as compared to  $I_2'$  as in Fig. (c), then  $\phi_1 = \phi_2$ . Moreover, under this assumption

$$N_1 I_2' = N_2 I_1 = N_1 I_2$$

$$\frac{I_2'}{I_2} = \frac{I_1}{I_2} = \frac{N_2}{N_1} = K$$

It shows that under full-load conditions, the ratio of primary and secondary currents is constant. This important relationship is made the basis of current transformer—a transformer which is used with a low-range ammeter for measuring currents in circuits where the direct connection of the ammeter is impracticable.

**15. Explain the Sumpner's or Back-to-Back Test.**

**Dec- 2016**

**Explain the back to back method of testing for two identical single phase transformer. May 2017**

This test is conducted simultaneously on two identical transformers and provides data for finding the efficiency, regulation and temperature rise. The main advantage of this test is that the transformers are tested under full-load conditions without much expenditure of power. The power required to conduct this test is equal to the losses of the two transformers. It may be noted that two identical transformers are needed to carry out this test.

**Circuit:**

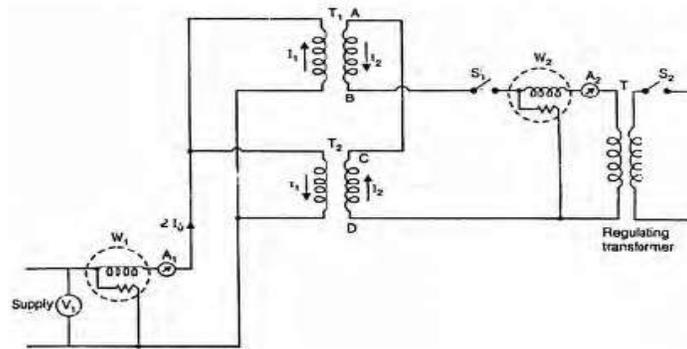
Figure shows the connections for back-to-back test on two identical transformers  $T_1$  and  $T_2$ . The primaries of the two transformers are connected in parallel across the rated voltage

$V_1$  while the two secondary's are connected in phase opposition. Therefore, there will be no circulating current in the loop formed by the secondary's because their induced e.m.f.s are equal and in opposition. There is an auxiliary low-voltage transformer which can be adjusted to give a variable voltage and hence current in the secondary loop circuit. A wattmeter  $W_1$ , an ammeter  $A_1$  and voltmeter  $V_1$  are connected to the input side. A wattmeter  $W_2$  and ammeter  $A_2$  are connected in the secondary circuit.

### Operation

(i) The secondary's of the transformers are in phase opposition. With switch  $S_1$  closed and switch  $S_2$  open (i.e., regulating transformer not in the circuit), there will be no circulating current ( $I_2 = 0$ ) in the secondary loop circuit. It is because the induced e.m.f.s in the secondary's are equal and in opposition. This situation is just like an open-circuit test. Therefore, the current drawn from the supply is  $2 I_0$  where  $I_0$  is the no-load current of each transformer. The reading of wattmeter  $W_1$  will be equal to the core losses of the two transformers.

$W_1 = \text{Core losses of the two transformers}$



Now switch  $S_2$  is also closed and output voltage of the regulating transformer is adjusted till full-load current  $I_2$  flows in the secondary loop circuit. The full-load secondary current will cause full-load current  $I_1 (= KI_2)$  in the primary circuit. The primary current  $I_1$  circulates in the primary winding only and will not pass through  $W_1$ . Note that full-load currents are flowing through the primary and secondary windings. Therefore, reading of wattmeter  $W_2$  will be equal to the full-load copper losses of the two transformers.

$W_2 = \text{Full-load Cu losses of two transformers}$

$\therefore W_1 + W_2 = \text{Total losses of two transformers at full load}$

The following points may be noted:

(a) The wattmeter  $W_1$  gives the core losses of the two transformers while wattmeter  $W_2$  gives the full-load copper losses (or at any other load current  $I_2$ ) of the two transformers. Therefore, power required to conduct this test is equal to the total losses of the two transformers.

(b) Although transformers are not supplying any load, yet full iron loss and full-load copper losses are occurring in them.

(c) There are two voltage sources (supply voltage and regulating transformer) and there is no interference between them. The supply voltage gives only  $2I_0$  while regulating transformer supplies  $I_2$  and hence  $I_1 (= K I_2)$ .

### Advantages

- (i) The power required to carry out the test is small.
- (ii) The transformers are tested under full-load conditions.
- (iii) The iron losses and full-load copper losses are measured simultaneously.
- (iv) The secondary current  $I_2$  can be adjusted to any current value. Therefore, we can find the copper loss at full-load or at any other load.
- (v) The temperature rise of the transformers can be noted.

## PROBLEMS

- 1) A 30 step down transformer is connected to 6.6 KV mains and takes 10 amps. calculate the secondary line voltage and line current for the i)  $\Delta/\Delta$  ii)  $Y/Y$  iii)  $\Delta/Y$  iv)  $Y/\Delta$  connection the ratio of turns per phase is 12 and neglect no load losses. (May 2012)

### Solution

#### $\Delta - \Delta$

$$\begin{aligned} \text{Primary phase voltage} &= 6600 \text{ v} \\ \text{Primary phase current} &= \frac{10}{\sqrt{3}} = 5.773 \text{ A} \\ \text{Secondary line voltage} &= \frac{6600}{12} = 550 \text{ V} \\ \text{Secondary line current} &= \sqrt{3} \times 5.733 \times 12 = 120 \text{ A} \end{aligned}$$

#### $Y - Y$

$$\begin{aligned} \text{Primary phase voltage} &= \frac{6600}{\sqrt{3}} = 3810.6 \text{ V} \\ \text{Primary Phase current} &= 10 \text{ A} \\ \text{Secondary Line voltage} &= \sqrt{3} * \frac{6600}{\sqrt{3}} \times \frac{1}{12} = 550 \text{ v} \\ \text{Secondary Line current} &= 10 \times 12 = 120 \text{ A} \end{aligned}$$

#### $\Delta - Y$

$$\begin{aligned} \text{Primary Phase voltage} &= 6600 \text{ v} \\ \text{Primary phase current} &= \frac{10}{\sqrt{3}} \text{ A} \\ \text{Secondary line voltage} &= \sqrt{3} \times 6600 \times \frac{1}{12} = 952.6 \text{ V} \\ \text{Secondary line current} &= \frac{10}{\sqrt{3}} \times 12 = 69.3 \text{ A} \end{aligned}$$

#### $Y - \Delta$

$$\text{Primary phase voltage} = \frac{6600}{\sqrt{3}} = 3810.6 \text{ V}$$

$$\begin{aligned} \text{Primary phase current} &= 10 \text{ A} \\ \text{Secondary line voltage} &= \frac{6600}{\sqrt{3}} \times \frac{1}{12} = 317.5 \text{ V} \\ \text{Secondary line current} &= \sqrt{3} \times 10 \times 12 = 208 \text{ A} \end{aligned}$$

- 2) Calculate the efficiency at half, full load of a 100 KVA transformer for p.f of unity and 0.8. The copper loss is 1000 w at full load and iron loss is 1000 w, KVA =1000 W(May - 2012),NOV-DEC 2015

**Given:** 100 KVA, p.f=0.8,  $P_i = 1000 \text{ w}$ ;  $P_c = 1000 \text{ w}$

**Formula used:**  $\% \eta = \frac{(KVA \times 10^3) \cos \phi}{(KVA \times 10^3) \cos \phi + P_{cu} + P_i} \times 100$

**Solution:**

$$P_i = 1000 \text{ w}; P_c = 1000 \text{ w}$$

**At 0.8 p.f**

$$\begin{aligned} \% \eta &= \frac{(KVA \times 10^3) \cos \phi}{(KVA \times 10^3) \cos \phi + P_{cu} + P_i} \times 100 \\ &= \frac{100 \times 10^3 \times 0.8}{100 \times 10^3 \times 0.8 + 1000 + 1000} \times 100 \\ &= 97.56\% \end{aligned}$$

**At unity p.f**

$$\begin{aligned} &= \frac{(KVA \times 10^3)}{(KVA \times 10^3) + P_{cu} + P_i} \times 100 \\ &= \frac{100 \times 10^3}{100 \times 10^3 + 1000 + 1000} \times 100 \\ &= 98.03\% \end{aligned}$$

- 3) A 40 KVA transformer has iron loss of 450 w and full load copper loss of 850 w , if the power factor of the load is 0.8 lagging , calculate i)full load efficiency ii) the load at which max n and iii) max efficiency (Dec 2012)

**Given:** KVA =40;  $P_i = 450 \text{ w}$ ;  $P_{cu} \text{ (F.L)} = 850 \text{ w}$ , pf =0.8

**Formula used:**  $\% \eta = \frac{(VArating) \cos \phi}{(VArating) \cos \phi + P_i + P_{cu} \text{ (F.L)}} \times 100$

**Solution**

$$KVA = 40; P_i = 450 \text{ w}; P_{cu} \text{ (F.L)} = 850 \text{ w}, \text{ pf} = 0.8$$

$$\begin{aligned} \% \eta &= \frac{(VArating) \cos \phi}{(VArating) \cos \phi + P_i + P_{cu} \text{ (F.L)}} \times 100 \\ &= \frac{40 \times 0.8 \times 10^3}{40 \times 10^3 \times 0.8 + 450 + 850} \times 100 \\ &= 96.09\% \end{aligned}$$

Load for  $\eta$  max

$$KVA \text{ at } \eta_{\max} = KVA \text{ rating} \times \sqrt{\frac{P_i}{P_{cu} \text{ (F.L)}}}$$

$$= 40 \times \sqrt{\frac{450}{850}}$$

$$= 29.1 \text{ KVA}$$

$$\% \eta_{\max} = \frac{\text{KVA at } \eta_{\max} \cos \phi}{\text{KVA at } \eta_{\max} \cos \phi + P_i + P_c}$$

$$= \frac{29.1 \times 10^3 \times 0.8}{29.1 \times 10^3 \times 0.8 + 900}$$

$$= 96.26 \%$$

4) The following data were obtained on a 20 KVA, 50 Hz, 2000/200 V distribution transformer

	Voltage V	Current A	Power w
OC test with HV open circuited	200	4	120
SC test with LV short circuited	60	10	300

Draw the approximate equivalent circuit of the transformer referred to the H.V and L.V sides respectively. (May 2013) Dec - 2016

Given: 20 KVA, 50 Hz, 2000/200 V

Solution:

From O.C test:

$$V_0 = 200 \text{ V}, I_0 = 4 \text{ A}, W_0 = 120 \text{ w}$$

$$\cos \phi = \frac{W_0}{V_0 I_0} = \frac{120}{200 \times 4} = 0.15 \text{ Lag}$$

$$K = \frac{V_2}{V_1} = \frac{200}{2000} = 0.1$$

$$I_c = I_0 \cos \phi = 0.6 \text{ A}$$

$$I_m = I_0 \sin \phi = 3.9547 \text{ A}$$

$$R_0 = \frac{V_1}{I_c} = \frac{2000}{0.6} = 3.33 \text{ K}\Omega$$

$$X_0 = \frac{V_1}{I_m} = \frac{2000}{3.9547} = 0.5057 \text{ K}\Omega$$

From SC test

$$V_{SC} = 60 \text{ V}, I_{SC} = 10 \text{ A}, W_{SC} = 300 \text{ w}$$

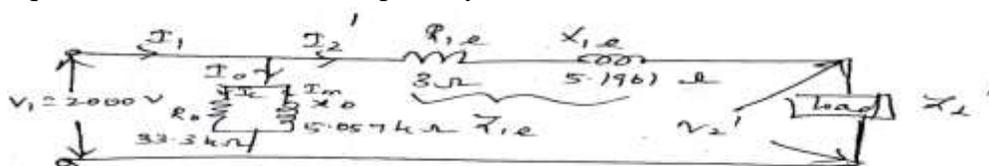
As L.V is short circuited, readings are on primary winding side

$$Z_{1e} = \frac{V_{SC}}{I_{SC}} = 6 \Omega \quad R_{1e} = \frac{W_{SC}}{I_{SC}^2} = 3 \Omega$$

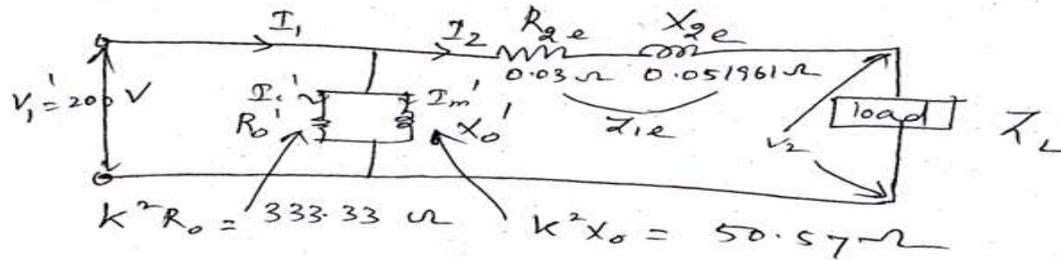
$$X_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2} = 5.1961 \Omega$$

$$R_{2e} = k^2 R_{1e} = 0.03 \Omega, \quad X_{2e} = k^2 X_{1e} = 0.051961 \Omega$$

Equivalent circuit referred to primary (H.V) side is



Equivalent circuit referred to secondary (L.V) side is



5) a 20 KVA , 2500/500 V , single phase transformer has the following parameter

HV winding :  $r_1 = 8 \Omega$  and  $x_1 = 17 \Omega$

LV winding :  $r_2 = 0.3 \Omega$  and  $x_2 = 0.7 \Omega$

Find the voltage regulation and the secondary terminal voltage at full load for a p.f of 0.8 lagging and 0.8 leading. The primary voltage is held constant at 2500 V (May 2011)

Given: 20 KVA , 2500/500 V,  $r_1 = 8 \Omega$  and  $x_1 = 17 \Omega$ ,  $r_2 = 0.3 \Omega$  and  $x_2 = 0.7 \Omega$ ,  $pf = 0.8$

Formula used:  $\%R = \frac{I_2(F.L)(R_{2e} \cos \phi + X_{2e} \sin \phi)}{V_2} \times 100$

**Solution:**

$$\text{Rating} = 20 \text{ KVA}, V_1 = 2500 \text{ V}; V_2 = 500 \text{ V}, K = \frac{V_2}{V_1} = 0.2$$

$$R_{2e} = r_2 + k^2 r_1 = 0.3 + (0.2)^2 \times 8 = 0.62 \Omega$$

$$X_{2e} = X_2 + k^2 X_1 = 0.7 + (0.2)^2 \times 17 = 1.38 \Omega$$

$$I_2 (F.L) = \frac{VA}{V_2} = \frac{20 \times 10^3}{500} = 40 \text{ A}$$

i)  $\cos \phi = 0.8 \text{ lag}$

$$\%R = \frac{I_2(F.L)(R_{2e} \cos \phi + X_{2e} \sin \phi)}{V_2} \times 100 = 10.592 \%$$

ii)  $\cos \phi = 0.8 \text{ lead}$

$$\%R = \frac{I_2(F.L)(R_{2e} \cos \phi - X_{2e} \sin \phi)}{V_2} \times 100$$

$$= -2.656 \%$$

$$V_2 = 500 - I_2 (F.L)(R_{2e} \cos \phi - X_{2e} \sin \phi)$$

$$= 513.28 \text{ V}$$

6) A transformer has its maximum efficiency of 0.98 at 15 KVA , at unity p.f .during the day it is loaded as follows ,

**For 12 hours: 2 k.w at 0.5 p.f lag**

**For 6 hours: 12 k.w at 0.8 p.f lag**

**For 6 hours: 18 k.w at 0.9 p.f lag**

**For 2 hours: no load efficiency Find All day efficiency?(Dec 2010)**

**Solution**

$$\eta_{max} = 0.98 \text{ at } 15 \text{ kVA}, \cos \phi = 1$$

At  $\eta_{max}$  copper loss are equal to iron losses

$$\text{Total losses} = P_i + P_{cu} = 2P_i$$

$$\text{as } P_i = P_{cu}$$

$$\% \eta_{max} = \frac{KVA \cdot \cos \phi}{kva \cos \phi + 2P_i} \times 100$$

$$0.98 = \frac{(15 \cdot 10^3 \times 1)}{(15 \cdot 10^3 \cdot 1) + 2P_i}$$

$$P_i = 153.0612 \text{ w}$$

$$P_{cu} = 153.0612 \text{ wat } 15 \text{ KVA}$$

Energy output in a day from given load =  $12 \times 2 + 6 \times 12 + 6 \times 18 = 204 \text{ kwh}$

Iron losses are constant for the entire day, so total energy spend due to Iron loss for 24 hours is

$$P_i = 153.0612 \times 24 = 3673.4688 \text{ wh}$$

i) Load at 2 kw at  $\cos \phi = 0.5$

$$\text{KVA supplied} = \frac{KW}{\cos \phi} = \frac{2}{0.5} = 4$$

$$n = \frac{\text{load KVA}}{\text{KVA for } \eta_{max}} = \frac{4}{15} = 0.266$$

copper losses are *ato* square of KVA ratio

$$\text{load 1 copper loss} = n^2 \times p_{cu} \text{ at } 15 \text{ KVA}$$

$$= (0.266)^2 * 153.0612$$

$$= 10.8543 \text{ w}$$

Energy spend =  $10.88 \times 12 = 130.61 \text{ wh}$

ii) Load at 12 KW at  $\cos \phi = 0.8$  lag

$$\text{KVA supplied} = \frac{KW}{\cos \phi} = \frac{12}{0.8} = 15$$

$$P_{cu} \text{ at } 15 \text{ KVA} = 153.0612 \text{ w}$$

$$\begin{aligned} \text{Energy spent} &= 153.0612 \times 6 \text{ hours} \\ &= 918.3672 \text{ wh} \end{aligned}$$

iii) Load at 18 kw at  $\cos\phi = 0.9$  lag

$$\text{KVA supplied} = \frac{\text{KW}}{\cos\phi} = \frac{18}{0.9} = 20$$

$$n = \frac{\text{loadKVA}}{\text{KVA for } \eta_{\max}} = \frac{20}{15} = 1.333$$

$$\begin{aligned} \text{Load 3 copper loss} &= n^2 P_{cu} \text{ at } 15 \text{ KVA} \\ &= 1.333^2 \times 153.06 = 272.108 \text{ w} \end{aligned}$$

$$\text{Energy spent} = 272.1088 \times 6 = 1632.1088 \text{ w}$$

iv) No load hence copper losses neglected

v) Total energy spend = Energy spent due to iron loss + energy spend due to copper loss

$$= 3673.4688 + 130.6116 + 918.3672 + 1632.65$$

$$= 6355.1004 \text{ wh}$$

$$\text{Total output} = 204 \text{ Kwh} = 202000 \text{ Wh}$$

$$\begin{aligned} \text{All days } \eta &= \frac{\text{total output for 24 hours}}{\text{total output} + \text{energy spend}} \\ &= \frac{204000}{204000 + 6355.1004} \times 100 \end{aligned}$$

$$\% \eta \text{ all} = 96.978 \%$$

7) A 100 KVA, 6600 v / 330 v 50hz single phase transformer took 10 A and 436 w at 100 v in a short circuit test , the figures referring to the high voltage side. Calculate the voltage to be applied to the high voltage side on full load at power factor 0.8 lagging when the secondary terminal voltage is 330 v.

Given: 100 KVA, 6600 v / 330 v 50hz, I=10, P=436W,

Formula used:  $X_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2}$ ,  $\text{Voltage drop} = I_1(\text{F.L.}) (R_{1e} \cos \phi + X_{1e} \sin \phi)$

### Solution

Rating = 100 KVA, 6600V / 330 V , 50 Hz ,  $\cos\phi = 0.8$ ,  $V_{SC} = 100\text{V}$ ,

$I_{SC} = 10 \text{ A}$ ,  $W_{SC} = 436 \text{ W}$  on HV side

$$R_{1e} = \frac{436}{I_{SC}^2} = \frac{436}{10^2} = 4.36 \Omega$$

$$Z_{1e} = \frac{V_{SC}}{I_{SC}} = \frac{100}{10} = 10 \Omega$$

$$X_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2}$$

$$= 8.99 \Omega$$

$$I_1(\text{F.L}) = \frac{VA}{V_1} = \frac{100 \times 10^3}{6600} = 15.15 \text{ A}$$

$$\text{Voltagedrop} = I_1(\text{F.L}) (R_{1e} \cos \phi + X_{1e} \sin \phi)$$

$$= 15.15 (4.36 \times 0.8 + 9 \times 0.6)$$

$$= 134.66 \text{ V}$$

To main  $V_2 = 330 \text{ V}$  the primary must be supply the drop

$$V_1 = 6600 + \text{drop} = 6600 + 134.66$$

$$V_1 = 6734.66 \text{ V}$$

8) Obtain the equivalent circuit of a 200/400 v, 50 hz, 1-phase transformer from the following test data :

O.C test : 200 v ,0.7 A ,70 W –on L.V side

S.C. test : 15v ,10 A ,85 W –on H.V side

Calculate the secondary voltage when delivering 5 KW at 0.8 p.f lagging, the primary voltage being 200 V.(May 2013)

Given:

O.C test : 200 v ,0.7 A ,70 W –on L.V side

S.C. test : 15v ,10 A ,85 W –on H.V side

$$\text{Formula used: } X_{02} = \sqrt{Z_{02}^2 - R_{02}^2}$$

$$= I_2 (R_{02} \cos \phi_2 + X_{02} \sin \phi_2)$$

### Solution

**O.C test(L.V side)**

Primary voltage  $V_1 = 200 \text{ V}$

No load input current  $I_0 = 0.7 \text{ A}$

No load input power  $P_0 = 70 \text{ W}$

$$P_0 = V_1 I_0 \cos \phi_0$$

$$10 = 200 \times 0.7 \times \cos \phi_0$$

$$\cos \phi_0 = 0.5$$

$$\phi_0 = \cos^{-1} 0.5 = 60^\circ$$

$$\sin \phi_0 = 0.8660$$

Watt full component (working component)

$$I_W = I_0 \cos \phi_0$$

$$= 0.7 \times 0.5 = 0.35 \text{ A}$$

Resistance representing the core loss

$$R_0 = \frac{V_1}{I_w} = \frac{200}{0.35} = 571.4 \Omega$$

Watt less component (magnetizing component )

$$I_\mu = I_0 \sin \phi_0 = 0.7 \times 0.866 \\ = 0.6062 \text{ A}$$

Magnetizing reactance

$$X_0 = \frac{V_1}{I_\mu} = \frac{200}{0.6062} = 329.92 \Omega$$

**S.C test (h.v side)**

Short circuit voltage  $V_{SC} = 15\text{V}$

Short circuit current  $I_{SC} = 10 \text{ A}$

Losses  $w_{sc} = 85 \text{ w}$

Impedance of transformer referred to h.v side

$$Z_{02} = \frac{V_{SC}}{I_{SC}} = \frac{15}{10} = 1.5 \Omega$$

$$R_{02} = \frac{W_{SC}}{I_{SC}^2} = \frac{85}{10^2} = \frac{85}{100} = 0.85 \Omega$$

$$X_{02} = \sqrt{Z_{02}^2 - R_{02}^2}$$

$$= \sqrt{1.5^2 - (0.85)^2}$$

$$X_{02} = 1.235 \Omega$$

Given power  $KW = 5$

Convert KW to KVA  $= \frac{5}{0.8} = 6.25 \text{ VA}$

$$I_2 = \frac{VA}{v_2} = \frac{6.25 \times 10^3}{400} = 15.625 \text{ A}$$

secondary voltage when delivering 5 Kw at 0.8 p.f

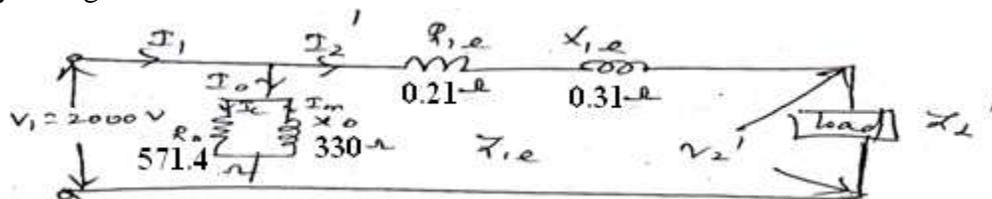
Where  $\cos \phi_2 = 0.8$ , therefore  $\sin \phi_2 = 0.6$

$$= I_2 (R_{02} \cos \phi_2 + X_{02} \sin \phi_2)$$

$$= 15.625 (0.85 * 0.8 + 1.235 * 0.6)$$

Drop across load  $= 22.2 \text{ V}$

Secondary voltage  $= 400 - 22.2 = 377.82 \text{ V}$



Equivalent circuit referred to primary

- 9) A 500 KVA transformer has 95% efficiency at full load and also at 60% of full load both at upf.  
 i) Separate out the transformer losses.  
 ii) Determine the transformer efficiency at 75% full load, Upf (Dec 2013)

Given:

$$P = 500 \text{ KVA}$$

$$\text{Efficiency at full load, upf} = \eta_{F.L} = 95\%$$

$$\text{Efficiency at 60\% full load, upf} = \eta_{0.6} = 95\%$$

Formula used:

$$\eta_{F.L} = \frac{P_{out}}{P_{out} + P_{cu} + P_i} \times 100$$

### Solution

At full load condition and upf

$$\eta_{F.L} = \frac{P_{out}}{P_{out} + P_{cu} + P_i} \times 100$$

$$0.95 = \frac{500 \times 1000}{500 \times 1000 + P_{cu} + P_i}$$

$$P_{cu} + P_i = 26315.79 \text{-----(1)}$$

At 60% full load condition and Upf,

$$\eta_{F.L} = \frac{nP_{out}}{nP_{out} + n^2P_{cu} + P_i} \times 100$$

$$0.95 = \frac{0.6 \times 500 \times 1000}{0.6 \times 500 \times 1000 + 0.6^2 P_{cu} + P_i} \times 100$$

$$0.6^2 P_{cu} + P_i = 15789.47 \text{-----(2)}$$

Solving (1) and (2)

$$P_i = 9868.415 \text{ watts}; \quad P_{cu} = 16447.375 \text{ watts}$$

Efficiency at 75% of full load and Upf, n=0.75

$$\eta_{0.75} = \frac{nP_{out}}{nP_{out} + n^2P_{cu} + P_i} \times 100$$

$$= \frac{0.75 \times 500 \times 1000}{0.75 \times 500 \times 1000 + 0.75^2 \times 16447.375 + 9868.415} \times 100; \quad \eta_{0.75} = 95.24\%$$

- 10) A 3phase step down transformer is connected to 11 KV mains and takes 10 amps.  
 calculate the secondary line voltage and line current for the i)  $\Delta/\Delta$  ii)  $Y/Y$  iii)  $\Delta/Y$  iv)  
 $Y/\Delta$  connection the ratio of turns per phase is 8 and neglect no load losses. (Dec 2014)

**Solution:**

k = 1/10; The given Voltage and current values are line values.

### $\Delta - \Delta$

$$\text{Primary phase voltage} = 11 \text{ KV}$$

$$\text{Primary phase current} = \frac{10}{\sqrt{3}} = 5.773 \text{ A}$$

$$\text{Secondary line voltage} = \frac{11000}{8} = 1375 \text{ V}$$

$$\text{Secondary line current} = \sqrt{3} \times 5.773 \times 8 = 79.5 \text{ A}$$

### $Y - Y$

$$\text{Primary phase voltage} = \frac{11000}{\sqrt{3}} = 6351 \text{ v}$$

$$\text{Primary Phase current} = 10 \text{ A}$$

$$\text{Secondary Line voltage} = \sqrt{3} \times \frac{11000}{\sqrt{3}} \times \frac{1}{8} = 1375 \text{ v}$$

$$\text{Secondary Line current} = 10 \times 8 = 80 \text{ A}$$

### $\Delta - Y$

$$\text{Primary Phase voltage} = 11000 \text{ v}$$

$$\text{Primary phase current} = \frac{10}{\sqrt{3}} = 5.773 \text{ A}$$

$$\text{Secondary line voltage} = \sqrt{3} \times 11000 \times \frac{1}{8} = 2381.5 \text{ V}$$

$$\text{Secondary line current} = \frac{10}{\sqrt{3}} \times 8 = 45.86 \text{ A}$$

### $Y - \Delta$

$$\text{Primary phase voltage} = \frac{11000}{\sqrt{3}} = 6351 \text{ V}$$

$$\text{Primary phase current} = 10 \text{ A}$$

$$\text{Secondary line voltage} = \frac{11000}{\sqrt{3}} \times \frac{1}{8} = 793.9 \text{ V}$$

$$\text{Secondary line current} = \sqrt{3} \times 10 \times 8 = 138.56 \text{ A}$$

---

**11) A single phase transformer has 180 turns and 90 turns respectively in its secondary and primary windings. The respective resistances are 0.233 and 0.067. Calculate the equivalent resistance of**

- 1. The primary in terms of the secondary winding.**
- 2. The secondary in terms of the primary winding, and**
- 3. The total resistance of the transformer in terms of the primary. (May 2014)**

### **Solution**

$$N_2 = 180, N_{031} = 90, R_2 = 0.233 \text{ } \Omega \text{ and } R_1 = 0.067 \text{ } \Omega; \quad K = N_2/N_1 = 2$$

As secondary turns are more, secondary winding side is high voltage. Hence secondary the resistance values are higher

$$1. R_1' = K^2 R_1 = 2^2 \times 0.067 = 0.268 \text{ } \Omega$$

$$2. R_2' = R_2/K^2 = 0.233/2^2 = 0.05825 \text{ } \Omega \text{ ( Referred to Secondary means } R_{2e} = R_2 + R_1')$$

$$3. R_{1e} = R_1 + R_2' = 0.067 + 0.05825 = 0.12525 \text{ } \Omega.$$

---

**12) The parameters of approximate equivalent circuit of a 4 KVA, 200/400V, 50 Hz, single phase transformer are  $R_p = 0.15 \text{ } \Omega$ ,  $X_p = 0.37 \text{ } \Omega$ ,  $R_0 = 600 \text{ } \Omega$ ,  $X_m = 300 \text{ } \Omega$ . When a rated**

voltage of 200V is applied to the primary, a current of 10A at lagging power factor of 0.8 flows in the secondary winding. Cal

1. The current in the primary,  $I_p$

2. The terminal voltage at the secondary side. (May-2014, 2017 (Part – C Question))

Given:  $R_p = R_{01} = 0.15\Omega$ ;  $X_p = X_{01} = 0.37\Omega$ ;  $R_0 = 600\Omega$ ;  $X_m = X_0 = 300\Omega$

Formula used:  $I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi$

**Solution:**

$$I_\mu = \frac{V_1}{X_0} = \frac{200}{300} = 0.666 \text{ A}$$

$$I_w = \frac{V_1}{R_0} = \frac{200}{600} = 0.333 \text{ A}$$

$$I_0 = \sqrt{I_\mu^2 + I_w^2} = 0.745 \text{ A}$$

From figure,  $\tan \theta = \frac{I_w}{I_\mu} = 0.$

$$\theta = 26.6^\circ$$

$$\phi_o = 90 - 26.6 = 63.4^\circ$$

it is the angle between  $I_0$  and  $V_1$

$$k = \frac{400}{200} = 2$$

$$I_2 = KI_1 = 2 \times 10 = 20 \text{ A}$$

Primary current  $I_1 = \sqrt{I_0^2 + I_2^2} \cos \theta = 20.67 \text{ A}$

$$R_{02} = k^2 R_{01} = 2^2 \times 0.15 = 0.6\Omega$$

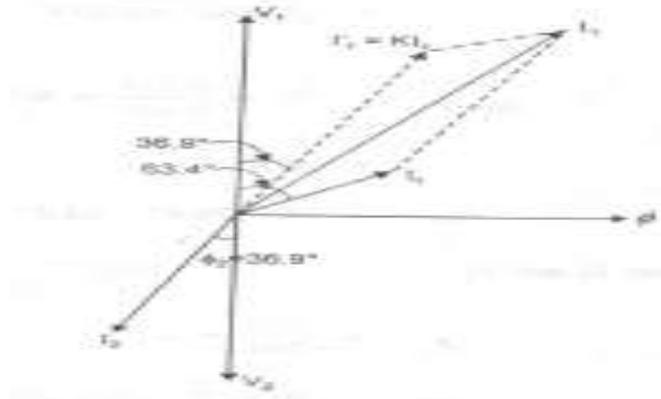
$$X_{02} = k^2 X_{01} = 2^2 \times 0.37 = 1.48\Omega$$

Approximate voltage drop =  $I_2 R_{02} \cos\phi + I_2 X_{02} \sin\phi$

$$= 10 \times 0.6 \times 0.8 + 10 \times 0.148 \times 0.6$$

$$= 13.7\text{V}$$

Secondary terminal voltage =  $400 - 13.7 = 386.3 \text{ V}$



**13) What is meant by inrush current in transformer? Specify the nature of inrush currents and its problem during transformer charging.(Apr-May 2015)(6 Marks)**

When transformer is first turned on, large current flows that exceed the steady-state current value. This current is called inrush current.

**Nature of inrush currents and its problem during transformer charging:**

When the transformer is charged, a transient current known as magnetizing inrush current (of magnitude as high as 10 times of rated current) flows in to the system. This is due to the nonlinear relationship of flux and magnetizing current as transformer core is in saturation mode. It is not only the high magnitude of inrush current but its composition (rich in DC component and harmonics) and duration are also the cause of concern which severely affects the stability of the system.

Factors contributing to the magnitude and duration of inrush current are Magnitude of residual flux in transformer core, on linear magnetizing characteristic of transformer core, Magnitude of source voltage at the switching instant and impedance and short circuit power of the source including VAR abortion capacity.

The key adverse effects include

- Mechanical and electrical stresses in windings
- Harmonic resonant over voltages
- Mall operation of protective relays
- Voltage dips.

**14) Specify the condition for parallel operation of transformer. Also explain the effect of load sharing due to impedance variation between transformer during parallel operation.(Apr-May 2015)(6 Marks)**

The condition for parallel operations of Transformer:

**1. For single phase Transformer:**

- Same polarity of transformer

- Same voltage ratio
- 2. For single phase Transformer:**
  - Same polarity of transformer
  - Same voltage ratio
  - Zero relative phase displacement
  - Same phase sequence

**Effect of load sharing due to impedance variation between transformers during parallel operation:**

- i. Same percentage impedance
- ii. X/R ratio

**Same percentage impedance:**

If the two transformers connected in parallel with similar per-unit impedances they will mostly share the load in the ratio of their KVA ratings. Here load is mostly equal because it is possible to have two transformers with equal per unit impedances but different X/R ratios. In this case the line current will be less than the sum of the transformer currents and the combined capacity will be reduced accordingly.

**X/R ratio:**

A difference in the ratio of the reactance value to resistance value of the per unit impedance result in a different phase angle of the current carried by the two paralleled transformers; one transformer will be working with a higher power factor and the other with a lower power factor than that of the combined output. Hence the real power will not be proportionally shared by the transformers.

---

**15) A 500 KVA transformer has a core loss of 2200 watts and a full load copper loss of 7500 watts. If the power factor of the load is 0.90 lagging. Calculate the full load efficiency and the KVA Load at which maximum efficiency occurs.(Apr-May 2015)**

**Given data:**

**KVA:** 500 KVA; **Core Loss ( $P_i$ ):** 2200 watts; **Copper Loss ( $P_c$ ):**7500 watts; **P.F:** 0.9 (Lag)

**Find:**

- i) Full Load efficiency
- ii) KVA load at which max efficiency occurs.

**Solutions:**

- i) **Full load efficiency:**

$$\eta_{\text{Full load}} = \frac{\text{KVA} \times \text{P.F}}{(\text{KVA} \times \text{P.F}) + P_i + P_{\text{cu}}}$$

$$= \frac{500 \times 10^3 \times 0.9}{(500 \times 10^3 \times 0.9) + 2200 + 7500}$$

$$\eta_{\text{Full load}} = 97.88\%$$

iii) KVA load at which max efficiency occurs.

$$= \text{Full load KVA} \times \sqrt{\frac{\text{core Loss}(P_i)}{\text{Copper Loss}(P_{\text{cu}})}}$$

$$= 500 \times \sqrt{\frac{2200}{7500}}$$

KVA load at which max efficiency occurs = 270.8 KVA

16) A 100 KVA, 3300V/240V.50 Hz, single phase transformer has 990 turns on the primary. Calculate the number of turns on secondary and the approximate value of primary and secondary full load currents? ( May-June 2015)(10 marks)

**Given Data:**

**Rating:** 100 KVA;  $V_1=3300$  V;  $V_2=240$  V.  $F=50$  Hz . $N_1=990$  Turns;

**Find:**  $N_2, I_1,$  and  $I_2$

**Solutions:**

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} (\text{Ideal})$$

$$\frac{3300}{240} = \frac{990}{N_2}$$

$$N_2 = 72 \text{ turns}$$

$$I_1 = \frac{\text{KVA}}{V_1} = \frac{100 \times 10^3}{3300} = 30.3 \text{ A}$$

$$I_2 = \frac{\text{KVA}}{V_2} = \frac{100 \times 10^3}{240} = 416.6 \text{ A}$$

17. Draw the equivalent circuit of a single phase 1100/220V transformer on which the following results were obtained.

(i) 1100V, 0.5A, 55W on primary side, secondary being open circuited

(ii) 10V, 80A, 400W on LV side, high voltage side being short circuited

Calculate the voltage regulation and efficiency for the above transformer when supplying 100A at 0.8 pf lagging. (May 2017)

**Given:**

(i) 1100V, 0.5A, 55W

(ii) 10V, 80A, 400W

**Formula used:** Regulation (%R) =  $\frac{I_2(FL)(R_{2e}\cos\phi + X_{2e}\sin\phi)}{V_2} \times 100$

**Solution:**

**From short circuit test**  $V_{sc} = 10V$ ,  $I_{sc} = 80 A$ ,  $W_{sc} = 400 w$

$$Z_{2e} = \frac{V_{sc}}{I_{sc}} = \frac{10}{80} = 0.125 \text{ w} \quad R_{2e} = \frac{W_{sc}}{I_{sc}^2} = \frac{400}{80^2} = 0.0625 \Omega$$

$$X_{2e} = \sqrt{Z_{2e}^2 - R_{2e}^2} = \sqrt{0.125^2 - (0.0625)^2} = 0.108 \Omega$$

Load current  $I_2 = 100A$ ,  $\cos\phi_2 = 0.8$ ,  $\sin\phi_2 = 0.6$ ,  $V_2 = 220 v$

$$\text{Regulation } (\%R) = \frac{I_2(FL)(R_{2e}\cos\phi + X_{2e}\sin\phi)}{V_2} \times 100$$

$$\%R = \frac{100 \times 0.0625 \times 0.8 + 100 \times 0.108 \times 0.6}{220} \times 100$$

$$\% R = 5.218 \%$$

Core loss from open circuit  $P_i = 55 w$

Full load copper loss from short circuit test  $P_{cuff} = 400 w$ ,  $n = 1$

$$\begin{aligned} \% \text{ efficiency} &= \frac{n V_2 I_2 \cos\phi_2}{n V_2 I_2 \cos\phi_2 + P_i + n^2 P_{cu}} \times 100 \\ &= \frac{200 \times 100 \times 0.8}{200 \times 100 \times 0.8 + 55 + 400} \times 100 \end{aligned}$$

$$\text{Efficiency} = 97.48 \%$$

18. The EMF per turn of a single phase 6.6KV/440V, 50HZ transformer is approximately 10V. calculate the number of turns in the HV and LV windings and the net cross sectional area of the core for a maximum flux density of 1.6T. (May-2018)

**Given:** 6.6KV/440V, 50HZ, 10V, flux density of 1.6T

Formula used:  $\phi_m = B_m \times a$ ,  $E_1 = 4.44f\phi_m N_1$

**Solution:**

$$\text{We know that } E_1 = N_1 \times \frac{emf}{turn}$$

$$6.6 \times 10^{-3} = N_1 \times 10$$

$$N_1 = \frac{6.6 \times 10^{-3}}{10} = 600 \text{ turns}$$

We know that  $E_2 = N_2 \times \frac{emf}{turn}$

$$\frac{440}{10} = N_2 = 44 \text{ turns}$$

$$\phi_m = B_m \times a$$

$$E_1 = 4.44f\phi_m N_1$$

$$6.6 \times 10^{-3} = 4.44 \times 50 \times \phi_m \times 660$$

$$\phi_m = 0.0450$$

$$B_m \times a = 0.0450$$

$$a = \frac{0.0450}{1.6} = 0.0281 \text{ m}^2$$

**19.A 11000/230V, 150KVA, 1-hase, 50 HZ transformer has core loss of 1.4 kw and F.L. Copper loss of 1.6 KW. Determine (i) The KVA load for maximum efficiency and the value of maximum efficiency at unity p.f.(ii) The efficiency at 0.8 pf leading.(May-2018)**

**Given:** 11000/230V, 150KVA, 1-hase, 50 HZ, core loss of 1.4 kw, Copper loss of 1.6 KW

**Formula used:**  $F. L. KVA \times \sqrt{\frac{Iron \text{ loss}}{F.L.Cu \text{ loss}}}$ ,  $\eta = \frac{x \times VA \text{ rating} \times pf}{x \times VA \text{ rating} \times pf + x^2 P_c} \times 100$

**Solution:**

$$\begin{aligned} \text{load KVA corresponding to max efficiency is} &= F. L. KVA \times \sqrt{\frac{Iron \text{ loss}}{F. L. Cu \text{ loss}}} \\ &= 250 \times \sqrt{\frac{1.6}{1.4}} = 160 \text{ KVA} \end{aligned}$$

At maximum efficiency

$$\text{Cu loss} = \text{Iron Loss}$$

$$\therefore \text{Total loss} = 1.4 + 1.4 = 2.8 \text{ KW}$$

$$\text{Out put power} = 160 \times 1 = 160 \text{ KW}$$

$$(i) \therefore \eta = \frac{160}{160 + 2.8} \times 100 = 98.2\%$$

$$\begin{aligned} (ii) \eta &= \frac{x \times VA \text{ rating} \times pf}{x \times VA \text{ rating} \times pf + x^2 P_c} \times 100 \\ &= \frac{1 \times 160 \times 10^3 \times 0.8}{1 \times 160 \times 10^3 \times 0.8 + 1 \times 1.4 \times 10^3} \times 100 \\ &= \frac{128000}{128000 + 3000} \times 100 \\ &= \frac{128000}{131000} \times 100 \end{aligned}$$

$$\eta = 97.70\%$$

**20. A 20 KVA,2000/200V,50Hz,single-phase transformer has the following parameters:**

$$r_1 = 2.8\Omega, r_2 = 0.02\Omega, X_{11} = 4.2\Omega, X_{12} = 0.6\Omega$$

**Calculate :**

**1)Equivalent resistance, Leakage reactance an impedance referred to HV side.**

**2) Equivalent resistance, Leakage reactance an impedance referred to LV side.**

**3)Full load copper loss.**

Solution:

$$(i)R_{01} = r_1 + \frac{r_2}{k^2}$$

$$k = \frac{E_2}{E_1} = 0.1$$

$$R_{01} = 2.8 + \frac{0.02}{0.1^2} = 4.8\Omega$$

$$X_{01} = X_1 + \frac{X_2}{k^2}$$

$$= 4.2 + \frac{0.6}{0.1^2} = 64.2\Omega$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{4.8^2 + 64.2^2} = 64.379\Omega$$

$$(i)R_{02} = r_2 + r_1k^2$$

$$= 0.02 + 2.8(0.1)^2 = 0.048\Omega$$

$$X_{02} = X_2 + X_1k^2 = 0.6 + 4.2(0.1)^2 = 0.642\Omega$$

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{(0.48)^2 + (0.642)^2} = 0.6437\Omega$$

$$(iii)\text{Full load copper loss} = I_1^2 R_{01} = 10^2 \times 4.8 = 480 \text{ W}$$

**21.A75 KVA transformer has 500 turns primary and 100 turns secondary .the primary and secondary resistances are 0.4Ω and 0.02Ωrespectively and the corresponding leakage reactance are 1.5Ω and 0.045Ω respectively. The supply voltage is 2200V.Calculate a) equivalent impedance referred to the primary circuit and b) the voltage regulation and secondary terminal voltage for full load at power factor of i) 0.8 lagging and ii) 0.8 leading.**

**Given:**

$$\begin{aligned}R_1 &= 0.4, R_2 = 0.02 \\X_1 &= 1.5, X_2 = 0.045 \\V &= 2200V\end{aligned}$$

**Formula used:**

$$\begin{aligned}Z_{01} &= R_{01} + jX_{01} \\&= I_2(R_{02}\cos\Phi - X_{02}\sin\Phi)\end{aligned}$$

**Solution:**

$$K = \frac{N_2}{N_1} = \frac{100}{500} = 0.2$$

$$\begin{aligned}R_1 &= 0.4, R_2 = 0.02 \\X_1 &= 1.5, X_2 = 0.045 \\V &= 2200V\end{aligned}$$

$$\begin{aligned}R_{01} &= R_1 + \frac{R_2}{K^2} \\&= 0.4 + \frac{0.02}{(0.2)^2} = 0.9\Omega\end{aligned}$$

$$\begin{aligned}X_{01} &= X_1 + \frac{X_2}{K^2} \\&= 1.5 + \frac{0.045}{(0.2)^2} = 2.625\end{aligned}$$

$$\begin{aligned}Z_{01} &= R_{01} + jX_{01} \\Z_{01} &= 0.9 + j2.625 \\&= 2.775\angle 71.05^\circ\end{aligned}$$

$$\begin{aligned}Z_{02} &= K^2 Z_{01} = (0.2)^2 \times (0.9 + j2.625) \\&= 0.036 + j0.105 \\&= 0.036\end{aligned}$$

$$\begin{aligned}\text{NO Load voltage} &= KV_1 \\&= (0.2) \times 2200 \\V &= 440V\end{aligned}$$

$$I_2 = \frac{\text{KVA}}{V} = \frac{75}{440} = 170.45A$$

Full load voltage drop ref to secondary

$$\begin{aligned}&= I_2(R_{02}\cos\Phi - X_{02}\sin\Phi) \\&= 170.45(0.036 \times 0.8 - 0.105 \times 0.6) \\&= 170.45(0.0288 - 0.063) \\&= 170.45(-0.0342) \\&= -5.83\end{aligned}$$

$$\begin{aligned}\% \text{ Reg} &= -5.83 \times \frac{75}{440} \\&= -0.993\end{aligned}$$

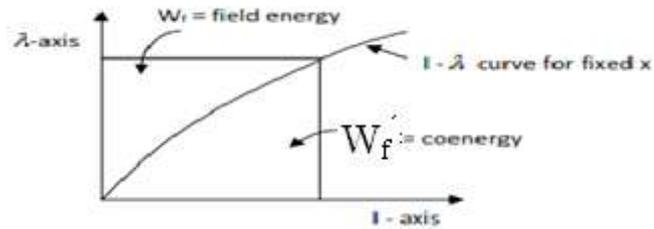
Sec terminal voltage at no load

$$\begin{aligned}\text{Leading } p.f &= 440 - (-5.833) \\&= 445.83V\end{aligned}$$

$$\begin{aligned}\text{Lgging } p.f &= 440 + (-5.833) \\&= 434.17V\end{aligned}$$

**UNIT -3**  
**PART-A**

1. In a linear system prove that field energy and co-energy are equal.[May/June-2010]



In the above figure, if the response is linear then the co-energy becomes equal to that of the field energy.

Therefore

$$W = W_f = \frac{1}{2} i(\lambda) = \frac{1}{2} i (Li)$$

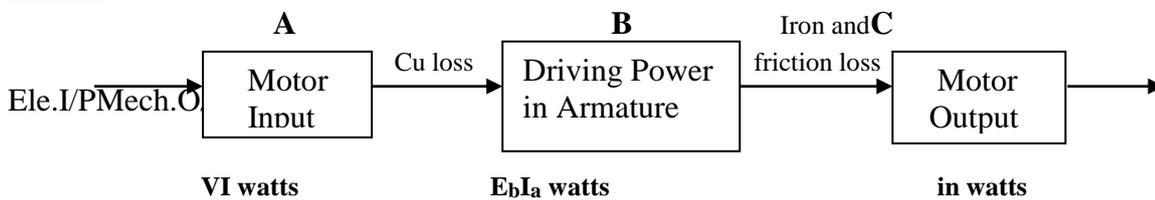
$$W_f' = W_f = \frac{1}{2} Li^2$$

For a linear relation

$$W_f' = W_f = \frac{1}{2} L(X) i^2$$

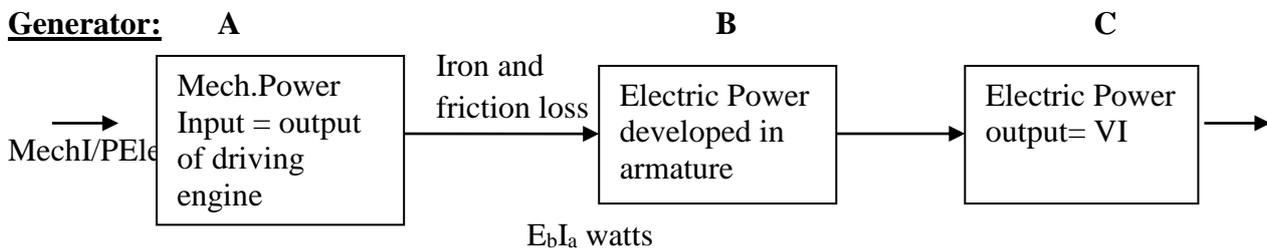
2. Draw the power flow diagram for motor and generator operation.[Nov/Dec-2010, 2012 & 2013]

**Motor:**



A – B = Copper losses & B – C = Iron and Friction losses

**Generator:**



3. Write the expression for the stored energy in the magnetic field.[May/June-2010 & Nov-Dec 2012]

$$\Delta W_f = \int_{\lambda_1}^{\lambda_2} i(\lambda) d\lambda \quad (\text{or}) \quad \Delta W_f = \int_{\phi_1}^{\phi_2} \mathcal{F}(\lambda) d\phi$$

4. In a magnetic circuit with a small air gap, in which part the maximum energy is stored and why?[Nov/Dec 2010]

Energy is stored in air gap. This occurs due to the following.

- In a magnetic material in case of iron core (or) steel core the saturation and ageing effects hinder storage.
- In air gap reluctance, as well as permeability are constant, the energy storage takes place linearly without any complexity.

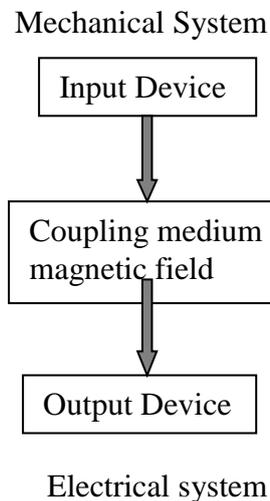
5. What are the advantages of analyzing the energy conversion devices by field energy concept? Or Write the Advantages of field energy method.[Nov/Dec-2011]

- It is applicable to all the types of devices such as translator, rotational, vibratory, linear etc.
- Both the steady state as well as transient behaviour of the devices can be analyzed.
- The approach forms the basis of generalized theory of electrical machines.

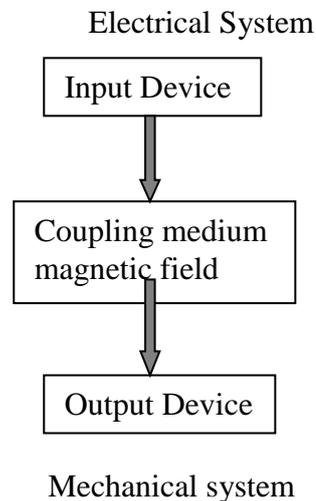
- The operation of all the electromechanical devices can be studied with great detail giving more physical insight.
- The various effect such as saturation, communication can be easily studied by introducing conventional approach at any stage.

**6. Draw the general block diagram of electromechanical energy conversion device.[Dec-2011& 2013]**

**(a) Generating device**



**(b) Motoring device**



**7. What is an electromechanical energy conversion system?**

- The electrical energy is converted into mechanical energy for motor and mechanical energy is converted to electrical energy for generator.
- The electromechanical energy conversion takes place via the medium of a magnetic or electric field is called electromechanical energy conversion system.

**8. What is multiply & double excited magnetic field system?[Nov/Dec-2005 & 2001]**

- In special devices more than one excitation coils are necessary such systems are called multiple excited systems.
- Very commonly used multiple excited systems use two excitation coils and are called doubly excited systems  
Ex: Synchronous motor, Alternator or Generator. Loud speakers, Tachometers, DC machines

**9. Write the reason for magnetic field as a coupling medium rather than electric field?[May/June-2007,2015](Nov/Dec-2013)**

- When compared to electric field, energy can easily stored and retrieved from a magnetic system with reduced losses comparatively.
- The energy storing capacity of the magnetic field is much higher than (about 25,600 times) than that of the electric field.

**10. Write the application of singly and doubly fed magnetic systems.[Nov/Dec-2006 & May- 2013]**

- Single: Electromagnetic relay, Reluctance motor, Toroid coil, & Solenoid coil.
- Multiple: Alternator, Electro mechanical transducer

**11. How energy is stored?[April/May-2004]**

- Energy can be stored or retrieved from the magnetic system by means of an exciting coil connected to an electric source.

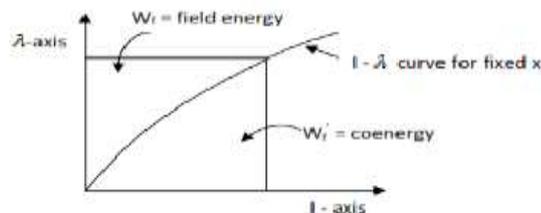
**12. Write the equation for mechanical force.**

$$F_f = - \frac{\partial W_f(\lambda, x)}{\partial x}$$

**13. Define Co-energy and field energy. [Nov/Dec-2009, April/May - 2012& 2013, 2016]**

- Co energy is an energy used for a linear system computation keeping current as constant. It will not be applied to the non-linear systems **Or** When armature is held open then almost entire mmf is required to drive the flux through air gap and hence magnetic saturation may not occur. – **The energy required for energy conversion (Elect - Mech and Mech – Elect).**
- The energy drawn by virtue of change in the distance moved by the rotor in electrical machines in field configurations are known as field energy.- **The conversion of energy(Elect - Mech and Mech – Elect) is called field energy.**

**14. Draw the graphical relation between field energy and coenergy. [Nov/Dec-2009, 2003]**



**15. Write the expression for the principle of energy conversion.[May/June-2011]**

Mechanical energy output= Electrical energy input– Increased or change in field energy.

$$F_f dx = i d\lambda - dW_f$$

**16. What is the significance of coenergy? [May/June-2006]**

The coenergy has no physical significance but it is important in obtaining magnetic forces.

**17. How the energy stored in magnetic field? [April/May-2004]**

When the moving part of any physical system is held fixed, and then the entire electrical energy input gets stored in the magnetic field.

**18. What are the requirements of the excitation systems?[April/May 2012],[NOV-DEC 2015]**

- Input device (Mechanical or Electrical)
- Output device
- Coupling field

**19. Enumerate the advantages of using short pitched winding in a synchronous machine. [ Dec - 2013]**

- Harmonics are reduced in induced voltage.
- Saving of copper
- End connections are shorter.

**20. Write the equation which relates rotor speeds in electrical and mechanical radians per second.(MAY/JUNE 2015)**

$$\omega_e = \omega_m (P/2)$$

Where,  $\omega_e$  – Speed is electrical radians per sec.

$\omega_m$ - Speed in mechanical radians per sec.

P – No. of poles.

**21. State the principle of electromechanical energy conversion.[May/June-2011,Dec 2017]**

- As electric energy is not readily available in nature.
- It has to be generated to meet the demands of electricity.
- The mechanical energy is converted to electrical energy vice versa of which is also possible is called electro-mechanical conversion which takes place through either electric field or magnetic field.

**22. State three types of electromechanical energy conversion. AU-NOV/DEC 2008**

- The various transducers such as microphones, loudspeakers and thermocouples. – **Limited Motion**
- The device which produce the mechanical force or torque based on translatory motion such as electromagnet and relays. – **Small Motion**
- The devices used for continuous energy conversion using rotational motion such as generators and motors. – **Continuous Motion**

**23. Write energy balance equation.**

$$dW_e = dW_m + dW_f + dW_{loss}$$

$$dW_m = dW_e + dW_f + dW_{loss}$$

**24. What is current excited system? Nov/Dec 2010**

This is the expression for system in which  $i$  is independent variable. This means input current constant such a system is current excited system

**25. How the direction of mechanical force or torque developed in any physical system?  
April/May 2009**

- decrease the magnetic stored energy at constant  $\lambda$
- increase the stored energy and co-energy at constant  $i$
- decrease the reluctance
- increase the inductance.

**26. What are the three basic type of rotating electrical Machine? (May-2011, Dec 2013)**

- DC Machines
- Poly phase synchronous machines
- Poly phase Induction machines

**27. What is meant by winding inductance?(May 2016)**

It is the property of an electric conductor or circuit that causes an electromotive force to be generated by a change in the current flowing.

**28. What is magnetic saturation? Dec – 2016**

Any further increase in flux density (B) will have no effect on the value of magnetic field strength (H), and the point on the graph where the flux density reaches its limit is called **Magnetic Saturation** also known as **Saturation of the Core**.

**29. What is meant by distributed winding? Dec - 2016**

If 'x' conductor per phase are distributed amongst the 3 slots per phase available under pole, the winding is called distributed winding.

**30. Define winding factor. [Nov – 2011]**

The winding factor  $K_w$  is defined as the ratio of phasor addition of emf induced in all the coils belonging to each phase winding of their arithmetic addition

**31. Define the synchronous speed. Write the expression also. May 2017**

In synchronous machine, the rotor and flux at stator rotate in  $N_s$  speed whereas in induction machine the stator flux alone moves at this speed.

$$N_s = 120f/p$$

**32. Define the term pole pitch and coil pitch. May 2017**

Pole Pitch – is the centre to centre distance between any two consecutive poles in a rotating machine, measured in terms of slots per pole.

**33. Predominant energy storage occurs in the air gap of an electromechanical energy conversion device. is this statement correct–true (Dec-2017)**

In airgap reluctance, as well as permeability are constant, the energy storage takes place linearly without any complexity.

**34. Why the relation between current and coil flux linkages of electromechanical energy conversion devices are linear (May 2018)**

When alternator is held open then almost entire MMF is required to device the flux through air gap and hence magnetic saturation may not occur. so the relationship between current and coil flux linkages of electromechanical energy conversion devices are linear.

**35. What are the causes for irrecoverable energy loss when the flux in the magnetic circuit undergoes a cycle (May 2018)**

When a magnetic circuit undergoes a cycle  $\phi_1 \rightarrow \phi_2 \rightarrow \phi_1$ , it undergoes a cycle of magnetization and demagnetization. The hysteresis and eddy current effects are dominant under such condition.

## Part – B

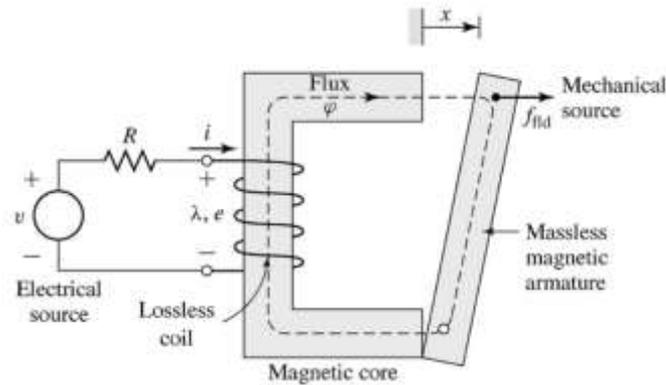
1. Explain the concept of singly-excited machines and Derive an expression for the magnetic Energy stored in a singly excited electromagnetic relay. [ Nov/Dec-2010] [May 2016]  
Discuss in detail the production of mechanical force for an attracted armature relay excited by an electric source (Dec 2017)

Or

Consider an attracted armature relay is excited by an electric source. Explain about the mechanical force developed and the mechanical energy output with necessary equations for linear and nonlinear cases. (May 2018)

Self-excited machines are used to produce magnetic field. E.g: electromagnetic relay, Reluctance Motor, Toroid coil etc.,

The attractive type armature relay is shown below,



It includes electrical input energy, mechanical stored field energy and mechanical force.

- The following assumptions are made while performing the analysis of single excited magnetic system.
- The resistance of the exciting coil is assumed to be present in lumped form, outside the coil. This coil is lossless and ideal.
- The leakage flux does not take part in energy conversion process (80H) is neglected as practically it is small.

Hence all the flux is confined to the iron core and links all the  $N$  turns of coil.

$$\lambda = N\Phi$$

Where,

$N$  - No: of turns of the coil

$\Phi$  - Total flux

Leakage inductance is negligible

There is energy loss in the magnetic core

The reluctance of the iron path is neglected.

### Electrical Energy Input:

Due to the flux linkages  $\lambda$ , the reaction emf exists, whose direction is so as to oppose the cause producing it.

$$\text{Induced emf } e = \frac{d\lambda}{dt}$$

Applying KVL to the coil circuit

$$V = ir + e$$

$$V = ir + \frac{d\lambda}{dt}$$

Multiplying on both sides by 'i', we get

$$\begin{aligned}
 Vi &= i^2r + i \frac{d\lambda}{dt} \\
 Vidt &= i^2r dt + id\lambda \\
 Vidt - i^2r dt &= id\lambda \\
 (V - ir)idt &= id\lambda \\
 eidt &= id\lambda \quad \text{[since } e=V-ir\text{]}
 \end{aligned}$$

Now the input electrical energy to the lossless coil is given by,

$$\begin{aligned}
 dWe &= eidt = id\lambda \\
 dWe &= id(N\Phi) \\
 dWe &= Ni d\Phi
 \end{aligned}$$

Where,

$$Ni = \text{mmf of the coil}$$

Thus the magnetic system extracts the electrical energy from the supply.

### Magnetic Field Energy stored:

Consider that the armature is held fixed at position 'x'.

As armature is not moving, the mechanical work done is zero.

According to energy balance equation,

$$\text{Input from supply} = \text{Mechanical output} + \text{Stored energy} + \text{loss}$$

The entire electric energy input gets stored in the magnetic field

$$\text{i.e., } dW_e = dW_f \quad \text{[since } dW_m = 0\text{]}$$

$$dW_f = eidt = id\lambda = Ni d\Phi \text{----- (1)}$$

The relationship  $i-\lambda$  is basically non-linear for a magnetic circuit, similar to the B-H relationship.

From the Eqn-(1), the energy absorbed for a finite change in flux linkages can be obtained.

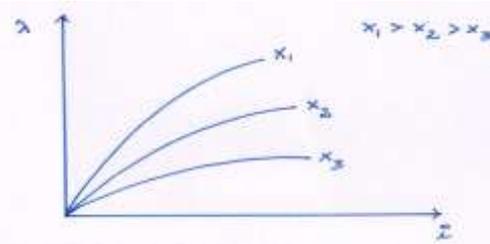
$$\Delta W_f = \int_{\lambda_1}^{\lambda_2} i(\lambda) d\lambda = \int_{\Phi_1}^{\Phi_2} Ni d\Phi$$

If the initial flux and flux linkages are zero,

i.e.  $\Phi_1 = \lambda_1 = 0$ , then the energy stored in the magnetic field is,

$$W_f = \int_0^{\lambda} i(\lambda) d\lambda = \int_0^{\Phi} Ni d\Phi$$

$I - \lambda$  relationship is similar to the magnetization curve for a magnetic material for various values of x.



**Mechanical Force:**

Magnetic field produces a mechanical force  $F_f$ .

Let armature move at a distance  $dx$  in positive direction (i.e. direction of force).

Therefore the mechanical work done by the magnetic field is given by,

$$dW_m = F_f dx$$

As per the energy balance equation,

$$dW_m = \text{Electrical energy} - \text{Change in stored energy}$$

$$F_f dx = i d\lambda - dW_f \quad \text{----- (2)}$$

**Case 1:** The independent variables are  $(i, x)$

i.e. current is constant, then

$$\lambda = \lambda(i, x) \quad (\lambda \text{ changes as } i \& x)$$

$$d\lambda = \frac{\partial \lambda}{\partial i} di + \frac{\partial \lambda}{\partial x} dx \quad \text{----- (3)}$$

While

$$W_f = W_f(i, x)$$

$$dW_f = \frac{\partial W_f}{\partial i} di + \frac{\partial W_f}{\partial x} dx \quad \text{----- (4)}$$

From (2),(3),(4),

$$F_f dx = i \frac{\partial \lambda}{\partial i} di + i \frac{\partial \lambda}{\partial x} dx - \frac{\partial W_f}{\partial i} di - \frac{\partial W_f}{\partial x} dx$$

$$F_f dx = i \left[ \frac{\partial \lambda}{\partial x} - \frac{\partial W_f}{\partial x} \right] dx + \left[ i \frac{\partial \lambda}{\partial i} - \frac{\partial W_f}{\partial i} \right] di$$

There is no term of 'di' on left side of the equation. It should be zero on the right hand side.

$$i \frac{\partial \lambda}{\partial i} - \frac{\partial W_f}{\partial i} = 0$$

Therefore

$$F_f = i \frac{\partial \lambda}{\partial x}(i, x) - \frac{\partial W_f}{\partial x}(i, x)$$

This is the expression for the mechanical force developed by the magnetic coupling field.

$$W_f^1 = i\lambda - W_f$$

$$W_f^1(i, x) = i\lambda(i, x) - W_f(i, x)$$

Where,  $(i, x)$  are independent variables.

$$F_f = \frac{W_f^1(i, x)}{\partial x}$$

This is the expression for system in which 'I' is independent variable This means input current constant. Such a system is called current excited system.

**Case:2** The independent variables are  $(\lambda, X)$  i.e,  $\lambda$ -constant

Thus 'I' changes as  $\lambda$  and  $x$ . Hence  $i=i(\lambda, X)$

While  $W_f=W_f(\lambda, X)$

$$dw_f = \frac{\partial w_f}{\partial \lambda} d\lambda + \frac{\partial w_f}{\partial X} dx$$

Sub  $dw_f$  in equ 2

$$F_f dx = i d\lambda - \frac{\partial w_f}{\partial \lambda} d\lambda - \frac{\partial w_f}{\partial X} dx$$

$$F_f dx = -\frac{\partial w_f}{\partial \lambda} dx + \left[ i - \frac{\partial w_f}{\partial \lambda} \right] d\lambda$$

There are no terms of  $d\lambda$  on the left hand side of the, the corresponding term on the right hand side must be zero.

$$i - \frac{\partial w_f(\lambda, x)}{\partial \lambda} = 0$$

$$i = \frac{\partial w_f(\lambda, x)}{\partial \lambda}$$

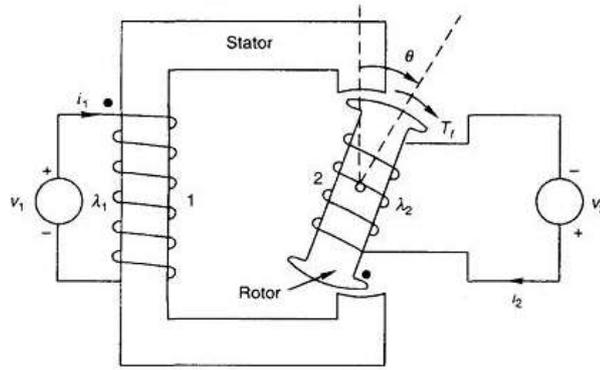
$$F_f = -\frac{\partial w_f(\lambda, x)}{\partial X}$$

This is the expression for system in which ' $\lambda$ ' is independent variable, i.e. flux producing voltage is constant. Such a system is voltage controlled system

-----&&&-----

**2. Explain with neat diagram and sufficient expressions, the multiply excited magnetic field systems and derive the expression for energy and force equation. [Dec-2012, 2015 & May- 2010, 2013,2015, 2016]**

- Singly excited devices are generally employed for motion through a limited distance or rotation through a prescribed angle.
- Electro-mechanical transducers have the special requirement of producing an electrical signal proportional to forces or velocities or producing force proportional to electrical signal (current or voltage).
- Such transducers require two excitations one excitation establishes a magnetic field of specified strength while the other excitation produces the desired signal (electrical or mechanical).
- Also continuous energy conversion devices motors and generators require multiple excitation.
- Figure shows a magnetic field system with two electrical excitations one on stator and the other on rotor.
- The system can be described in either of the two sets of three independent variables  $(\lambda_1, \lambda_2, \theta)$  or  $(i_1, i_2, \theta)$ .
- For continuous energy conversion devices like alternators, synchronous motors etc., multiply excited magnetic systems are used.
- This system has two independent sources of excitations. One source is connected to coil on stator while other is connected to coil on rotor.



- Let
- $i_1$  = Current due to source 1
  - $i_2$  = Current due to source 2
  - $\lambda_1$  = Flux linkages due to  $i_1$
  - $\lambda_2$  = Flux linkages due to  $i_2$
  - $\theta$  = Angular displacement of rotor
  - $T_f$  = Torque developed

Due to two sources, there are two sets of three independent variables i.e.  $(\lambda_1, \lambda_2, \theta)$  or  $(i_1, i_2, \theta)$ .

**Case 1:** Independent variables  $\lambda_1, \lambda_2, \theta$  i.e.  $\lambda_1, \lambda_2$  are constants

From the earlier analysis it is known,

$$T_f = \frac{-\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} \quad \dots \text{currents variables (1)}$$

While the field energy is,  $\partial \theta$

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} i_1 d\lambda_1 + \int_0^{\lambda_2} i_2 d\lambda_2 \quad \dots \dots \dots (2)$$

- Now let
- $L_{11}$  = Self inductance of stator
  - $L_{22}$  = Self inductance of rotor
  - $L_{12} = L_{21}$  = Mutual inductance between stator and rotor

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \quad \dots (3)$$

and

$$\lambda_2 = L_{12} i_1 + L_{22} i_2 \quad \dots (4)$$

Solve (3) and (4) to express  $i_1$  and  $i_2$  in terms of  $\lambda_1$  and  $\lambda_2$  as  $\lambda_1$  and  $\lambda_2$  are independent variables.

Multiply (3) by  $L_{12}$  and (4)  $L_{11}$ ,

$$L_{12} \lambda_1 = L_{11} L_{12} i_1 + L_{12}^2 i_2$$

and

$$L_{11} \lambda_2 = L_{11} L_{12} i_1 + L_{11} L_{22} i_2$$

Subtracting these two equations we get,

$$L_{12} \lambda_1 - L_{11} \lambda_2 = [L_{12}^2 - L_{11} L_{22}] i_2$$

$$i_2 = [L_{12} / L_{12}^2 - L_{11} L_{22}] \lambda_1 - [L_{11} / L_{12}^2 - L_{11} L_{22}] \lambda_2$$

... (5)

$$i_2 = \beta_{12} \lambda_1 + \beta_{22} \lambda_2$$

Note that negative sign is absorbed in defining  $\beta$ .

Similarly  $i_1$  can be expressed in terms of  $\lambda_1$  and  $\lambda_2$  as,

$$i_1 = \beta_{11}\lambda_1 + \beta_{12}\lambda_2 \quad \dots (6)$$

Where,

$$\beta_{11} = \frac{L_{22}}{L_{11}L_{22} - L_{12}^2}$$

$$\beta_{22} = \frac{L_{11}}{L_{11}L_{22} - L_{12}^2}$$

$$\beta_{21} = \frac{-L_{12}}{L_{11}L_{22} - L_{12}^2}$$

Using in equation (2),

$$W_f(\lambda_1, \lambda_2, \theta) = \int_0^{\lambda_1} [\beta_{11}\lambda_1 + \beta_{12}\lambda_2] d\lambda_1 + \int_0^{\lambda_2} [\beta_{12}\lambda_1 + \beta_{22}\lambda_2] d\lambda_2$$

Integrating the terms we get,

$$W_f(\lambda_1, \lambda_2, \theta) = \frac{1}{2}\beta_{11}\lambda_1^2 + \beta_{12}\lambda_1\lambda_2 + \frac{1}{2}\beta_{22}\lambda_2^2 \quad \dots (7)$$

The self and mutual inductances of the coils are dependent on the angular position  $\theta$  of the rotor.

**Case 2:** Independent variables  $i_1, i_2, \theta$  i.e.  $i_1$  and  $i_2$  are constants.

The torque developed can be expressed as,

$$T_f = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta} \quad \dots (8)$$

The co-energy is given by,

$$W_f'(i_1, i_2, \theta) = \int_0^{i_1} \lambda_1 d i_1 + \int_0^{i_2} \lambda_2 d i_2 \quad \dots (9)$$

Using  $\lambda_1 = L_{11}i_1 + L_{12}i_2$

and  $\lambda_2 = L_{12}i_1 + L_{22}i_2$

$$W_f'(i_1, i_2, \theta) = \int_0^{i_1} [L_{11}i_1 + L_{12}i_2] di_1 + \int_0^{i_2} [L_{12}i_1 + L_{22}i_2] di_2$$

$$W_f'(i_1, i_2, \theta) = \frac{1}{2}L_{11}i_1^2 + L_{12}i_1i_2 + \frac{1}{2}L_{22}i_2^2$$

**Force in doubly excited system:**

$$F = \frac{\partial W_f'}{\partial \theta}(i_1, i_2, \theta)$$

Where  $i_1$  and  $i_2$  are constants which are the stator and rotor current respectively.

$$F = \frac{\partial}{\partial \theta} \left[ \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2 \right]$$

$$\therefore F = \frac{1}{2} i_1^2 \frac{\partial L_{11}}{\partial \theta} + i_1 i_2 \frac{\partial L_{12}}{\partial \theta} + \frac{1}{2} i_2^2 \frac{\partial L_{22}}{\partial \theta}$$

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### 3. Explain the MMF of Distributed AC Windings. April/May-2010, 2017

- The armature winding used for three phase alternators is distributed in nature.
- The efforts are made to place the coils in all the slots available per pole per phase. Such coils are then interconnected such that the magnetic field produced by armature after carrying current has same number of poles as that of the field winding.
- let us obtain the mmf space distribution of the current carrying armature by superimposing the mmf space waves of individual coils.

#### MMF Space Wave of a Single Coil

- Consider single multi turn coil, carrying N turns and is a full pitch coil. it carries a current i such that the total ampere turns produced by the coils(Ni).
- The direction of current i and hence direction of flux lines around a anda' can be obtained by using right hand thumb rule.
- The stator is wound for two poles same as the number of poles for which field winding is wound.
- Due to the flux lines produced by the coil, the north and south poles are induced on the stator periphery. The magnetic axis of the stator coil is form N pole to S pole.

#### The assumptions to obtain MMF space wave are

- It is cylindrical rotor machine.
- The armature and rotor are made up of high grade magnetic material hence permeability of these parts is much higher than air. Hence reluctance is low so entire reluctance can be assumed to be due to two air gaps.
- Thus if total mmf is Ni then half the mmf is required to create flux from rotor to stator in the air gap while half is required to create flux from stator to rotor in the air gap.
- The flux lines rapidly crosses the air gap between rotor and stator twice, normal to the stator and rotor iron surfaces.
- Consider a developed diagram of the coil that rotor surface is over the stator which is laid down flat.
- The mmf and flux rapidly outwards from rotor to stator is assumed positive while that from stator to rotor is assumed negative. Thus mmf distribution is stepwise giving a rectangular waveform. themmf +Ni\2 is used in setting flux from rotor to stator in the air gap .
- No mmf required for iron path .similarly the mmf–Ni/2 is used in setting flux from stator to rotor in the air gap.
- Thus mmf changes suddenly from –Ni/2 to Ni/2 at one slot while from +Ni/2 to –Ni/2 at other slot which is a pole pitch away. Total change in mmf is abrupt and equql to Ni in crossing from one side to other of a coil.

MMF which changes at any slot= $Ni$

- The direction depends on the current direction. The rectangular mmf space can be resolved into its Fourier series which includes its fundamental component and a series of odd harmonics. The fundamental component of mmf wave is given by,

$$F_{a1} = (4/\pi)Ni/2 \cos\theta = F_{1P}\cos\theta$$

Where

$\theta$ =electrical angle measured from stator magnetic axis

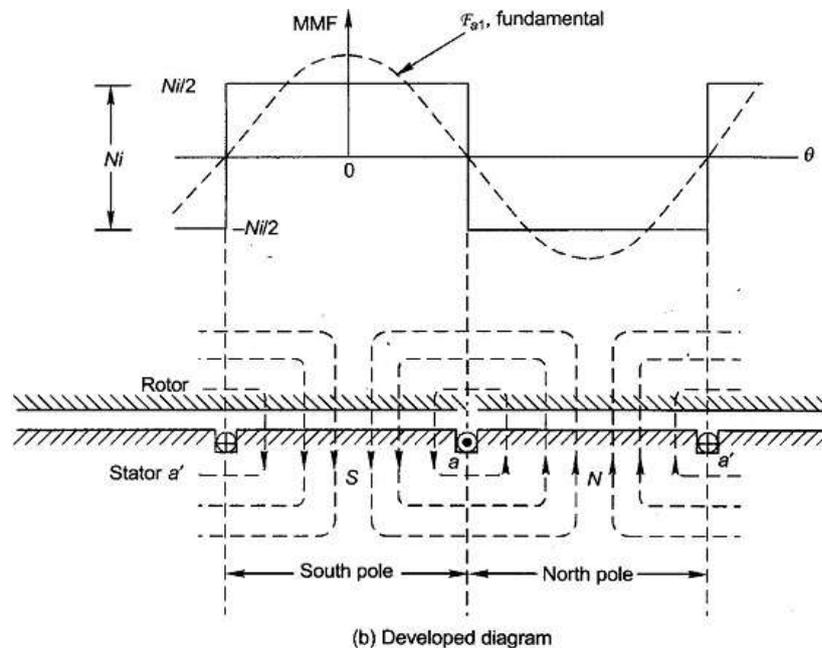


Fig. 5.24 Mmf space wave of a single coil

### MMF Space Wave of one phase of distributed winding

Consider a pole, cylindrical rotor with,

$$M = \text{slots/pole/phase} = 5$$

$$N = \text{slots/pole} = m \times 3 = 15$$

The distributed winding for phase a, occupying all 5 slots per Pole per phase

Let  $N_c$  = turns in a coil = conductor/layer

$I_c$  = conductor current

Total slot mmf =  $2 \times (\text{mmf/layer})$

$$= 2N_c I_c$$

- As the number of slots are odd, half of ampere conductors of middle slot of a and a' constitute s-pole and remaining half contribute N pole. The m.m.f wave has shown a step jump at the slot equal to  $2N_c I_c$ . The developed diagram of the stator and m.m.f wave.
- Total ampere-conductors produced by conductors of group 'a' are corresponding to 5 slots, each slot producing  $2N_c I_c$ . Hence total ampere-conductors are  $10N_c I_c$ .
- Half of it are used to set the flux from rotor to stator and half for setting flux from stator to rotor. So  $+5N_c I_c$  used to set flux from rotor to stator while  $-5N_c I_c$  used to set flux from stator to rotor.

- But these  $5N_c i_c$  ampere-conductors are also available in step form at the respective slots to obtain peak of the fundamental component of mmf wave, assume that the winding is concentrated.

Let

$T_{ph}$  = series turns per parallel path of a phase

$A$  = Number of parallel paths

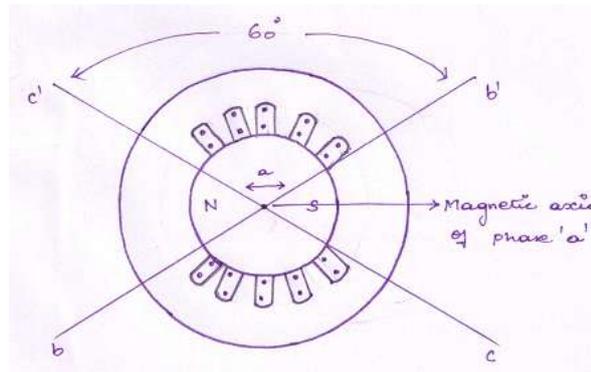
$AT/\text{parallel path} = T_{ph}(\text{series}) \times i_c$

$AT/\text{phase} = A [T_{ph}(\text{series}) \times i_c]$

$A \times i_c = i_a$  = Total phase current

$AT/\text{Phase} = T_{ph}(\text{series}) \times i_a$

$AT/\text{Pole/Phase} = T_{ph}(\text{series}) \times i_a / P$



- But this is peak value if the winding is concentrated. When the winding is distributed, the peak value will be reduced by  $K_d$

$$F_{1p} (\text{Distributed}) = \frac{4}{\pi} K_d \left[ \frac{T_{ph} \times i_a}{p} \right]$$

- Thus the equation of the mmf wave in space is given by,

$$F_{a1} = F_{1p} \cos \theta = \frac{4}{\pi} K_d \frac{T_{ph}}{p} i_a \cos \theta$$

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**4. Prove that a three phase set of currents, each of equal magnitude designed and differing in space by  $120^\circ$  applied to a three phase winding spaced  $120^\circ$  electrical degrees apart around the surface of the machine will produce a rotating magnetic field of constant magnitude. (Nov/Dec-2010, 2016)**

By practices as well as by research works it has been observed that the magnitude field produced when a balanced three phase supply with  $120^\circ$  electrical phase angle separation is given to a balance  $3\phi$

Winding the phases distributed in space so that relative space difference is  $2\pi/3$  (real-elect) causes a resultant mmf to rotate in the air gap between stator and rotor at a synchronous speed of

$$N_s = \frac{120f}{p} \text{ (rpm)}$$

But to justify this concept analytically is very important from both design and analysis points of views.

Supply allows the following balanced currents to flow through the windings as

$$i_a = I_m \cos wt : i_b = I_m \cos(wt - 120) : i_c = I_m \cos(wt - 240)$$

By the principle of mutual inductance three currents develop a magnetic field which are also separated by 120° with respect to the magnetic axes.

These develop mmf's that can be expressed as follows,

$$F_a = F_m \cos wt \cos \theta$$

$$F_b = F_m \cos(wt - 120) \cos(\theta - 120)$$

$$F_c = F_m \cos(wt - 240) \cos(\theta - 240)$$

Where  $\theta$  is the angle from the reference axis or that the axis of phase 'a' shown in figure.

The resulting mmf is given by

$$F = F_a + F_b + F_c$$

$$F(\theta, t) = F_m \cos wt \cos \theta + F_m \cos(wt - 120) \cos(\theta - 120) + F_m \cos(wt - 240) \cos(\theta - 240)$$

By trigonometric relations,

$$F(\theta, t) = \frac{3}{2} F_m \cos(\theta - wt) + \frac{1}{2} F_m \cos(\theta + wt) + \cos(\theta + wt - 240) + \cos(\theta + wt - 480) \text{-----(A)}$$

$$\begin{aligned} \text{But } \cos(\theta + wt - 480) &= \cos(\theta + wt - 480 + 360) \\ &= \cos(\theta + wt - 120) \end{aligned}$$

equation (A) can be written as

$$F(\theta, t) = \frac{3}{2} F_m \cos(\theta - wt) + \frac{1}{2} F_m \{(\cos \theta + w + \cos(\theta + wt - 120) + \cos(\theta + wt - 240))\}$$

Algebraic sum of three vectors with a progressive phase difference of 120° equals to zero

$$F(\theta, t) = \frac{3}{2} F_m \cos(\theta - wt) \text{-----(B)}$$

The equation (B) gives a quantity which is distributed both in space and time. It is standard form representation of traveling wave where phase angle changes linearly with time as  $\theta \propto wt$ .

Hence the resultant mmf represented by equation (B) rotates at a constant speed of  $w$  rad/sec.

Hence the peak value of mmf developed,

$$F_b = \frac{3}{2} F_m \text{-----(C)}$$

Substituting the expression of  $F_m$

$$F_m = 4 \frac{\sqrt{2} Nph}{\pi p} K_w \text{.Irms}$$

In the equation (C),

$$F_p = 3 \frac{2\sqrt{2} Nph}{\pi p} K_w \text{.Irms}$$

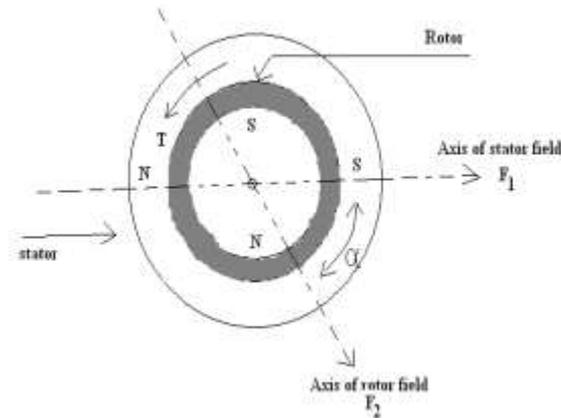
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**5. How is torque developed in round rotor machine? Derive an expression for the same. State the assumptions made. [Nov/Dec-2013]**

**Torque in Round Rotor Machine:**

When current carrying conductor is placed in a magnetic field, the force is exerted on the conductor to cause motoring action. This force is the basic cause of the production of torque. Practically current carrying conductor produces its own field while there exists the magnetic field in which it is kept. The two magnetic fields interact with each other and the torque is produced so as to align these two fields.

In a.c. machines both stator and rotor carry currents and produce their own magnetic fields. These fields are shown in the Fig.



### Torque in round rotor machine

The stator and rotor are wound for 2 poles, producing north and south poles respectively on the stator and rotor surfaces. The direction of axis of each field is as shown in the fig. The angle between the two axes is  $\alpha$ . Due to this angle, the torque appears so as to align the two axes. Note that for the torque to exist the two fields must have same number of poles and the relative motion between them should be zero i.e. they must be stationary with respect to each other. Then the torque depends on the amplitudes of the stator and rotor m.m.f. waves and the angle between the two fields.

Assumptions for Deriving the Torque Equation

The assumptions made for deriving the torque equation are,

- 1.The machine is round rotor machine having cylindrical rotor. Thus air gap is uniform.
- 2.The m.m.f. waves produced by stator and rotor are sinusoidal. This is practically achieved using distributed winding.
- 3.The tangential component of the magnetic field in the air gap is negligible compared to the radial component. The mutual flux goes straight across the gap. The radial length 'g' of the gap is very small compared to the radius of stator and rotor. Hence flux density does not vary along the radial path of airgap.

Hence the field intensity H along the air gap is constant. The m.m.f. across the air gap is,

$$F_{\text{air gap}} = H \times g \text{ ----- 1}$$

Where  $g$  = Radial length of air gap

4. Most of the flux produced by stator and rotor windings crosses the air gap and links with both the windings. This entire flux is mutual flux. The flux linking with the respective windings is leakage flux but this does not affect the torque and only affects the voltages. Mutual flux is important from torque point of view.

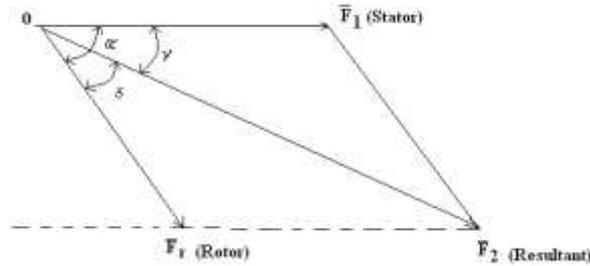
5. The reluctance of the iron path is negligibly small.

6. The sinusoidal m.m.f. space wave produces sinusoidal flux density (B) wave in phase with it.

### Derivation of Torque Equation for Round Rotor Machine

- Let  $F_1$  = Peak value of sinusoidal m.m.f. stator wave  
 $F_2$  = Peak value of sinusoidal m.m.f. rotor wave  
 $\alpha$  = Angle between  $F_1$  and  $F_2$  called torque angle  
 $\delta$  = Angle between  $F_2$  and  $F_r$  called load angle  
 $\gamma$  = Angle between  $F_1$  and  $F_r$  called torque angle

To find the resultant of  $F_1$  and  $F_2$ , these can be represented in the phasor form as shown in the fig. The angle between  $\bar{F}_1$  and  $\bar{F}_r$  is  $\gamma$  while the angle between  $\bar{F}_2$  and  $\bar{F}_r$  is  $\delta$



Phasor sum of  $\bar{F}_1$  and  $\bar{F}_2$

The resultant  $\bar{F}_r$  can be obtained using law of parallelogram and is also sinusoidal m.m.f. space wave.

Using cosine rule, the peak value of resultant m.m.f.  $F_r$  is given by,

$$F_r^2 = F_1^2 + F_2^2 + 2F_1F_2 \cos \alpha \text{ -----2}$$

The entire resultant m.m.f. is required to cross the flux across the air gap as reluctance of iron path is neglected.

$$\therefore F_{\text{air gap}} = F_r = H_r g$$

Where  $H_r$  = Peak value of resultant field intensity

$$H_r = \frac{F_r}{g} \text{ -----3}$$

The co-energy density of the co-energy stored in the air gap at a point is given by  $(1/2)\mu_0 H^2$ . The average value of co-energy density over the volume of air gap is  $\mu_0/2$  times the average value of  $H^2$ .

$$\text{Average value of co-energy density} = \frac{1}{2} \mu_0 (\text{Average value of } H^2) \text{ -----4}$$

But  $H$  is purely sinusoidal hence,

$$\text{Average value of } H^2 = \frac{1}{2} \times [\text{peak value of } H]^2 = \frac{1}{2} H_r^2 \text{ -----5}$$

Using in equation (4)

$$\text{Average co-energy density} = \frac{1}{4} \mu_0 H_r^2 \text{ -----6}$$

$$\text{Volume of air gap} = \pi D l g \text{ -----7}$$

Where  $D$  = Average diameter of air gap

$l$  = Axial length of air gap

$g$  = Air gap length

$\mu_0$  = Permeability of free space =  $4\pi \times 10^{-7} \text{ H/m}$

Total co-energy of the field = Average co-energy density  $\times$  Volume

$$W_f' = \frac{1}{4} \mu_0 H_r^2 \times \pi D l g = \frac{\pi}{4} \mu_0 \left[ \frac{F_r}{g} \right]^2 D l g$$

$$W_f' = \frac{\mu_0 \pi D l}{4g} F_r^2 \text{ -----8}$$

Using equation (2) in equation (8),

$$W_f' = \frac{\mu_0 \pi D l}{4g} [F_1^2 + F_2^2 + 2F_1F_2 \cos \alpha] \text{ -----9}$$

The torque developed is the partial derivative of the field energy with respect to angle  $\alpha$ .

$$\therefore T = \frac{\partial W_f'}{\partial \alpha} = -\frac{\mu_0 \pi D l}{4g} 2F_1 F_2 \sin \alpha$$

$$\therefore T = -\frac{\mu_0 \pi D l}{2g} F_1 F_2 \sin \alpha \text{-----10}$$

The torque given by the equation (10) is torque per pair of poles as 2 pole machine is considered.

For p pole machine, the torque is given by,

$$T = -\frac{P}{2} \frac{\mu_0 \pi D l}{2g} F_1 F_2 \sin \alpha \text{----- 11}$$

The negative sign indicates that the torque acts in the direction so as to reduce  $\alpha$  i.e. so as to align the two fields. Thus equal and opposite torques are excited on the stator and rotor.

**6. The doubly excited coil has self inductances of  $(2 + \cos 2\theta)$  and the mutual inductances be  $\cos \theta$  where  $\theta$  is the angle between arcs of the coils. The coils are connected in series and carry a current of  $i = \sqrt{2} I \sin \omega t$ . Derive an expression for the average torque as function of angle 'q'.**

Soln:

$$L_{11} = L_{22} = 2 + \cos 2\theta; L_{12} = \cos \theta; i = \sqrt{2} I \sin \omega t$$

The co-energy can be expressed as,

$$\begin{aligned} w'f &= \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2 \\ &= \frac{1}{2} (2 + \cos 2\theta) i_1^2 + \cos \theta i_1 i_2 + \frac{1}{2} (2 + \cos 2\theta) i_2^2 \\ w'f &= [(2 + \cos 2\theta) + \cos \theta] i^2 \quad \left[ \begin{array}{l} i_1 = i_2 = i \\ \text{series ckt} \end{array} \right] \end{aligned}$$

$$\text{Torque } T_f = \frac{\partial w'f}{\partial \theta} = \frac{\partial}{\partial \theta} [(2 + \cos 2\theta) + \cos \theta] (\sqrt{2} I \sin \omega t)^2$$

$$= 2I^2 \sin^2 \omega t \left[ \frac{\partial}{\partial \theta} \cos 2\theta + \frac{\partial}{\partial \theta} \cos \theta \right]$$

$$= 2I^2 \sin^2 \omega t \left[ -\frac{\sin 2\theta}{2} - \sin \theta \right]$$

$$T_f = -I^2 \sin^2 \omega t [\sin 2\theta + 2 \sin \theta]$$

$\therefore$  The average of the torque development is

$$T_{av} = \frac{1}{T} \int_0^T T_f dt$$

$$= -\frac{1}{T} I^2 (\sin 2\theta + 2 \sin \theta) \int_0^T \sin^2 \omega t dt$$

$$= -\frac{1}{T} I^2 (\sin 2\theta + 2 \sin \theta) \left[ \int_0^T \frac{1 - \cos 2\omega t}{2} dt \right]$$

$$= -\frac{1}{T} I^2 (\sin 2\theta + 2 \sin \theta) \frac{1}{2} \left[ t - \frac{\sin 2\omega t}{2} \right]$$

$$T_{av} = -\frac{I^2}{2} [\sin 2\theta + 2 \sin \theta] \frac{1}{T} [T]$$

$$\therefore T_{av} = -\frac{I^2}{2} [\sin 2\theta + 2 \sin \theta]$$

7. Two coupled coils have self and mutual inductances of  $L_{11} = 2 + \frac{1}{2x}$ ;  $L_{22} = 1 + \frac{1}{2x}$ ;  $L_{12} = L_{21} = \frac{1}{2x}$ . Over certain range of linear displacement X the first coil is excited by a constant current of 20A and the second by a constant current of -10A. Find

- i) Mechanical workdone if x changes from 0.5 to 1m
- ii) Energy supplied by each electrical source in part (a)
- iii) Change in field energy.

Hence verify that energy supplied by the sources is equal to increase in field energy plus the mechanical. [Apr/May-2010.Nov/Dec-2012]

Soln:

$$L_{11} = 2 + \frac{1}{2x}; L_{22} = 1 + \frac{1}{2x}; L_{12} = L_{21} = \frac{1}{2x}$$

$$i_1 = 20A, i_2 = -10A$$

1. Mechanical workdone:  $\int_{x_1}^{x_2} F_f dx$

$$F_f = \frac{d\omega_f}{dx}$$

$$\omega_f(i_1, i_2, x) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

$$= \frac{1}{2} \left( 2 + \frac{1}{2x} \right) (20)^2 + \frac{1}{2x} (20)(-10) + \frac{1}{2} \left( 1 + \frac{1}{2x} \right) (-10)^2$$

$$= \frac{4x+1}{4x} (400) - \frac{100}{x} + 50 \left( \frac{2x+1}{2x} \right)$$

$$\omega_f = 450 + \frac{25}{x}$$

$$F_f = \frac{d\omega_f}{dx} = \frac{d(450 + \frac{25}{x})}{dx} = -\frac{25}{x^2}$$

$$\text{Mechanical work done } (\Delta w_m) = \int_{0.5}^1 F_f dx$$

$$= \int_{0.5}^1 -\frac{25}{x^2} dx = \frac{25}{x} = \left[ +\frac{25}{1} + \left( -\frac{25}{0.5} \right) \right] = -25J$$

$\Delta w_m = -25J$

ii. Energy supplied by source 1

$$(w_{e1}) = \int_{\lambda_1(x_1=0.5)}^{\lambda_2(x_2=1)} i_1 d\lambda_1$$

$$\lambda_1 = L_{11} i_1 + L_{12} i_2$$

$$= \left( 2 + \frac{1}{2x} \right) (20) + \frac{1}{2x} (-10)$$

$$\lambda_1 = 40 + \frac{5}{x}$$

$$\lambda_1 = 40 + \frac{5}{x}$$

$$w_{e1} = \int_{0.5}^1 d\lambda_1 (20) = \left[ [i_1 \lambda_1] \right]_{0.5}^1$$

at x=1

$$= 20[\lambda_2 \text{ at } x_2 - \lambda_1 \text{ at } x_1]$$

$$\lambda_1 = 40 + 5 = 45$$

$$= 20[45 - 50] w_{e1} = -100J$$

at X=0.5

$$w_{e1} = -100J \quad \lambda_1 = 40 + \frac{5}{0.5}$$

$$= 40 + 10 = 50$$

$$w_{e2} = \int_{\lambda_1 \text{ at } x_1}^{\lambda_2 \text{ at } x_2} i_2 d\lambda_2 = [i_2 \lambda_2]_{\lambda_1 \text{ at } x_1 \Rightarrow x=0.5}^{\lambda_2 \text{ at } x_2 \Rightarrow x=1}$$

$$\lambda_2 = L_{11}i_1 + L_{22}i_2 \quad x=1$$

$$= \frac{1}{2x}(20) + \left(1 + \frac{1}{2x}\right)(-10)$$

$$\lambda_2 = -10 + \frac{5}{1} = -5$$

$$\lambda_2 = -10 + \frac{5}{x} \quad X=0.5$$

$$w_{e2} = i_2[-5 - 0]\lambda_2 = -10 + \frac{5}{0.5} = -10 + 10$$

$$= -10(-5)$$

$$w_{e2} = 50J \quad \lambda_2 = 0$$

$$\text{Net electrical energy input } \Delta w_e = \Delta w_{e1} + \Delta w_{e2} \\ = -100 + 50$$

$$\Delta w_e = -50J$$

iii) Change in field energy

$$\Delta w_f = [w_f]_{0.5}^1$$

$$= \left[450 + \frac{25}{x}\right]_{0.5}^1$$

$$= 450 + \frac{25}{1} - (450 + 50)$$

$$= 475 - 500 \quad \Delta w_f = -25J$$

Ans:

i) Mechanical work done ( $\Delta w_n$ ) = -25J

ii) Energy supplied by source 1 ( $w_{e1}$ ) = -100J

Energy supplied by source 2 ( $w_{e2}$ ) = 50J

iii) Change in field energy  $\Delta w_f = -25J$

iv)  $w_{e1} + w_{e2} = \Delta w_f + \Delta w_n$

$$-100 + 50 = -25 + (-25)$$

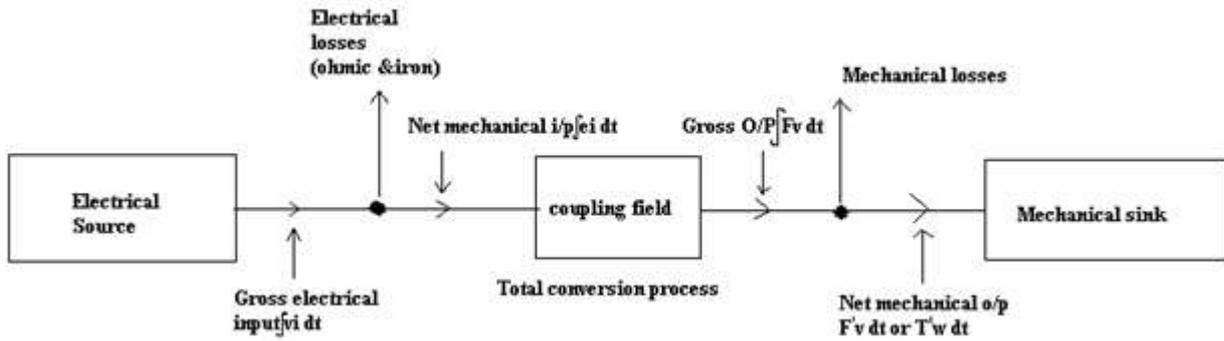
$$-50J = -50J$$

Hence  $\Delta w_f + \Delta w_n = \Delta w_e$  verified

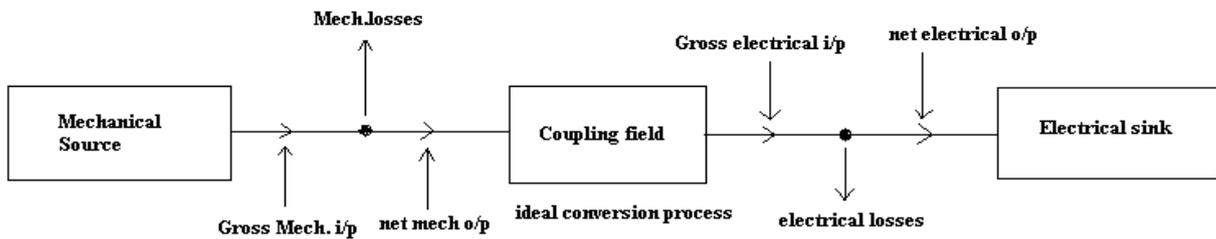
### 8. Discuss the flow of energy in electromechanical device in detail. [April - 2010] (May 2017)

- Electrical mechanical energy conversion is a reversible process. If the armature is allowed to moved on positive x direction under the influence of  $F_f$
- Electrical energy is converted to mechanical from via the coupling field.
  - If instead the armature is moved in the negative x direction under the influence of external force, mechanical energy is converted to electrical from the coupling field.
- This conversion process is not restricted to translating devices as illustrated but is equally applicable to rotary devices.

- Electrical and mechanical losses causes irreversible flow of energy out of a practical conversion device. The flow of energy in electromechanical conversion in either direction along with irrecoverable energy losses is shown in figure.



### Motor

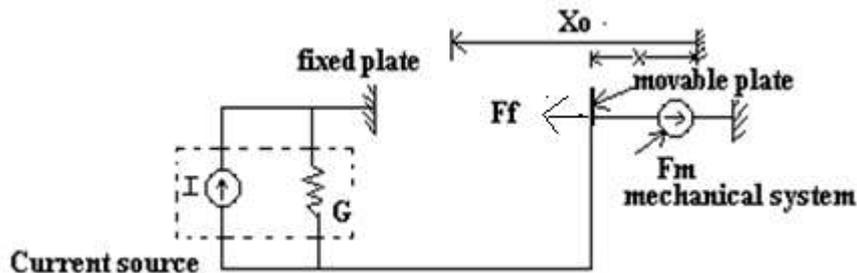


### Generator

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## 9. Derive an expression for mechanical force terms of field energy. (Apr-May 2015), Dec - 2016

- To realize the electro-mechanical energy conversion via electric field, we consider a model analogues to a condenser.
- The condenser model is shown in figure is considered to possess two parallel plates one of which is fixed and other is a movable one.
- The model is fed from a constant current source. The leakage current of the condenser is visualized as a conductance from exterior portion of the circuit.



- The movable plate is capable of moving in the horizontal direction as indicate by the variable  $X_o$ .
- For a given displacement 'x' the movable plate is considered to be fixed, a charging current 'I' flows into the condenser building a voltage 'V' between the fixed & movable plate.

- Hence some electrical energy is expected to be fed into the system resulting in a rise in potential.

This is expressed as,

$$dE_e = dW_e \times \text{time}$$

$$dW_e = V \frac{dq}{dt} \times \text{time.}$$

$$dW_e = V dq = dW_f = \text{Differential energy stored.}$$

Now the voltage 'V' is constant while the charge increases from 0 to q hence,

$$W_f = \int_0^q V dq$$

For a consider 'V' and 'q' are linearly related given by

$$C = \frac{q}{V} = \text{capacitance}$$

$$W_f = \int_0^q \frac{q}{C} dq = \frac{1}{2} \frac{q^2}{C}$$

$$\therefore W_f = \frac{1}{2} Vq$$

The capacitance of the parallel plate capacitor with a distance of (X<sub>0</sub>-X) b/w the plates is given by

$$C = \epsilon_0 \frac{A}{(X_0 - X)} \quad \text{when A-Cross-sectional area of plate in m}^2$$

$$\epsilon_0 - \text{permittivity of free space} = 8.8524 \times 10^{-12} \text{ F/M}$$

Thus W<sub>f</sub> is function of two independent variables q and x.

$$W_f(q, x) = \frac{1}{2} \frac{q^2}{C(x)}$$

From this eqn electric field energy can be changed

- i) Electrically by changing the charge 'q'.
- ii) Mechanically by changing the distance 'X' with moving plate

$$W_f(v, x) = \frac{1}{2} C(X) V^2 = \frac{1}{2} Vq$$

This energy density in the electric field is given by

$$W_f = \int_0^D E dD$$

$$= \frac{1}{2} \frac{D^2}{\epsilon_0}$$

$$= \frac{1}{2} \epsilon_0 E^2$$

Where,

$$D = \epsilon_0 E = \text{Electric flux density in C/m}^2$$

$$E = \text{Electric field intensity or potential gradient in V/m.}$$

Mechanical energy output = Electrical energy i/p – Increase or change in field energy

$$\therefore F_f dx = V dq - dW_f$$

-----&-----&-----

**10. The relay shown in fig. is made from infinitely permeable magnetic material with a movable plunger also of infinitely permeable material. The height of the plunger is much**

greater than the air gap length ( $h > g$ ). cal. the magnetic energy stored as a function of plunger position ( $x$ ) for  $N=1000$  turns.  $G=2.0\text{mm}$ ,  $d=0.5\text{m}$ ,  $l=0.1\text{m}$  and  $I=10\text{A}$ . [Nov/Dec-2010]

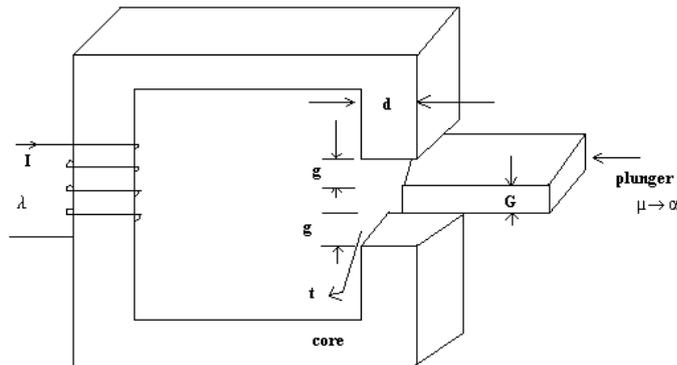
Given:  $N=1000$  turns.  $G=2.0\text{mm}$ ,  $d=0.5\text{m}$ ,  $l=0.1\text{m}$  and  $I=10\text{A}$

Formula used:

$$W_{\text{frd}} = \frac{1}{2} L(x) i^2$$

$$L(x) = \frac{\mu_0 N^2 A g_{\text{ap}}}{2g}$$

Solution:



$$W_{\text{frd}} = \frac{1}{2} L(x) i^2$$

$$L(x) = \frac{\mu_0 N^2 A g_{\text{ap}}}{2g}$$

$$A g_{\text{ap}} = l (d-x)$$

$$= l d (1-x/d)$$

$$L(x) = \frac{\mu_0 N^2 l d (1-x/d)}{2g}$$

$$W_{\text{frd}} = \frac{1}{2} \frac{\mu_0 N^2 l d (1-x/d)}{2g} (i^2)$$

$$= \frac{\frac{1}{2} \times 1000^2 \times 4\pi \times 10^{-7} \times 0.1 \times 0.5 \times 10^2}{2 \times (0.002)} \left(1 - \frac{x}{d}\right)$$

$$W_{\text{frd}} = \frac{0.0314}{2 \times 0.002} = 7.85 \left(1 - \frac{x}{d}\right)$$

$$\therefore W_{\text{frd}} = 7.85 \left(1 - \frac{x}{d}\right)$$

- &-----&-----
11. Two windings one mounted on the stator and the other mounted on a rotor have self and mutual inductances of  $L_{11}=4.5\text{H}$ ,  $L_{12}=2.5\text{H}$  and  $L_{21}=2.8\cos\theta\text{H}$ , where ' $\theta$ ' is the angle b/w the axes of the windings. The resistances of the windings may be neglected. Winding 2 is short circuited and the current in winding 1 as a function of time is  $i=10\sin\omega t\text{A}$ . Derive an expression for the numerical value of the instantaneous torque on the rotor in N.m in terms of the angle  $\theta$ . [Nov/Dec-2010, 2015]

**Given:**  $L_{11}=4.5H, L_{12}=2.5h$  and  $L_{12}=2.8\cos \theta, i=10\sin wt$

**Formula used:**  $T_f = \frac{\partial w_{mf}}{\partial \theta}; = \frac{i_1^2}{2} \frac{dL_{11}(\theta)}{d\theta} + i_1 i_2 \frac{dL_{12}(\theta)}{d\theta} + \frac{i_2^2}{2} \frac{dL_{22}(\theta)}{d\theta}$

**Soln:**

$$\begin{aligned} T_f &= \frac{\partial w_{mf}}{\partial \theta} \\ &= \frac{i_1^2}{2} \frac{dL_{11}(\theta)}{d\theta} + i_1 i_2 \frac{dL_{12}(\theta)}{d\theta} + \frac{i_2^2}{2} \frac{dL_{22}(\theta)}{d\theta} \\ &= \frac{10^2}{2} \frac{d(4.5)}{dt} + 10^2 \frac{d(2.5)}{dt} + i_1 i_2 \frac{dL_{12}(\theta)}{d\theta} \\ &= 0 + 0 + \frac{10 \times 10}{2} (-2.8 \sin \theta) \\ &= \frac{100}{2} (-2.8 \sin \theta) \\ T_f &= -140 \sin \theta \text{ N.m} \end{aligned}$$

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**12. The magnetic flux density on the surface of an iron face is 1.6T which is a typical saturation level value for ferromagnetic material. Find the force density on the iron face. (May/June-2011)**

**Soln:**

Let the area of iron face =  $A$  in  $m^2$

Consider the field energy in the volume contained b/w two forces with normal distance 'X'

$$W_f(B, x) = \frac{1}{2} \frac{\beta^2 Ax}{\mu_0} \qquad W_f = \frac{1}{2} s \phi^2 \quad (\phi, x)$$

Mechanical force due to

$$\begin{aligned} \text{The field is } F_f &= -d \frac{W_f(B, x)}{dx} = \\ &= \frac{1}{2} \left( \frac{x}{\mu_0 A} \right) \cdot B^2 A^2 \\ &= -\frac{1}{2} \frac{\beta^2 Ax}{\mu_0} \end{aligned}$$

(-ive sign indicates that the force acts in a direction so as to reduce X.)

$$\begin{aligned} \therefore \text{Force/unit area} = |F_f| &= \frac{1}{2} \frac{\beta^2}{\mu_0} \\ &= \frac{1}{2} \frac{(1.6)^2}{4\pi \times 10^{-7}} \end{aligned}$$

$$\begin{aligned} |F_f| &= \frac{2.037}{2} \times 10^6 \text{ N/m}^2 \\ |F_f| &= 1.0186 \times 10^6 \text{ N/m}^2 \end{aligned}$$

**13. The doubly-excited magnetic field has coil self-and mutual- inductances of  $L_{11}=L_{22}=2H$  and  $L_{12}=L_{21}= \cos 2\theta$ , Where  $\theta$  is the angle between the axes of the coils.**

**i) The coils are connected in parallel to a voltage source  $V=V_m \sin wt$ . Derive an expression for the instantaneous torque as a function of the angular position  $\theta$ . Find the time-average torque. Evaluate for  $\theta=30^\circ, V=100\sin 314t$ .**

**ii) If coil 2 is shorted while coil 1 carries a current  $i_1=I_m \sin wt$ , derive expression for the instantaneous and time-average torques. Compute  $\theta=45^\circ$  and  $i_1=\sqrt{2} \sin 314t$ . [Nov/Dec-2013]**

**Given:**  $L_{11}=L_{22}=2H$  and  $L_{12}=L_{21}= \cos 2\theta, V=V_m \sin wt$

**Soln:**

$$w'f(i_1, i_2, \theta) = \frac{1}{2}L_{11}i_1^2 + L_{12}i_1i_2 + \frac{1}{2}L_{22}i_2^2$$

$$= \frac{1}{2}(2i_1^2) + \cos\theta i_1i_2 + \frac{1}{2}(2i_2^2)$$

$$w'f(i_1, i_2, \theta) = i_1^2 + \cos\theta i_1i_2 + i_2^2$$

$$T_f = \frac{\partial w'f}{\partial \theta} = 0 - 2\sin 2\theta i_1i_2 + 0$$

$$= -2\sin 2\theta i_1i_2$$

Assuming that the coils are connected in parallel across the supply

$$e_1 = \frac{d\lambda_1}{dt} = L_{11} \frac{di_1}{dt} + L_{12} \frac{di_2}{dt} = 2 \frac{di_1}{dt} + \cos \theta \frac{di_2}{dt}$$

$$e_2 = \frac{d\lambda_2}{dt} = L_{21} \frac{di_1}{dt} + L_{22} \frac{di_2}{dt} = 2 \frac{di_1}{dt} + \cos \theta \frac{di_2}{dt}$$

$$e_1 = 2 \frac{di_1}{dt} + \cos 2\theta \frac{di_2}{dt}$$

$$e_2 = \cos 2\theta \frac{di_1}{dt} + 2 \frac{di_2}{dt}$$

$$0 = \cos 2\theta \frac{di_1}{dt} + 2 \frac{di_2}{dt} \quad [\text{coil 2 shorted } e_2 = 0V]$$

$$\frac{di_2}{dt} = -\frac{1}{2} \cos 2\theta \frac{di_1}{dt}$$

Integrating on both sides

$$i_2 = -\frac{1}{2} \cos 2\theta i_1 \quad [\text{but } i_1 = I_m \sin \omega t]$$

$$i_2 = -\frac{1}{2} I_m \cos 2\theta \sin \omega t$$

Sub  $i_1$  &  $i_2$  values in  $T_f$  equ. We get,

$$T_f = -\sin 2\theta (I_m \sin \omega t) \left( -\frac{1}{2} I_m \cos 2\theta \sin \omega t \right)$$

$$T_f = I_m^2 \sin 2\theta \cos 2\theta \sin^2 \omega t \quad [\text{Instantaneous torque}]$$

$$\therefore T_{(av)} = \frac{1}{2} I_m^2 \sin 2\theta \cos 2\theta \left[ \frac{1}{T} \int_0^T \sin^2 \omega t \, dt = 1/2 \right]$$

i) Both coils in parallel across  $V = 100 \sin 314 V$

$$V = e_1 = e_2$$

$$2 \frac{di_1}{dt} + \cos 2\theta \frac{di_2}{dt} = 100 \sin 314 V$$

$$\cos 2\theta \frac{di_1}{dt} + 2 \frac{di_2}{dt} = 100 \sin 314 V$$

$$2 \frac{di_1}{dt} + \cos 2\theta \frac{di_2}{dt} = 100 \sin 314 t \text{-----} 1$$



$$\text{self inductance } L = \frac{N\Phi}{i} = \frac{N}{i} \cdot \frac{Ni}{s} = \frac{N^2}{s}$$

$$l = \frac{N^2}{s}$$

$$\Phi = \frac{Ni}{s} = \frac{N\sqrt{2}V}{\sqrt{R^2 + w^2L^2}} \sin\left(\omega t - \tan^{-1}\left(\frac{wL}{R}\right)\right)$$

$$\Phi = \frac{N\sqrt{2}V}{\sqrt{R^2 + w^2L^2}} \sin\left(\omega t - \tan^{-1}\left(\frac{w}{R} \cdot \frac{N^2}{s}\right)\right)$$

$$= \frac{N\sqrt{2}V}{\sqrt{R^2S^2 + S^2w^2\left(\frac{N^2}{s}\right)^2}} \sin\left(\omega t - \tan^{-1}\left(w \cdot \frac{N^2}{RS}\right)\right)$$

$$\Phi = \frac{N\sqrt{2}V}{\sqrt{R^2S^2 + w^2N^4}} \sin\left(\omega t - \tan^{-1}\left(\frac{wN^2}{RS}\right)\right)$$

$$\therefore F_f = -\frac{1}{2}\Phi^2b$$

$$= -\frac{1}{2}\Phi^2b \left[ \frac{N\sqrt{2}V}{\sqrt{R^2S^2 + w^2N^4}} \sin\left[\omega t - \tan^{-1}\left(\frac{wN^2}{RS}\right)\right] \right]^2$$

$$= -\frac{\frac{1}{2}bN^2V^2}{R^2S^2 + w^2N^4} \sin^2\left(\omega t - \tan^{-1}\left(\frac{wN^2}{RS}\right)\right)$$

Time average force is  $F_{f(av)} = \frac{1}{T} \int_0^T F_f(t) dt$

$$= -\frac{1}{T} \int_0^T \frac{N^2V^2b}{R^2S^2 + w^2N^4} \left[ \frac{1 - \cos 2\left[\omega t - \tan^{-1}\left(\frac{wN^2}{RS}\right)\right]}{2} \right] dt$$

$$= -\frac{1}{T} \frac{N^2V^2b}{R^2S^2 + w^2N^4} \left[ t - \frac{\sin 2\left[\omega t - \tan^{-1}\left(\frac{wN^2}{RS}\right)\right]}{2} \right]_0^T$$

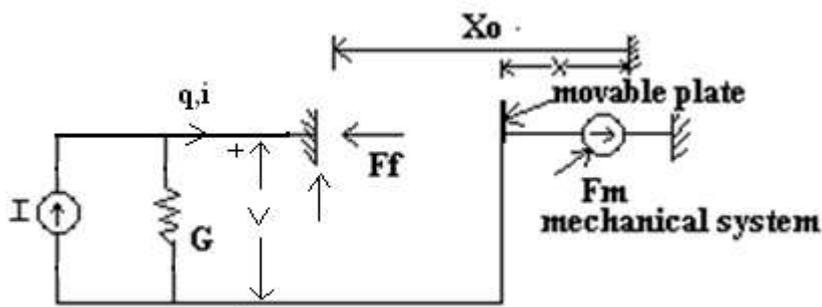
$$= -\frac{N^2V^2b}{2(R^2S^2 + w^2N^4)} \cdot \frac{1}{T} \left[ t - \frac{\sin 2\left[\omega t - \tan^{-1}\left(\frac{wN^2}{RS}\right)\right]}{2} \right]_0^T$$

$$\therefore T_f(av) = -\frac{1}{2} \cdot \frac{bN^2V^2}{R^2S^2 + w^2N^4} \left[ \frac{\omega T = 2\pi}{T = 2\pi/\omega} \right]$$

$$= \frac{1}{T} \left[ -\frac{\sin 2\left[\omega t - \tan^{-1}\left(\frac{wN^2}{RS}\right)\right]}{2} \right]_0^T \lambda = 0$$

-----&-----&-----

**15. Find an expression for the force per unit area between the plates of a parallel plate condenser in terms of the electric field intensity. Use both the energy and co-energy methods. Find the value of the force per unit area when  $E=3 \times 10^6 \text{V/m}$ , the breakdown strength of air. [May/June-2013]**



$$w_f(q, x) = \frac{1}{2} \frac{q^2}{\epsilon} = \frac{1}{2} \frac{q^2 (x_0 - x)}{A \epsilon_0}$$

$$F_f = \frac{-\partial w_f(q, x)}{\partial x} = \frac{1}{2} \frac{\epsilon^2}{A \epsilon_0}$$

$$Q = DA = \epsilon_0 EA$$

$$F_f = \frac{1}{2} \epsilon_0 E^2 A \text{ or } \frac{F_f}{A} = \frac{1}{2} \epsilon_0 E^2$$

$$= \frac{1}{2} \times (3 \times 10^6)^2 \times 8.85 \times 10^{-12}$$

$$= 39.8 \text{ N/m}^2$$

∴ The co energy is

$$w'_f(V, x) = 1/2 CV^2 = 1/2 V^2 \frac{A \epsilon_0}{(x_0 - x)}$$

$$F_f = \frac{\partial w'_f(V, x)}{\partial x} = 1/2 V^2 \frac{A \epsilon_0}{(x_0 - x)^2}$$

But  $V = E(x_0 - x)$

$$F_f = \frac{1}{2} \epsilon_0 E^2 A \text{ or } \frac{F_f}{A} = \frac{1}{2} \epsilon_0 E^2$$

-----&-----&-----

**16. The doubly-excited magnetic field has coil self-and mutual- inductances of  $L_{11}=L_{22}=2H$  and  $L_{12}=L_{21}= \cos\theta$ , Where  $\theta$  is the angle between the axes of the coils.**

**i) The coils are connected in parallel to a voltage source  $V = V_m \sin \omega t$ . Derive an expression for the instantaneous torque as a function of the angular position  $\theta$ . Find the time-average torque. Evaluate for  $\theta = 30^\circ$ ,  $V = 100 \sin 314t$ . [Nov/Dec-2013]**

$$T_f = \frac{\partial w'_f(i_1, i_2, \theta)}{\partial \theta}$$

$$w'_f(i_1, i_2, \theta) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

$$= \frac{1}{2} 2 i_1^2 + \cos \theta i_1 i_2 + \frac{1}{2} 2 i_2^2$$

$$= i_1^2 + \cos \theta i_1 i_2 + i_2^2$$

$$T_f = \frac{\partial (i_1^2 + \cos \theta i_1 i_2 + i_2^2)}{\partial \theta}$$

$$T_f = -\sin \theta i_1 i_2$$

From circuit eqn

$$e_1 = \frac{d\lambda_1}{dt} = L_{11} \frac{di_1}{dt} + L_{12} \frac{di_2}{dt} = 2 \frac{di_1}{dt} + \cos \theta \frac{di_2}{dt}$$

$$e_2 = \frac{d\lambda_2}{dt} = L_{21} \frac{di_1}{dt} + L_{22} \frac{di_2}{dt} = \cos \theta \frac{di_1}{dt} + 2 \frac{di_2}{dt}$$

$$V_m \sin \omega t = 2 \frac{di_1}{dt} + \cos \theta \frac{di_2}{dt} \text{-----} 1$$

$$V_m \sin \omega t = \cos \theta \frac{di_1}{dt} + 2 \frac{di_2}{dt} \text{-----} 2$$

Solving 1 & 2

$$\frac{di_1}{dt} = \frac{di_2}{dt} = \frac{V_m \cos \omega t}{(2 + \cos \theta)}$$

Integrating

$$i_1 = i_2 = \frac{V_m \sin \omega t}{\omega(2 + \cos \theta)}$$

Sub  $T_f$

$$T_f = -\frac{V_m^2 \sin \theta}{(2 + \cos \theta)^2 \omega^2} \sin^2 \omega t$$

$$T_f(\text{av}) = -\frac{V_m^2 \sin \theta}{(2 + \cos \theta)^2 \omega^2}$$

Given  $\theta = 30^\circ, V = 100 \sin 314t$

$$T_f(\text{av}) = -\frac{100^2 \sin 30}{2(2 + \cos 30^\circ)^2 \times (314)^2} = -\frac{5000}{1619755.125}$$

$$= -0.003086 \text{ Nm}$$

-----&-----&-----

**17. Derive an expression for co-energy density of an electromechanical energy conversion derive. (Nov/Dec-2013)**

Consider that the armature is held fixed at position 'x'.

As armature is not moving, the mechanical work done is zero.

According to energy balance equation,

$$\text{Input from supply} = \text{Mechanical output} + \text{Stored energy} + \text{loss}$$

The entire electric energy input gets stored in the magnetic field

$$\text{i.e., } dW_e = dW_f \quad [\text{since } dW_m = 0]$$

$$dW_f = e i dt = i d\lambda = N i d\phi \quad \text{----- (1)}$$

The relationship  $i-\lambda$  is basically non-linear for a magnetic circuit, similar to the B-H relationship.

From the Eqn-(1), the energy absorbed for a finite change in flux linkages can be obtained.

$$\Delta W_f = \int_{\lambda_1}^{\lambda_2} i(\lambda) d\lambda = \int_{\phi_1}^{\phi_2} f(\phi) d\phi$$

Where  $i(\lambda)$  is a function of  $\lambda$  and  $f(\phi)$  is a function of  $\phi$

If the initial flux and flux linkages are zero,

i.e.  $\phi_1 = \lambda_1 = 0$ , then the energy stored in the magnetic field to establish the flux  $\phi$  is given by,

$$W_f = \int_0^\lambda i \lambda d\lambda = \int_0^\phi f(\phi) d\phi$$

This is the energy stored in the magnetic field when the flux  $\phi$  is established in it.

In our analysis it is assumed that these losses are separated out and supplied by the electric source used. Hence the coil is ideal lossless coil. Such a magnetic system is called conversation system.

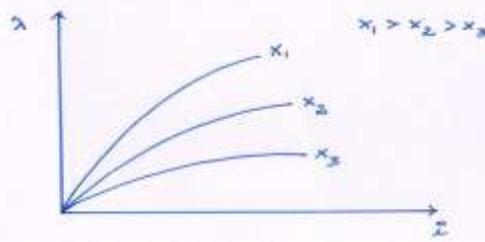
$i - \lambda$  relationship: The  $i - \lambda$  is similar to the magnetization curve for a magnetic material for various values of  $x$ , the relationship is shown in the fig

Practically  $\lambda$  may vary according to  $I$  or  $I$  may vary according to  $\lambda$ . So mathematically this relationship is expressed as,

$$i=i(\lambda, x)$$

$$\lambda =\lambda(i, x)$$

$\lambda$ =Independent variable  
 $i$ =Independent variable



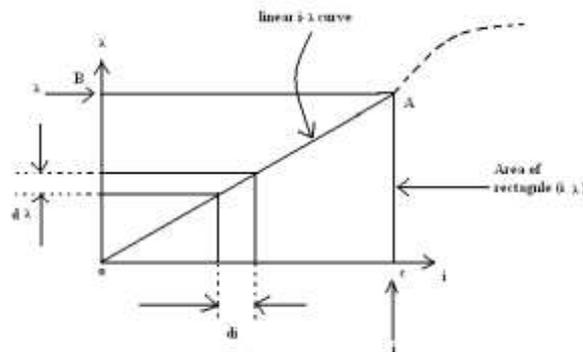
Depending upon the independent variable the stored field energy is also the function of  $i, x$  or  $\lambda, x$ .

$$W_f=W_f(\lambda, x) \quad \text{or} \quad W_f(i, x)$$

**Concept of co-energy:** When armature is held open then almost entire m.m.f. is required to derive the flux through air gap and magnetic saturation may not occur.

So  $i, x$  or  $F-\phi$  relationship is linear in nature as shown in the Fig

As per the equation



$$W_f=\int_0^\lambda i(\lambda)d\lambda = \text{Area OABO} = \text{Field energy}$$

$$\text{While Area OACO}=\int_0^\lambda \lambda di$$

This area OACO is complementary area of the  $i-\lambda$  rectangle and is defined as co-energy denoted as  $w'_f$ .

$$w'_f = \int_0^\lambda \lambda di = \text{Area OACO}=\text{co-energy}$$

For linear relationship between  $i$  and  $\lambda$  without magnetic saturation,

$$\text{Area OABO}=\text{Area OACO}$$

$$\text{i.e } W_f=w'_f$$

$$\therefore eW_f+w'_f = \text{Area OABO} = \text{Area OACO} = i\lambda$$

$$W_f=\frac{1}{2} i\lambda = \frac{1}{2} F\phi = \frac{1}{2} S\phi^2$$

Where  $S=\frac{F}{\phi}$  =Reluctance of magnetic circuit

The self inductance  $L$  of the coil is defined as magnetic flux linkages per ampere.

$$\therefore L = \frac{\lambda}{i}$$

$$\therefore W_f=\frac{1}{2} \frac{\lambda^2}{L} = \frac{1}{2} Li^2 \text{ joules}$$

The co-energy can be expressed for linear case as,

$$w'_f = \frac{1}{2} i\lambda = \frac{1}{2} F\phi = \frac{1}{2} PF^2 \text{ joules}$$

Where  $p=\frac{\phi}{F}$  =Permeance of magnetic circuit

From equation it can be observed that field energy  $W_f$  is function of two independent variables  $\lambda$  and  $x$ .

$$W_f(\lambda, x) = \frac{1}{2} \frac{\lambda^2}{L(x)}$$

where  $L(x)$  = inductance as a function of  $x$

From equation it can be observed that co-energy  $w'_f$  is function of two independent variables  $I$  and  $x$ .

$$\therefore w'_f = \frac{1}{2} L(x) i^2$$

The equations and are general expressions for energy and co-energy.

Magnetic stored energy density: The magnetic energy density is the magnetic stored energy per unit volume. It is denoted as  $W_f$

$$\begin{aligned} \therefore W_f &= \frac{W_f}{Volume} = \frac{W_f}{(Length\ of\ circuit)(Area\ normal\ to\ flux)} \\ &= \frac{W_f}{lA} = \frac{1}{2} \frac{i\lambda}{lA} = \frac{1}{2} \frac{f\phi}{lA} \\ \therefore W_f &= \frac{1}{2} \frac{F\phi}{lA} \end{aligned}$$

Now  $\frac{f}{l}$  = Magnetic field intensity =  $H$  in AT/m

While  $\frac{\phi}{A}$  = Magnetic flux density =  $B$  in Wb/m<sup>2</sup> or tesla

$$W_f = \frac{1}{2} BH \text{ J/m}^3 \quad \text{Stored field energy density}$$

For a magnetic circuit,  $B = \mu H$

Where  $\mu = \mu_0 \mu_r$

$$\therefore W_f = \frac{1}{2} \frac{B^2}{\mu} = \frac{1}{2} \mu H^2$$

Similarly the co-energy density can be written as,

$$\therefore w'_f = W_f = \frac{1}{2} \mu H^2 = \frac{1}{2} \frac{B^2}{\mu}$$

-----&-----&-----

**18. A double excited system has  $L_{11} = (4 + \cos 2\theta) \times 10^{-3} \text{ H}$ ,  $L_{12} = 0.15 \cos\theta \text{ H}$ ,  $L_{22} = (20 + 5 \cos 2\theta) \text{ H}$ . Find the torque developed if  $i_1 = 1 \text{ A}$ ,  $i_2 = 0.02 \text{ A}$ .**

**Given:**  $L_{11} = (4 + \cos 2\theta) \times 10^{-3} \text{ H}$ ,  $L_{12} = 0.15 \cos\theta \text{ H}$ ,  $L_{22} = (20 + 5 \cos 2\theta)$ ,  $i_1 = 1 \text{ A}$ ,  $i_2 = 0.02 \text{ A}$ .

**Formula used:**

$$w'_f(i_1, i_2, \theta) = \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2$$

**Solution:**

$$\begin{aligned} w'_f(i_1, i_2, \theta) &= \frac{1}{2} L_{11} i_1^2 + L_{12} i_1 i_2 + \frac{1}{2} L_{22} i_2^2 \\ w'_f(i_1, i_2, \theta) &= \frac{1}{2} (4 + \cos 2\theta) \times 10^{-3} i_1^2 + (0.15 \cos\theta) i_1 i_2 + \frac{1}{2} (20 + 5 \cos 2\theta) i_2^2 \\ T_f &= \frac{\partial w'_f(i_1, i_2, \theta)}{\partial \theta} \\ &= -\sin 2\theta \times 10^{-3} i_1^2 - 0.15 \sin\theta i_1 i_2 - 5 \sin 2\theta i_2^2 \\ &= -10^{-3} \sin 2\theta - 3 \times 10^{-3} \sin\theta \end{aligned}$$

The negative sign indicates that the torque is restoring in nature. i.e. it opposes the displacement  $\theta$ .

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19. Two coupled coils have self and mutual inductance of  $L_{11} = 3 + 0.5x$ ;  $L_{22} = 2 + 0.5x$ ;  $L_{12} = 0.3x$  over a certain range of linear displacement  $x$ . The first coil is excited by a constant current of 15A and the second by a constant current of -8A. Determine (i) Mechanical work done if  $x$  changes from 0.6m to 1m (ii) Energy supplied by each electrical source. (May 2018)

Given:  $L_{11} = 3 + 0.5x$ ;  $L_{22} = 2 + 0.5x$ ;  $L_{12} = 0.3x$ ,  $A = 15A$

Formula used:

$$W_f' = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + L_{12}i_1i_2$$

Solution:

$$\begin{aligned} &= \frac{1}{2} \times 3 + 0.5x \times 15^2 + \frac{1}{2} \times 2 + 0.5x \times (-8)^2 + 0.3x \times 15 \times (-8) \\ &= 401.5 + 36.25x \end{aligned}$$

Mechanical work done = 172.2J

Energy supplied by source 1  $W_{e1} = \int_{0.6}^1 i_1 d\lambda_1$

$$\lambda_1 = L_{11}i_1 + L_{12}i_2$$

$$\lambda_1 = 45 + 5.1x$$

$$W_{e1} = 30.6J$$

Energy supplied by source 2  $W_{e2} = \int_{0.6}^1 i_2 d\lambda_2$

$$\lambda_2 = L_{22}i_2 + L_{12}i_1$$

$$\lambda_2 = -16 + 0.5x$$

$$W_{e2} = -1.6J$$

## UNIT 4

### PART-A

**1. Write down the emf equation for D.C generator.(Nov-Dec 2013, 2016)**

$$E_g = (P \Phi ZN / 60A) V$$

Where P = number of poles,  $\Phi$  = flux per pole

Z= Total number of conductors

A= number of parallel paths

N= speed in rpm

**2. Define the term armature reaction in dc machines.(Dec - 2012, May-2013, 2017)**

The interaction between the flux set up by the current carrying armature conductors with the main field flux is defined as armature reaction.

**3. What are the conditions for parallel operation of dc generator?(Nov-2009)**

- 1.The voltage of both the generator must be equal.
- 2.The change of voltage with change of load should be of same character.
- 3.The polarities of the generator must be same.
- 4.Prime movers driving the generator should have stable & similar rotational speed characteristics.

**4. What is the function of yoke?(May-2010,Dec-2017)**

It serves the purpose of outermost cover of the dc machine. So that the insulating material get protected from harmful atmospheric elements the moisture, dust and various gases like CO<sub>2</sub>.

**5. How to reduce the effect of armature reaction?(May-2010)**

- 1.Armature reaction neutralized by compensating windings.
- 2.Increasing length of air gap, increase the path of cross magnetizing field.
3. By use of interpoles.

**6. What is the function of compensating winding?(June - 2006)**

The compensating windings are basically used to neutralize the armature flux in the pole arc region which will otherwise cause severe distortion of main field flux.

**7. Name the various method of decreasing the effect of armature reaction.**

- 1.The reluctance of the path of the cross magnetizing field is increased.
- 2.If reluctance at pole tips are increased.
- 3.The effect of armature reaction can be neutralized by use of compensating winding.

**8. Define commutator?(June2012)**

The commutator is a device which converts alternating induced emf in a generator to dc. In case of motor it produces unidirectional torque.

**9. Define commutation?(June- 2012)(Dec- 2014)**

The process by which current in the short circuited coil is reversed while it crosses the MNA is called commutation. The time during which the coil remains short circuited is known as commutation period.The period is 0.0005 to 0.002S.

**10. State the causes of failure to excite self excited generated?(Dec-2010) May 2017**

1. Absence of residual magnetism due to ageing
2. The field winding may not be properly connected with armature
3. Under no load condition, the shunt field resistance should be greater than the critical field resistance.
4. Under load condition, the shunt field resistance should be less than the critical field resistance.

**11. State the application of various types of generator?(Apr-2010)**

Series Generator: Booster on dc feeders, as a constant current generator for welding.  
Shunt generator: Commonly used in battery charging and ordinary lighting purpose  
Compound generator: Domestic lighting and electric arc welding.

**12. What is coil span?(Dec- 2005)**

The two coil sides of a coil are embraced by number of teeth which is known as coil span.

**13. Define pole pitch?(Dec- 2008)**

It is defined as the distance between the two adjacent poles.

**14. Name the methods of improving commutation?(May2003,2006,Dec2009)**

1. Resistance commutation.
2. Giving a brush shift
3. Use of interpoles.

**15. What are the major parts of dc generator?(May2009)**

Yoke, pole, pole shoe, field winding, armature winding, interpoles, brushes and commutators.

**16. Why are carbon brushes preferred for dc machines?**

The high contact resistance carbon brushes help the current in the coil undergoing commutation to attain its full value in the reverse direction at the end of commutation. The carbon brushes also lubricate and give less wear and tear on commutator surface.

**17. What do you mean by armature reaction?(Nov-Dec 2012,2011,May-2013)**

The effect of armature flux on the distribution of main field flux is called armature reaction.

**18. Compare Lap and Wave winding?(May-2014, 2016)**

According to the connection armature winding classified into

<b>Lap winding</b>	<b>Wave winding</b>
Finishing end of one coil is connected with starting end of next coil	Finishing end of one coil is connected with starting end of another coil which is always away from the first coil.
Number of parallel paths (A)=poles(P)	Number of parallel paths (A)=poles(P)
Number of brush sets required is equal to number of poles	Number of brush sets required is always equal to two
Preferable for high current ,low voltage capacity generators	Preferable for high voltage ,low current capacity generators
Normally used for generators of capacity more than 500 A.	Preferred for generators of capacity less than 500A.

**19. List the factors involved in the voltage build up of a shunt generator?(May 2014)**

1. There must be some residual magnetism in the generator poles.
2. For the given direction of rotation, the shunt field coils should be correctly connected to the armature i.e. they should be so connected that the induced current reinforces the mmf produced initially due to residual magnetism.
3. If excited on open circuit, its shunt field resistance should be less than the critical resistance (which can be found from its O.C.C.)
4. If excited on load, then its load resistance should be more than a certain minimum value of resistance which is given by internal characteristic

**20. Why the external characteristics of DC shunt generator is more drooping than that of a separately excited generator?(May 2014)**

$V_t + I_a R_a = E$ , neglecting other drops. If load current  $I_L$  increases,  $I_a$  increases. Thus the drop  $I_a R_a$  increases and terminal voltage  $E - I_a R_a = V_t$  decreases. But the value of armature resistance is very small, the drop in terminal voltage as  $I_L$  changes from no load to full load is very small. But its characteristic is drooping in nature.

**21. Differentiate compensating winding and interpoles. Dec - 2016**

Both are connected in series and will neutralize armature reaction. But inter poles in addition supplying reversing emf to improve commutation. The position of inter poles is near the commutating area.

**22. What is Critical Resistance of a D.C shunt generator(Dec-2017)**

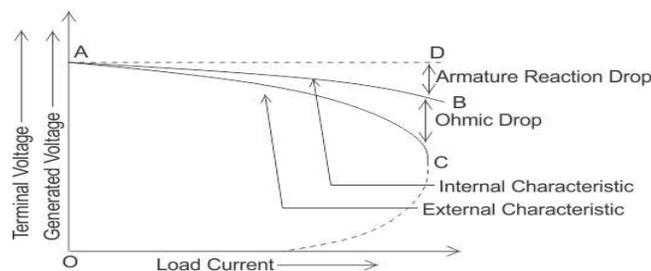
The critical field resistance at that resistance of the field circuit at a given speed at which generator just excites and starts voltage buildings. Beyond this value of resistance the machine will fail to build voltage.

**23. An 8-pole, wave-connected armature has 600 conductors and is driven at 625 rev/min. If the flux per pole is 20m Wb, determine the generate e.m.f. (Nov/Dec 2013)**

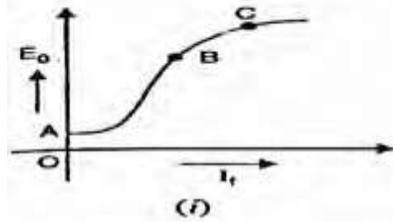
$$Z = 600, A = 2 \text{ for wave winding, } P = 4 \text{ pairs} = 4 \times 2 = 8, N = 625, \phi = 20 \times 10^{-3}$$

$E_g = \frac{\Phi P N Z}{60A} \text{ in volts}$	= 500 Volts	$E_g = \frac{20 \times 10^{-3} \times 8 \times 625 \times 600}{60 \times 2}$
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**24. Draw various characteristics of DC shunt generator. (May 2016)**



**25. Draw and explain magnetizing characteristics of DC shunt generator (May-2018)**



26. Why the armature core in the D.C machine is constructed with laminated steel sheets instead of solid steel sheets? (May-2018)

To minimize the cost of construction the machine and keep eddy current loss as low as possible.

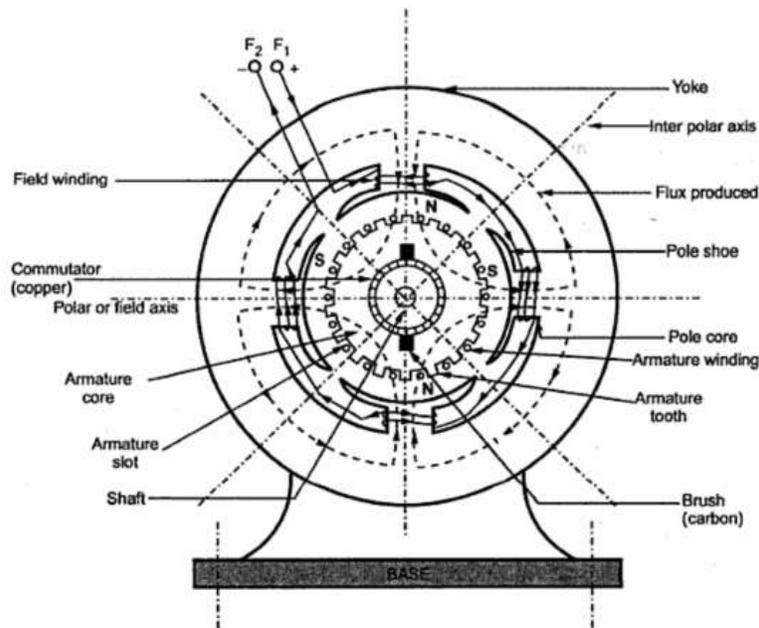
**Part – B**

1. Explain the construction of dc machines. [May-2010] Dec - 2016

It consists of the following parts

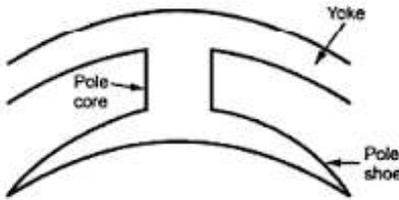
- 1.Yoke
- 2.Poles
- 3.Field winding
- 4.Armature
- 5.Commutator
- 6.Brushes and brush gear
- 7.Bearings.

1. Yoke:



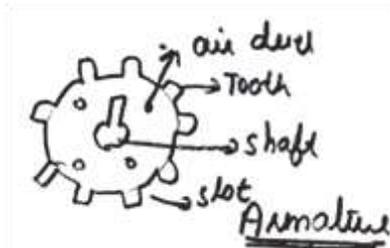
- 1.It serves the purpose of outermost cover of the dc machine. So that insulating materials get protected from harmful atmospheric elements like moisture dust and various gases like SO<sub>2</sub>
- 2.It provider mechanical support to the poles.

2. Poles:



1. Poles core basically carrier a field winding which is necessary to produce the flux.
2. it directs the flux produced through air gap to armature core, to the next pole.

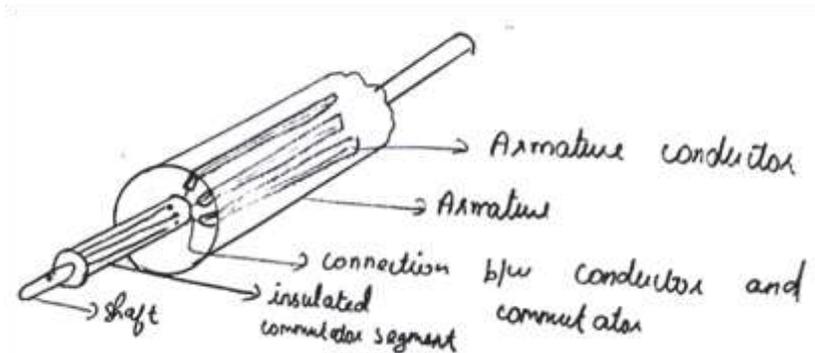
### 3. Field windings:



To carry current due to which pole core, on which the field winding is placed, behaves as an electromagnet, producing necessary flux.

As it helps in producing the magnetic field it is called field windings.

### 4. Armature:



Armature core provides house for armature windings.

To provide a path of low reluctance to the magnetic flux produced by the field windings.

### 5. Commutator:

To facilitate the collection of current from the armature conductors

To convert internally developed alternating emf to individual emf.

To produce unidirectional torque in case of motors.

### 6. Brush and Brush gear:

To collect current from commutator and make it available to the stationary external circuit.

### 7. Bearings:

Ball bearings are usually used as they are more reliable. For heavy duty machines, roller bearings are preferred.

## Generator Principle

An electric generator is a machine that converts mechanical energy into electrical energy. An electric generator is based on the principle that whenever flux is cut by a conductor, an e.m.f. is induced which will cause a current to flow if the conductor circuit is closed. The direction of induced e.m.f. (and hence current) is given by Fleming's right hand rule. Therefore, the essential components of a generator are:

- (a) a magnetic field
- (b) conductor or a group of conductors
- (c) motion of conductor w.r.t. magnetic field.

### **Simple Loop Generator**

Consider a single turn loop ABCD rotating clockwise in a uniform magnetic field with a constant speed as shown in Fig.(1.1). As the loop rotates, the flux linking the coil sides AB and CD changes continuously. Hence the e.m.f. induced in these coil sides also changes but the e.m.f. induced in one coil side adds to that induced in the other.

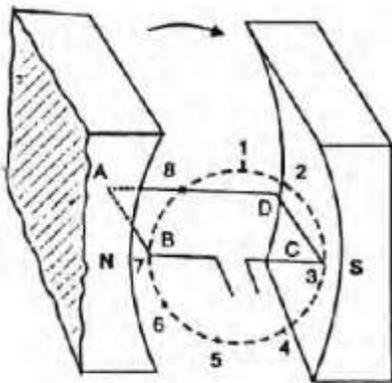


Fig. (1.1)

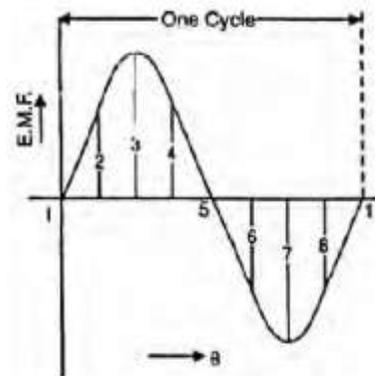


Fig. (1.2)

- (i) When the loop is in position no. 1 [See Fig. 1.1], the generated e.m.f. is zero because the coil sides (AB and CD) are cutting no flux but are moving parallel to it.
- (ii) When the loop is in position no. 2, the coil sides are moving at an angle to the flux and, therefore, a low e.m.f. is generated as indicated by point 2 in Fig. (1.2).
- (iii) When the loop is in position no. 3, the coil sides (AB and CD) are at right angle to the flux and are, therefore, cutting the flux at a maximum rate. Hence at this instant, the generated e.m.f. is maximum as indicated by point 3 in Fig. (1.2).
- (iv) At position 4, the generated e.m.f. is less because the coil sides are cutting the flux at an angle.
- (v) At position 5, no magnetic lines are cut and hence induced e.m.f. is zero as indicated by point 5 in Fig. (1.2).
- (vi) At position 6, the coil sides move under a pole of opposite polarity and hence the direction of generated e.m.f. is reversed. The maximum e.m.f. in this direction (i.e., reverse direction, See Fig. 1.2) will be when the loop is at position 7 and zero when at position 1. This cycle repeats with each revolution of the coil.

Note that e.m.f. generated in the loop is alternating one. It is because any coil side, say AB has e.m.f. in one direction when under the influence of N-pole and in the other direction when under the influence of S-pole. If a load is connected across the ends of the loop, then alternating current will flow through the

load. The alternating voltage generated in the loop can be converted into direct voltage by a device called commutator. We then have the d.c. generator. In fact, a commutator is a mechanical rectifier

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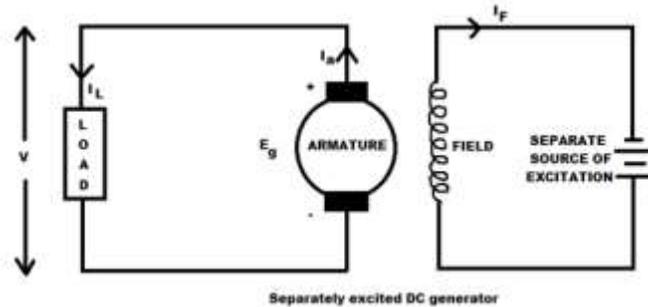
**2. Various methods of excitation of dc machines (Dec – 2006)**

Depending on the method of excitation used the dc generator are classified as

1. Separately excited generator
2. Self excited generator

**1. Separately excited generator**

When the field winding is supplied from external separate dc supply i.e. excitation of field winding is separate then the generator is called separately excited generator.



Voltage and current relation:

For armature side, we can see that it is supplying a load, demanding a load current of  $I_c$  at a voltage of  $V_t$  which is called terminal voltage.

$$I_a = I_L$$

So voltage equation for separately excited generator can be written as

$$E = V_t + I_a R_a + V_{brush}$$

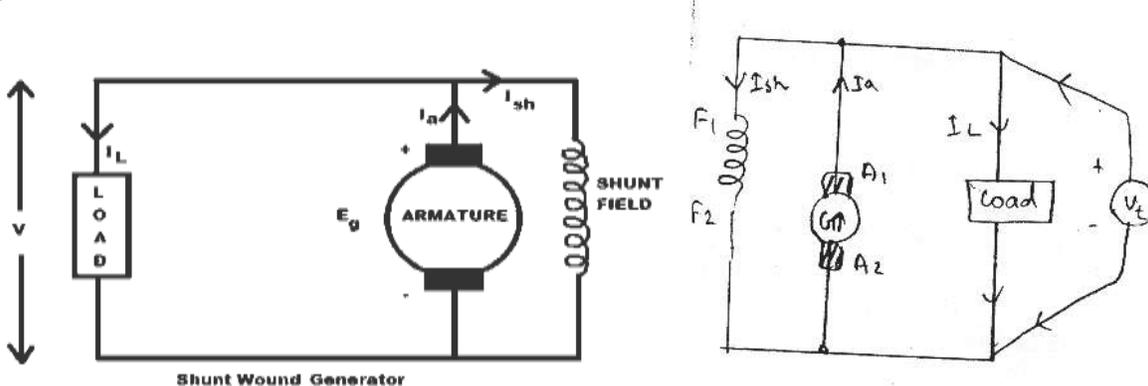
Where  $E = \frac{\phi p n z}{60 A}$

Self-excited generator:

Based on field winding is connected to the armature to derive its excitation

1. Shunt generator
2. Series generator
3. Compound generator

**Shunt generator:**



When the field winding is connected in parallel with the armature and the combination across the load is called shunt generator.

Voltage and current relation:

$$I_a = I_L + I_{sh}$$

Now voltage across load is  $V_t$  which is same across field winding as both in parallel with each other

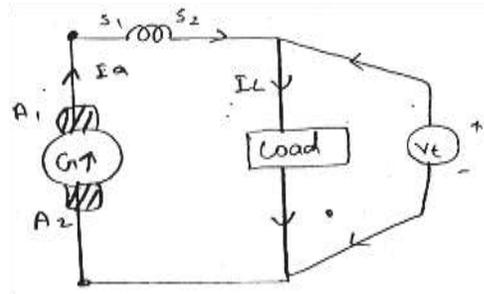
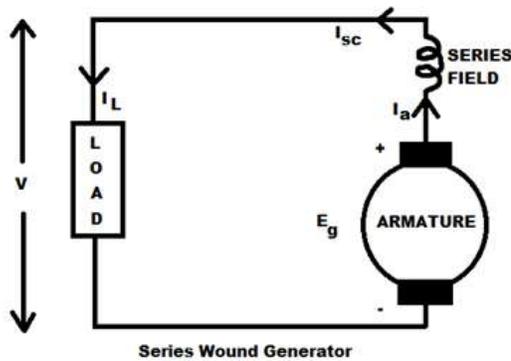
$$I_{sh} = \frac{V_t}{R_{sh}}$$

While induced emf,

$$E = V_t + I_a R_a + V_{brush}$$

Where, 
$$E = \frac{\phi p n z}{60 A}$$

**Series generator:**



When the field winding is connected in series with the armature winding while supplying the load is called series generator

Voltage and current relation:

$$I_a = I_{sc} = I_L$$

Now in addition to drop  $I_a R_a$ , induced emf has to supply voltage drop across series field winding too

$$E = V_t + I_a R_a + I_a R_{sc} + V_{brush}$$

$$E = V_t + I_a (R_a + R_{sc}) + V_{brush}$$

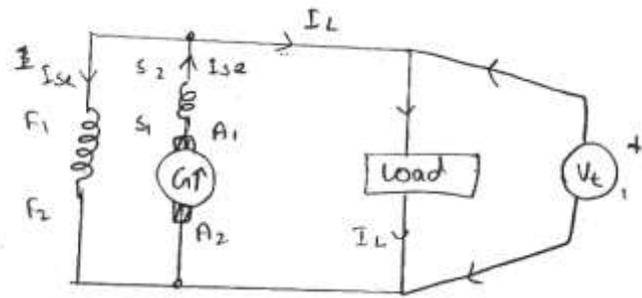
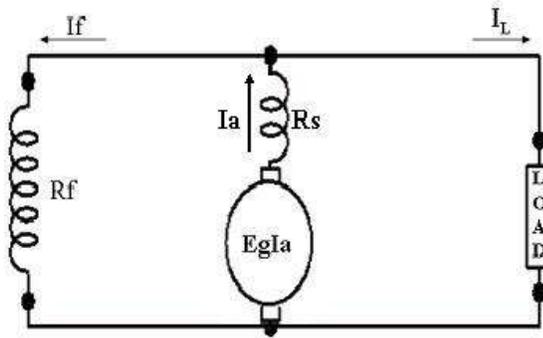
Where 
$$E = \frac{\phi p n z}{60 A}$$

**Compound generator:**

- 1. Long shunt compound generator
- 2. Short shunt compound generator

**Long shunt compound generator:**

In this type, shunt field winding connected across the series combination of armature a series field winding.



$$I_a = I_L + I_{sh}$$

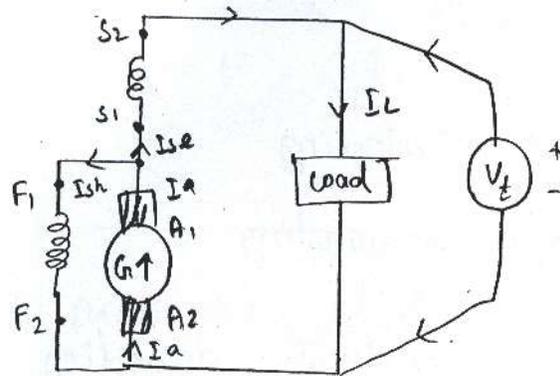
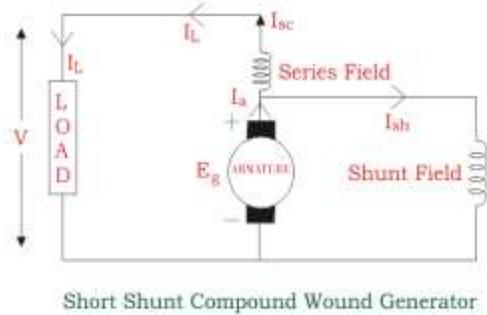
Voltage across shunt field winding is  $V_t$

$$I_{sh} = \frac{V_t}{R_{sh}}$$

Voltage equation is

$$E = V_t + I_a R_a + R_a R_{sc} + V_{brush}$$

**Short shunt compound generator:**



Voltage and current relation are as follows

$$I_a = I_{sc} + I_{sh}$$

$$I_{sc} = I_L$$

$$I_a = I_L + I_{sh}$$

The drop across field winding is drop across the armature

$$I_{sh} = \frac{E - I_a R_a}{R_{sh}}$$

Now the voltage equation

$$E = V_t + I_a R_a + I_{sc} R_{sc} + V_{brush}$$

$$I_{sc} = I_L$$

$$E = V_t + I_a R_a + I_L R_{sc} + V_{brush}$$

Neglecting  $V_{brush}$

$$E = V_t + I_a R_a + I_L R_{sc}$$

$$E - I_a R_a = V_t + I_L R_{sc}$$

$$I_{sh} = \frac{V_t - I_a R_a}{R_{sh}}$$

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### 3. Derive the expression for generated voltage in DC machines [Dec-2011, 2013][May – 2015, 2016]

The emf induced in a dc machine.

Let  $\Phi$  = flux per pole

Assume that full flux per pole links with the armature coil to produce the induced emf

$N$  = speed in rpm

So in one revolution, the total flux cut by one conductor is,

$d\Phi$  = Total flux cut in one revolution =  $\Phi \times P$  in Wb

$dt$  = Total time required for one revolution

=  $\frac{60}{N}$  sec

Hence average emf induced in the conductor is,

$$\begin{aligned} e_{av} &= \frac{d\Phi}{dt} \\ &= \frac{d\Phi}{\left(\frac{60}{N}\right)} \\ &= \frac{\Phi P N}{60} \end{aligned}$$

The armature winding is wound with two layer winding forming a closed path. The two conductors constitute one turn and there are  $Z$  number of conductors arranged in  $A$  number of parallel paths.

$\frac{Z}{A}$  = Conductors per parallel path

All conductors per parallel path are in series.

Thus the total induced emf per parallel path,

$$E = \frac{\Phi P N \times \frac{Z}{A}}{60}$$

The voltage across all parallel paths remains same which is total induced emf 'Eg' in a dc machine

$E_g = \frac{\Phi P N Z}{60 A} \text{ in volts}$
--

The number of parallel paths depends on type of armature winding

$A = P$  for lap type winding

$A = 2$  for wave type winding

As two conductor constitute one turn, Series turns/parallel path =  $\frac{Z}{2A}$

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### 4. Explain the process of commutator in dc generator. (May - 2006, 2010) Dec - 2016

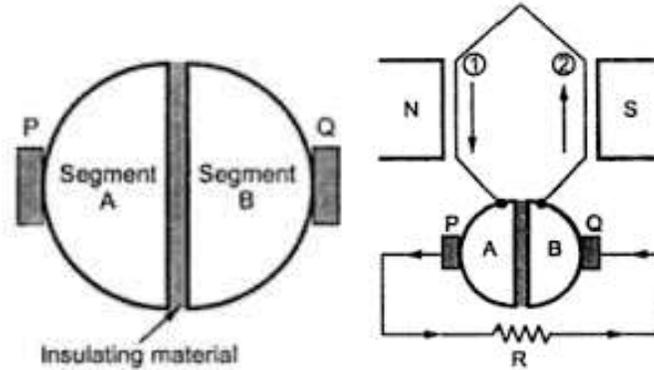
The induced emf in the conductor is always sinusoidal and commutator converts this sinusoidal emf to unidirectional emf.

Commutator is divided into number of copper segments insulated from each other. So in its simplest form it is a ring with halves separated by insulation

Such a ring is called split ring. The brushes P and Q are stationary and presented on the surface of split ring. Split ring is mounted on the shaft and rotates as armature rotates.

Consider a single turn generator with conductors (1) and (2). The armature conductors are connected to the two segments of split ring. The external resistance R is connected across brushes P and Q. Under instant 1, the current flowing through resistance R is flowing from left to right as shown in figure.

This is by assuming the direction of current through conductor (1) downwards which is under N pole and through conductor (2) upwards which is under S pole, at the instant considered. At this instant brush P behaves positive and Q as negative.

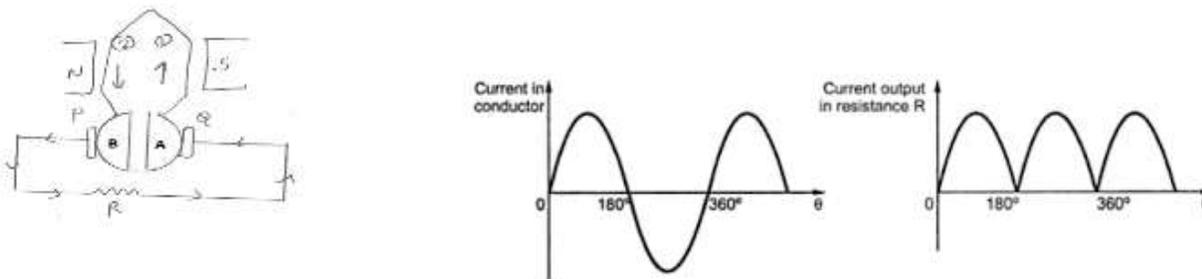


After a next half revolution, we have seen that direction of emf in the individual conductors reverses.

Hence conductor (1) now will carry a current which will be upwards and due to half revolution it will be under S pole. Similarly conductor (2) individually will carry a current downwards now, and will be under N pole as shown in figure.

Now split ring i.e. commutator is mounted on shaft and rotates with armature so when conductors will reverse their position, the split ring sections will also reverse their positions. But brushes P and Q are stationary and tapping the current from the commutator segments which are in contact with them.

Hence under instant 2, segment B will be in contact with brush P and segment A will be in contact with brush Q. Due to this current through resistance R maintains its direction from left to right as shown. Brush P remains positive and Q as negative.



The wave form of current in the Individual conductor and currents in external resistance R. So one brush remains always positive and other always negative and the load current is unidirectional.

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**5. Describe the effects of armature reaction on the operation of dc machines. Also remedies employed for decreasing the effect of armature reaction. (Dec- 2009) (May 2016)**

**Effects of armature reaction:**

- 1.The armature reaction always results in reduction of generated emf due to decrease in value of flux per pole.
- 2.Due to distortion in main field flux the maximum density at load increases above no load. Thus iron losses are observed to be more on load than on no load.
- 3.Due to armature reaction the maximum value of gap density increases.
- 4.Thus flux density in the interpoles axis is not zero but having same value.
- 5.Thus there will be an induced emf in the coil undergoing commutation which will try to maintain the current in original direction.
- 6.This will make commutation difficult and will cause delayed commutation.

**Reduction of effects armature reaction:**

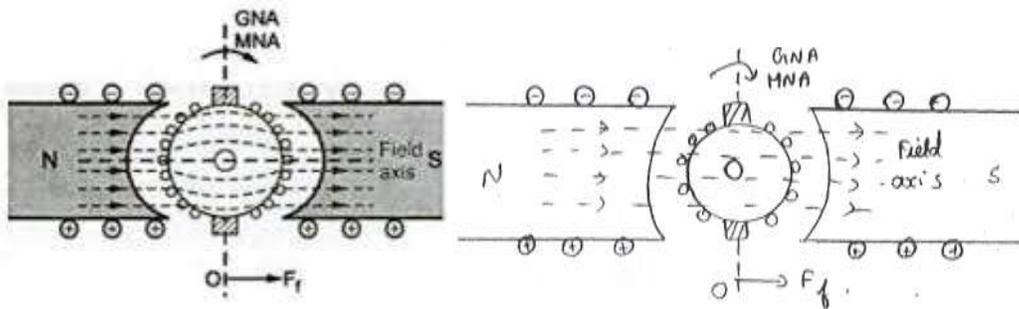
- 1.Armature reaction neutralized by compensating windings.
- 2.Increasing length of air gap, increase the path of cross magnetizing field.
- 3.By use of interpoles this induce mmf

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**6. Explain Armature Reaction (May- 2016, 2017)**

**Armature Reaction:**

Assuming that the generator is not driving any load. So that there is no current in the armature conductor. The flux is distributed symmetrical with respect to axis called polar axis.



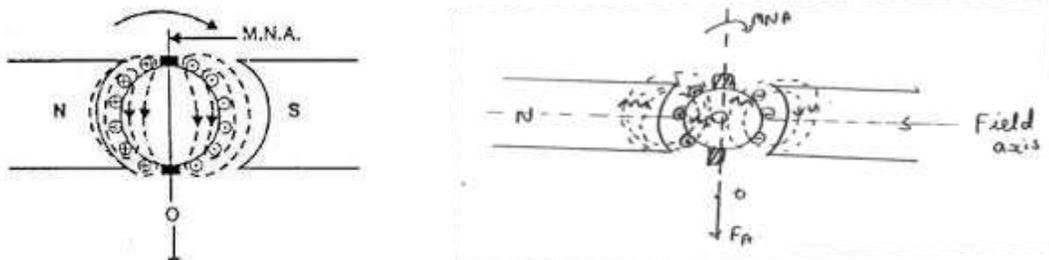
The axis along there is no emf induced in the armature conductors is called magnetic neutral axis it can be seen that magnetic neutral axis and geometric neutral axis coincides with each other.

The brushes are always kept along MNA.

Now we will consider that the field coils are unexcited whereas the armature conductors are carrying current.

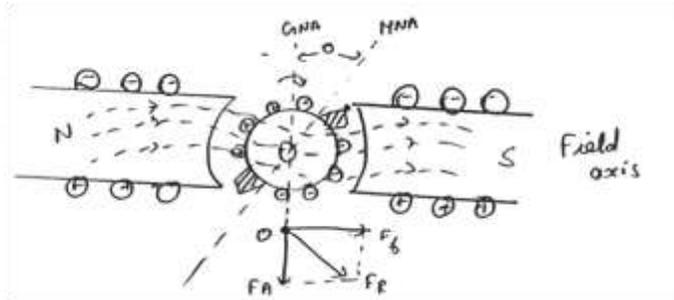
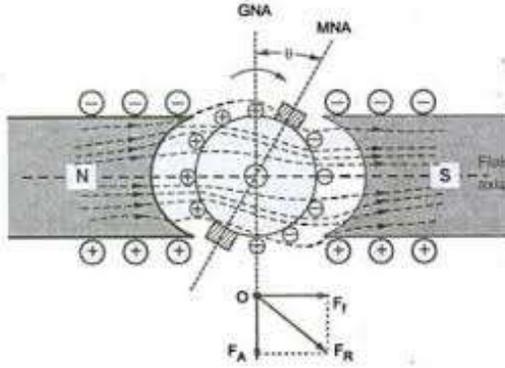
The direction of the armature conductors can be found by applying Fleming’s right hand rule

The direction of the flux produced by current carrying conductors is vertically downward in the armature core. This flux is symmetrically about a brush axis.



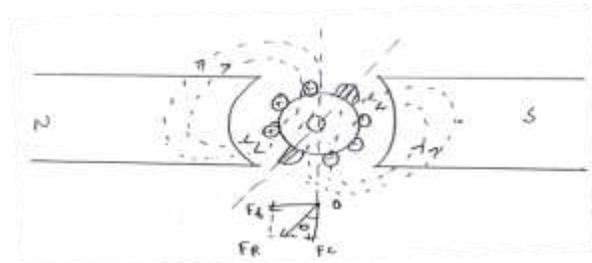
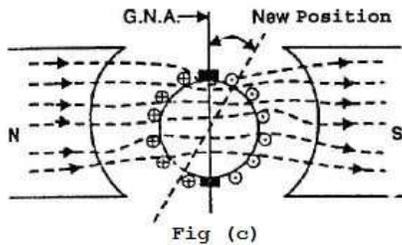
The vector of a represents the armature direction both in magnitude. This mmf depends on the magnitude of the armature current.

Now the flux through the armature is not uniform and symmetrical. The flux gets distorted due to interaction of two fluxes; the resultant flux distribution is changed.



The flux is crowded or concentrated at the traveling pole tips but weeded out at the leading pole tips.

The new position of MNA is to the resultant mmf vector of R



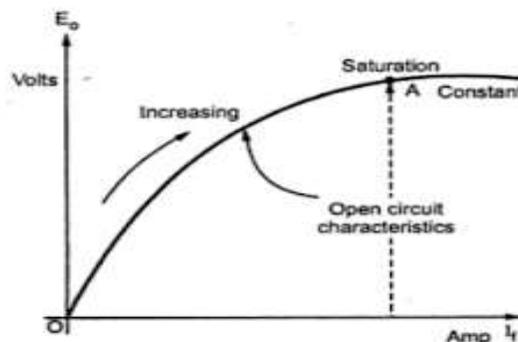
The conductors on the left of new position of MNA carry current downward and those to the right carry upwards.

The component of d is in direct opposition with field mmf vector off. Hence this component is called demagnetizing component.

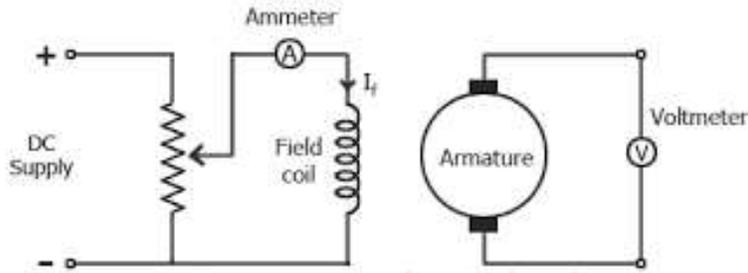
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## 7. OC characteristics and external characteristics of dc generator. (May-2010)

**OC characteristics:**



The rheostat as a potential divider is used to control the field current and the flux. It is varied from zero and is measured on ammeter connected.



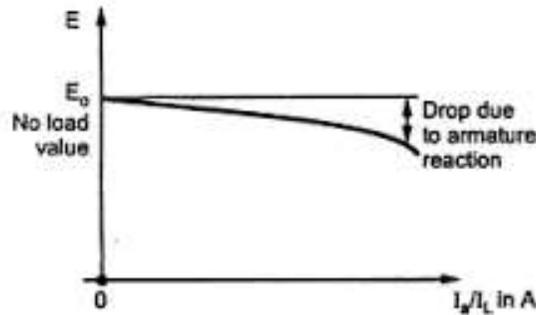
electricaleasy.com

As  $I_f$  is varied, then  $\phi$  changes and hence induced emf.  $E_o$  also varies. It is measured on voltmeter connected across armature.

No load is connected to machine hence characteristics are also called no load characteristics.

As  $I_f$  increases, flux  $\phi$  increases and  $E_o$  increases. After point A, saturation across when  $\phi$  becomes constant and hence  $E_o$  saturates.

**External characteristics:**



$E = V_t + I_a R_a$  neglecting other drops. So as load current  $I_c$  increases,  $I_a$  increases. Thus the drop  $I_a R_a$  increases and terminal voltage

$V_t = E - I_a R_a$  decreases.

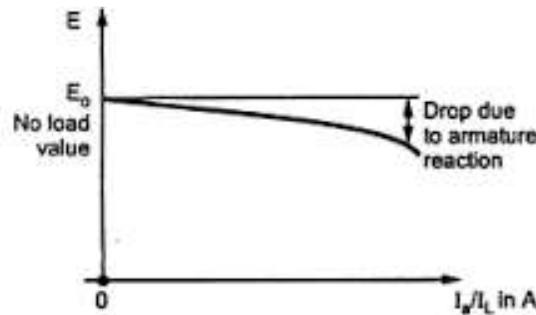
But the value of armature resistance is very small, the drop in terminal voltage as  $I_c$  changes from no load to full load is very small.

Hence dc shunt generator is called constant voltage generator.

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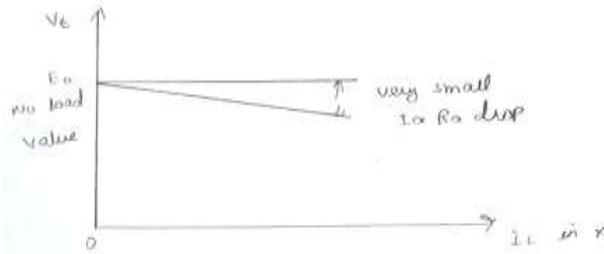
**8. Draw the load characteristics (Dec-2010, May 2016)**

**Internal Characteristics:**

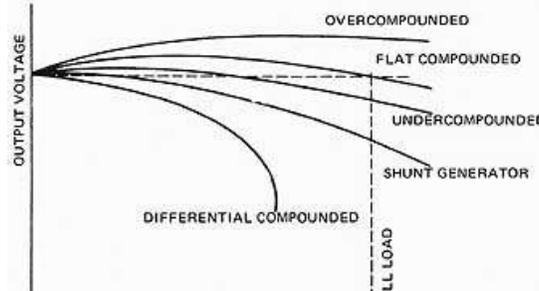


The effect of flux produced by armature on the main flux produced by through field winding is called an armature reaction.

**External Characteristics:**

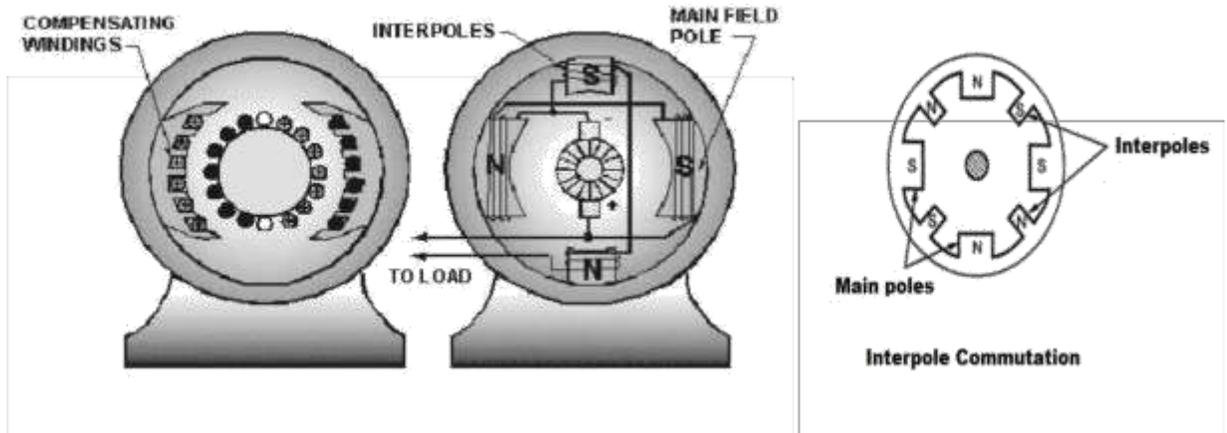


**DC compound generator:**



**9) Explain interpoles?**

Interpoles are used to neutralize reactance voltage induced in the coil undergoing commutation by using small poles fixed to the yoke and placed in between the main poles along geometrical neutral axis. These poles are called interpoles.



The interpoles have a few turns of large wire and are connected in series with the armature. Interpoles are wound and placed so that each interpole has the same magnetic polarity as the main pole ahead of it, in the direction of rotation. The field generated by the interpoles produces the same effect as the compensating winding. This field, in effect, cancels the armature reaction for all values of load current by causing a shift in the neutral plane opposite to the shift caused by armature reaction. The amount of shift caused by the interpoles will equal the shift caused by armature reaction since both shifts are a result of armature current.

The emf induced in the interpoles is called commutating or reversing emf which will neutralize reactance voltage and making sparkles commutation.

The other advantage of interpoles is neutralizing cross magnetizing effects of armature reaction.  $OF_f$ -mmf due to main pole,  $OF_c$  cross magnetizing mmf,  $OF_i$ -mmf due to interpole it is in opposition to the  $OF_c$ .

**10) Explain in detail about commutation and list various Methods to improve commutation in details with a neat sketch. (Dec-2017)**

There are two main **methods of improving commutation**. These are

- a. Resistance commutation
- b. E.M.F. commutation

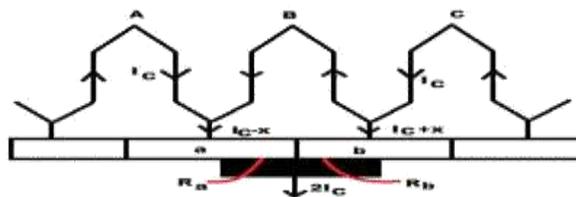
**Resistance Commutation**

In this method of commutation we use high electrical resistance brushes for getting spark less commutation. This can be obtained by replacing low resistance copper brushes with high resistance carbon brushes.

We can clearly see from the picture that the current  $I_C$  from the coil C may reach to the brush in two ways in the commutation period. One path is direct through the commutator segment b and to the brush and the 2<sup>nd</sup> path is first through the short-circuit coil B and then through the commutator segment to the brush. When the brush resistance is low, then the current  $I_C$  from coil C will follow the shortest path, i.e. the 1st path as its electrical resistance is comparatively low because it is shorter than the 2<sup>nd</sup> path.

The advantages of carbon brushes are that (i) they are to some degree self-lubricating and polish the commutator and (ii) should sparking occur, they would damage the commutator less than when Cu brushes are used.

When high resistance brushes are used, then as the brush moves towards the commutator segments, the contact area of the brush and the segment b decreases and contact area with the segment increases. Now, as the electrical resistance is inversely proportional to the contact area of then resistance  $R_b$  will increase and  $R_a$  will decrease as the brush moves. Then the current will prefer the 2<sup>nd</sup> path to reach to the brush. Thus by this **method of improving commutation**, the quick reversal of current will occur in the desired direction.



**E.M.F. Commutation**

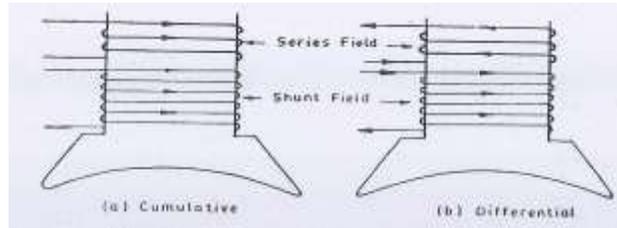
The main reason of the delay of the current reversing time in the short circuit coil during commutation period is the inductive property of the coil. In this type of commutation, the reactance voltage produced by the coil due to its inductive property, is neutralized by producing a reversing emf in the short circuit coil during commutation period.

**Reactance voltage:** The voltage rise in the short circuit coil due to inductive property of the coil, which opposes the current reversal in it during the commutation period, is called the reactance voltage.

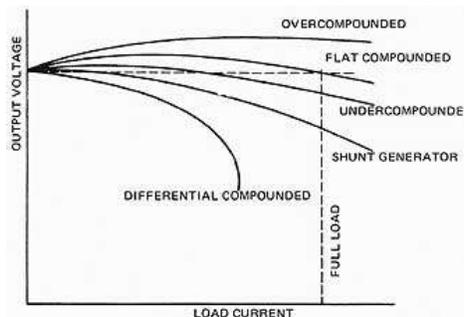
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### 11. Draw and explain the load characteristics of differentially and cumulative compound DC generators? (May-June 2015), (NOV-DEC 2015)

In a compound generator, the shunt field predominates and is much stronger of the two. When the series field m.m.f. aids the shunt field m.m.f., the generator is said to be ‘cumulatively compounded’ When the series field m.m.f. opposes the shunt field m.m.f., generator is said to be ‘differentially compounded’



Depending on the relative additional aiding m.m.f., produced by the series field there are three types of load characteristics possible for the cumulative compound generator.



These types are called:

- (i) Over-compound
- (ii) Flat-compound
- (iii) Under-compound.

**Over compound generator.** An over-compound generator is one whose terminal voltage rises with the application: of load so that its full-load voltage exceeds its no-load voltage (negative regulation).

**Flat-compound generator.** A flat compound generator has a load-voltage characteristic in which the no-load and full-load voltages are equal (zero per cent regulations).

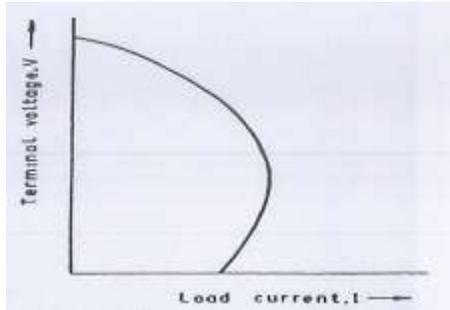
**Under-compound generator.** An under-compound generator has a load characteristic in which the full load voltage is somewhat less than no-load voltage, but whose aiding series-field ampere-turns cause its characteristic to have better regulation than an equivalent shunt generator.

Most commercial compound D.C. dynamos, whether used as generators or motors, are normally supplied by the manufacturer as over-compound machines. The degree of compounding (over, flat or under) may be adjusted by means of diverter which shunts the series field.

#### Characteristic of Differential Compound Generator:

The differential compound generator is defined as that compounding produced when the series field m.m.f. opposes the shunt field m.m.f. The difference in current direction of the two windings is

where for the sake of clarity, the series field winding is shown above (rather than directly around) the shunt field winding.



When the differential compound generator is without load it builds up and self-excites its shunt field in much the same manner as the shunt generator. However, when a load is applied, the generated voltage  $E_g$  is now reduced by the reduction in the main field flux created by the opposing m.m.f. of the series field. This reduction in  $E_g$  occurs in addition to the armature and series circuit voltage drop, the armature reaction, and the reduction in field current produced by reduction of the armature voltage. The result is a sharp drop in the terminal voltage with load as shown in Fig. and the field is below saturation and rapidly unbinds.

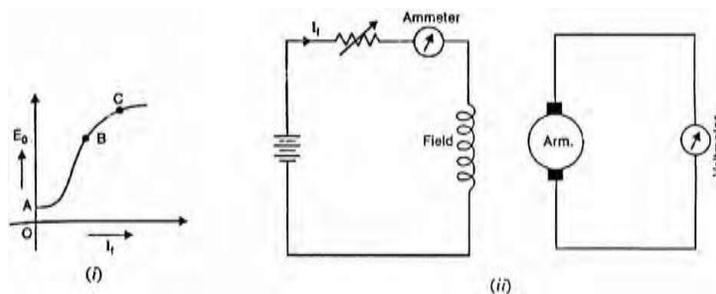
- The differential compound generator is used as a constant-current generator for the same constant-current applications as the series generator.

## 12. Explain the characteristics D.C. Generator.

### 1. Open Circuit Characteristic (O.C.C.)

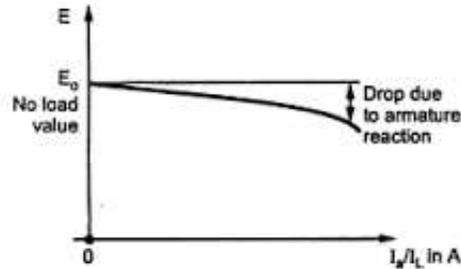
This curve shows the relation between the generated e.m.f. at no-load ( $E_0$ ) and the field current ( $I_f$ ) at constant speed. It is also known as magnetic characteristic or no-load saturation curve. Its shape is practically the same for all generators whether separately or self-excited. The data for O.C.C. curve are obtained experimentally by operating the generator at no load and constant speed and recording the change in terminal voltage as the field current is varied.

The O.C.C. for a d.c. generator is determined as follows. The field winding of the d.c. generator (series or shunt) is disconnected from the machine and is separately excited from an external d.c. source as shown in Fig. (ii). The generator is run at fixed speed (i.e., normal speed). The field current ( $I_f$ ) is increased from zero in steps and the corresponding values of generated e.m.f. ( $E_0$ ) read off on a voltmeter connected across the armature terminals. On plotting the relation between  $E_0$  and  $I_f$ , we get the open circuit characteristic as shown in Fig. (i).



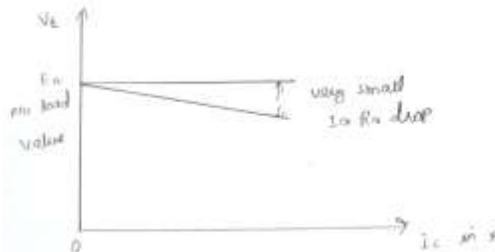
## 2. Internal or Total characteristic (E/I<sub>a</sub>)

This curve shows the relation between the generated e.m.f. on load (E) and the armature current (I<sub>a</sub>). The e.m.f. E is less than E<sub>0</sub> due to the demagnetizing effect of armature reaction. Therefore, this curve will lie below the open circuit characteristic (O.C.C.). The internal characteristic is of interest chiefly to the designer. It cannot be obtained directly by experiment. It is because a voltmeter cannot read the e.m.f. generated on load due to the voltage drop in armature resistance. The internal characteristic can be obtained from external characteristic if winding resistances are known because armature reaction effect is included in both characteristics.



## 3. External characteristic (V/I<sub>L</sub>)

This curve shows the relation between the terminal voltage (V) and load current (I<sub>L</sub>). The terminal voltage V will be less than E due to voltage drop in the armature circuit. Therefore, this curve will lie below the internal characteristic. This characteristic is very important in determining the suitability of a generator for a given purpose. It can be obtained by making simultaneous measurements of terminal voltage and load current (with voltmeter and ammeter) of a loaded generator.



1. A DC machine has 'p' no of poles with curved poles facing having 'Z' no of conductors around the rotor armature of radius 'r' and the flux/pole is given as,  $\phi$ . The rotor rotates at a speed of 'n' rpm. Obtain the induced emf of the DC machine assuming a no of parallel paths (Nov/Dec-2010)

Let  $\phi$  be the flux per pole flux per pole in weber.

P be the number of poles

Z be the total no of conductors in the armature.

All the Z conductors are connected in series. They are divided into groups and let A be the no of parallel paths into which these conductors are grouped.

So each parallel path will have Z/A conductors in series.

Let N be the speed of rotation in revolutions per minute (rpm)

Consider one conductor on the periphery of the armature. As this conductor makes one complete revolution. It cuts  $P\phi$  Weber's. As the speed is N rpm. The time taken for one revolution is  $60/N$  secs.

Since the emf induced in conductor = Rate of change of flux cut

$$e \propto \frac{d\phi}{dt} = \frac{p\phi}{60/N}$$

$$e = \frac{N.P.\phi}{60} \text{ volts.}$$

Since there are  $Z/A$  conductors in series in each parallel path the emf induced.

$$E_g = \frac{N.P.\phi}{60} \left(\frac{Z}{A}\right) = \frac{\phi Z N P}{60 A} \text{ volts.}$$

The armature conductors are generally connected into two different ways. Its lap winding and wave winding. For lap wound armatures, the no of parallel paths is equal to the no of poles ( $A=p$ ). In wave wound machine  $A=2$  always.

-----&&&&&-----

**2. A 12 pole Dc generator has a simplex wave wound armature containing 144 coils of 10 turns each. The resistance of each turn is  $0.011\Omega$ . Its flux per pole is  $0.05$  wb and it is running at a speed of 200rpm. Obtain the induced armature voltage and the effective armature resistance. [May/June-2010]**

**Given:**  $P=12$ ,  $A=2$ ,  $N=200$ rpm,  $\Phi=0.05$ wb

**Formula used:**  $E_g = \frac{p\phi Z N}{60}$

**Soln:**

Total no of conductors,  $Z=144 \times 10 \times 2 = 2880$

$P=12$ ,  $A=2$ ,  $N=200$ rpm,  $\Phi=0.05$ wb

Induced armature voltage

$$E_g = \frac{p\phi Z N}{60}$$
$$= \frac{12 \times 0.05 \times 2880 \times 200}{60 \times 2}$$

$$E_g = 2880 \text{ Volts}$$

$$\text{Effective armature resistance} = \frac{2880 \times 0.011}{\frac{31.68\Omega}{10}} = 3.1658\Omega$$

-----&&&&&-----

**3. A 4 pole lap-wound dc machine has 728 armature conductors. Its field wdg is excited from a dc source to create an air gap flux of  $32$  m wb/pole. The machine (generator) is run from a prime mover (diesel engine) at 1600 rpm. It supplies a current of  $100$ A to an electric load.**

**i) Cal.the electromagnetic power developed.**

**ii) What is the mechanical power that is fed from the prime-mover to the generator?**

**iii) What is the torque provided by the prime mover? [April/May -2011]**

**Given:**  $Z=728$ ,  $\phi = 32$ ,  $N=1600$ ,  $I=100$ A

**Formula used:**  $E_a = \frac{\phi NZ}{60} \left(\frac{p}{A}\right)$ ,  $T = \frac{p_m}{w_m} w_m$ , Electromagnetic power developed =  $E_a I_a$

**Soln:**

$$\begin{aligned} \text{i) } E_a &= \frac{\phi NZ}{60} \left(\frac{p}{A}\right) \\ &= \frac{32 \times 10^{-3} \times 1600 \times 728}{60} (4/4) \\ E_a &= 621.2V \end{aligned}$$

Electromagnetic power developed =  $E_a I_a$

$$= 621.2 \times 100$$

$$P_e = 62.12 \text{ kw}$$

ii) Mechanical power provided by primemover ( $P_m$ ) = electromagnetic power developed

$$P_m = 62.12 \text{ kw}$$

iii) But  $P_m = T W_m$

$$\begin{aligned} \text{Prime mover torque } T &= \frac{p_m}{w_m} w_m = \frac{2\pi N}{60} = \frac{2\pi \times 1600}{60} \\ &= \frac{62.12 \times 10^3}{167.55} \end{aligned}$$

$$T = 370.75 \text{ Nm}$$

-----&&&&-----

**4. A 4 pole DC shunt generator with Lap connected armature supplies 5 KW at 230 V. The armature and field copper losses are 360 W and 200 W respectively. Calculate the armature current and generated EMF (May-June 2015)**

**Given Data:**

No Of Pole: 4; Generator Capacity: 5 Kw; V-230V; Armature Cu Loss: 360W; Field Cu Loss : 200 W;

**Find:**

i) Armature Current ( $I_a$ )

ii) Generated Emf ( $E_g$ )

**Solution:**

i) **To Find Armature Current ( $I_a$ ):**

Input = output + losses

$$E_g \times I_a = 5000 + 560 = 5600W$$

$$E_g = V + I_a R_a$$

Hence

$$(V + I_a R_a) \times I_a = 5560 W$$

$$V I_a \times I_a^2 R_a = 5560W$$

$$V I_a = 5560 - (360) = 5200W$$

$$I_a = \frac{5200}{230} = 21.74 W$$

ii) **To Find Generated Emf ( $E_g$ )**

$$I_a^2 R_a = 360W$$

$$R_a = \frac{360}{(21.74^2)}$$

$$E_g = V + I_a R_a = 230 + 16.559$$

$$E_g = 246.56V$$

5. In a 400 V, Dc compound Generator, The resistance of the armature, series and shunt windings are 0.10 ohm, 0.05 ohm and 100 ohm respectively. The machine supplies power to 20 Nos. resistive heaters each rated 500 watts, 400 Volts. Calculate the induced Emf and armature currents when the generator is connected in i) Short Shunt ii) Long Shunt. Allow brush contact drop of 2 volts per brush. (May-June 2015)

**Given Data:**

V=400 V;  $R_a=0.10$  ohm ;  $R_{se}=0.05$  ohm ;  $R_{sh}=100$  ohm respectively ; machine supplies 20 heaters each rated 500 W, 400 V

**Find:**

Induced EMF When

- i) Short Shunt    ii) Long Shunt

**Solutions:**

i) Short Shunt:

$$I_L = \frac{500 \text{ w} \times 20(\text{Heaters})}{400 \text{ V}} = 25 \text{ A}$$

$$I_a = I_L + I_{Sh}$$

$$I_{Sh} = \frac{V}{R_{Sh}} = \frac{400}{100} = 4 \text{ A}$$

$$I_a = 25 + 4 = 29 \text{ A}$$

$$\text{Induced Emf } E_g = V + I_a(R_a + R_{se})$$

$$E_g = 400 + 29(0.10 + 0.05)$$

$$E_g = 404.35 \text{ V}$$

ii) Long Shunt:

$$E_g = V + I_a R_a + I_{se} R_{se}$$

$$\therefore I_{se} = I_L$$

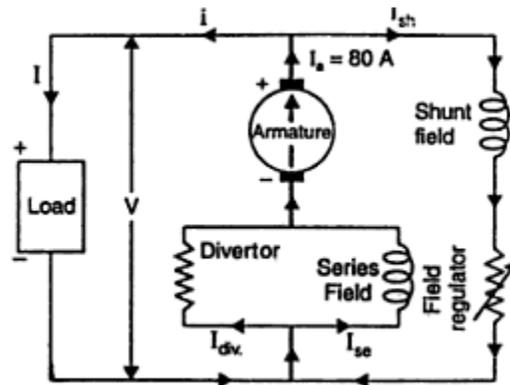
$$E_g = 400 + [29 \times 0.10] + [25 \times 0.05] = 400 + 2.9 + 1.25$$

$$E_g = 404.15 \text{ V}$$

6. A long shunt compound generator has a shunt field winding of 1000 turns per pole and series field winding of 4 turns per pole and a resistance of 0.05 ohm. In order to obtain the speed voltage both at load and full load for operating as shunt generator. It is necessary to increase the field current by 0.2A. the full load armature current of the compound generator is 80A. Calculate the diverter resistance connected in parallel of series field to obtain flat compound operations? (NOV-DEC 2015)

**Solution:**

**Additional ampere-turns required to maintain rated voltage at full load operation as a D.C shunt generator= No of turns on shunt field winding×additional shunt field current.**  
**= 1000 × 0.2 = 200AT**



No of series turns per pole,  $N_{se} = 4$

Current required to produce 200AT by the series field,

$$I_{sh} = \frac{200}{N_{se}} = \frac{200}{4} = 50A$$

Armature current,  $I_a = 80A$  given

Current through the diverter,  $I_{div} = I_a - I_{se}$

$$= 80 - 50 = 30A$$

If  $R_{div}$  is the resistance of diverter, then

$$R_{div} = \frac{I_{div} \times R_{div} = I_{se} \times R_{se}}{I_{se} \times R_{se}} = \frac{50 \times 0.05}{30} = 0.0833\Omega$$

**7. Two shunt generators are connected in parallel to supply a load of 5000A each machine has armature resistance of 0.03 ohm and field resistance of 60 ohm. EMF on one machine is 600V and in other machine is 640V. what power does each machine supply? (NOV-DEC 2015)**

**Given data :**

$E_{g1}=600$  V;  $E_{g2}=640$  V;  $R_a=0.03$  ohm;  $R_{sh}=60$  ohm;  $I=5000$  A;

**Find:** Power of each machine?

**Solutions:**

**Generator 1:**

$$I_{sh1} = \frac{V}{60} A$$

$$V = E_{g1} - I_{a1}R_{a1}$$

$$V = 600 - \left[ I_1 + \frac{V}{60} \right] \times 0.03 \dots \dots \dots (1)$$

**Generator 2:**

$$I_{sh1} = \frac{V}{60} A$$

$$V = 640 - \left[ I_2 + \frac{V}{60} \right] \times 0.03 \dots \dots \dots (2)$$

Equating Equation (1) and (2)

$$V = 600 - \left[ I_1 + \frac{V}{60} \right] \times 0.03 = 640 - \left[ I_2 + \frac{V}{60} \right] \times 0.03$$

$$0.03(I_2 - I_1) = 40$$

$$I_2 - I_1 = 1333.3 A \dots \dots \dots (3)$$

$$I_2 + I_1 = 5000A \dots \dots \dots (4)$$

$$2I_2 = 6333.3A$$

$$I_2 = 3166.65A$$

$$I_1 = 1833.35A$$

Now substituting the value of  $I_1$  in equation (1) we get

$$V = 504.7V$$

Output of Generator 1:  $P_1 = \frac{VI_1}{1000} KW$

$$P_1 = \frac{504.7 \times 3166.65}{1000} = 1598.2kw$$

Output of Generator 2:  $P_2 = \frac{VI_2}{1000} KW$

$$P_2 = \frac{504.7 \times 3166.65}{1000} = 1598.2 KW$$

**8. A 4 pole dc motor is lap-wound with 400 conductors. The pole-shoe is 20cm long and the average flux density over one pole-pitch is 0.4T, the armature diameter being 30cm. Find the torque and gross-mechanical power developed when the motor is drawing 25A and running at 1500 rpm.(May 2016)**

**Flux / pole ( $\phi$ )** =  $\frac{\pi}{4} \times 30 \times 10^{-2} \times 20 \times 10^{-2} \times 0.4 = 0.0188 \text{ wb}$

**Induced emf  $E_g = \frac{\phi NZ}{60} \left( \frac{p}{A} \right) = 188 \text{ V}$**

**Gross mechanical power developed** =  $E_a I_a = \frac{188 \times 25}{1000} = 4.7 \text{ kw}$

**Torque developed** =  $\frac{4.7 \times 1000}{2\pi \times 1500/60} = 29.9 \text{ Nm}$

**9.A separately excited generator when running at 1000 r.p.m. supplied 200A at 125V. What will be the load current when the speed drops to 800r.p.m. if  $I_f$  is unchanged? Given that armature resistance=0.04 $\Omega$  and brush drop=2V.Derive the necessary equations.(Dec-2017)**

Solution:

$$\text{Load resistnce } R = \frac{125}{200} = 0.625\Omega$$

$$E_{g1} = 125 + 200 \times 0.04 + 2 = 135V$$

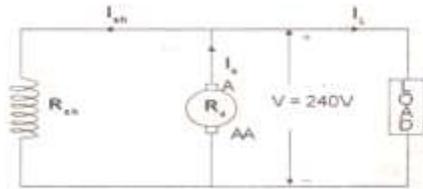
$$N_1 = 1000 \text{ rpm}$$

$$\text{At } 1000 \text{ rpm } E_{g2} = 135 \times \frac{800}{1000} = 108V$$

If  $I$  is new load current, terminal Voltage  $V$  is given by,

$$\begin{aligned} V &= 108 - 0.04I - 2 \\ &= 106 - 0.04I \\ \therefore I &= \frac{V}{R} = \frac{(106 - 0.04I)}{0.635} \\ I &= 159.4A \end{aligned}$$

**10. A four pole lap wound shunt generator supplies 60 lamps of 100 W, 240V, each; the field and armature resistances are 55  $\Omega$  and 0.18  $\Omega$  respectively. If the brush drop is 1V for each brush find (i) Armature current (ii) Current per path (iii) Generated emf (iv) Power output of DC machine. (May 2017)**



**Solution:**

Total power supplied  $P_o = 60 \times 100 = 6000W$

$$\text{Load current } I_L = \frac{P_o}{V} = \frac{6000}{240} = 25 \text{ A}$$

$$\text{Shunt field current } I_{Sh} = \frac{V}{R_{Sh}} = \frac{240}{55} = 4.36 \text{ A}$$

$$\text{Armature current } I_a = I_L + I_{Sh} = 25 + 4.36 = \mathbf{29.36 \text{ A}}$$

Number of parallel paths = Number of poles = 4 (for lap)

$$\text{Current per path} = \frac{29.36}{4} = \mathbf{7.34 \text{ A}}$$

$$\begin{aligned} \text{Generated emf } (E_g) &= V + I_a R_a + \text{brush drop} \\ &= 240 + 29.36 + 0.18 + 2 \times 1 \end{aligned}$$

$$E_g = \mathbf{247.28 \text{ V}}$$

$$\text{Power output of DC machine } P_o = V \times I_L$$

$$= 240 \times 25$$

$$P_o = \mathbf{6000 \text{ W}}$$

**11. Two 500V DC shunt generators rated at 100kW and 200kW respectively are operating in parallel. Both of them have linearly drooping external characteristics. Voltage regulation of the first generator is 4% and second is 6%. Determine the common bus voltage and current shared by each of the generators when their parallel combination is to supply a current of 300A. (May-2018)**

**Given:**  $V=500V, P_1, P_2=100, 200kW, \text{drop}=4\%, 5\%, I=300A$

**Solution:**

100Kw generator

$$\text{full load voltage drop} = 500 \times \frac{4}{100}$$

$$\text{full load current} = \frac{100 \times 10^3}{500} = 200A$$

$$\text{drop per ampere} = \frac{20}{200} = \frac{1}{10} \left( \frac{V}{A} \right)$$

200Kw generator

$$\text{full load voltage drop} = 500 \times \frac{6}{100}$$

$$\text{full load current} = \frac{200 \times 10^3}{500} = 400A$$

$$\text{drop per ampere} = \frac{30}{400} = \frac{3}{40} \left( \frac{V}{A} \right)$$

If  $I_1$  and  $I_2$  are currents supplied by the two generators and  $V$  is the bus bar voltage, than

$$V = 500 - \frac{1}{10} I_1 \quad \text{first generator}$$

$$V = 500 - \frac{3}{40} I_2 \quad \text{second generator}$$

$$\frac{I_1}{10} = \frac{3I_2}{40}$$

$$40I_1 = 30I_2$$

$$40I_1 - 30I_2 = 0 \dots\dots\dots (1)$$

$$I_1 + I_2 = 300 \dots\dots\dots (2)$$

Solving above equation  $I_1 = 128.57A, I_2 = 171.42A$

$$\therefore V = 500 - \frac{I_1}{10}$$

$$= 500 - \frac{128.57}{10}$$

$$= 500 - 12.857$$

$$V = 487.143V$$

**12.A 4-pole, 50KW, 250V, wave wound shunt generator has 400armature conductors. Brushes are given a lead of 4 commutator segments. Calculate the demagnetization ampere-turns per pole if shunt field resistance is 50 ohm. Also calculate extra shunt field turns per pole to neutralize the demagnetization.(May-2018)**

**Given:** 4-pole, 50KW, 250V,  $Z=400, R_{sh},$

**Solution:**

$$\text{Load current supplied } I_L = \frac{500 \times 10^3}{250} = 200A$$

$$\text{Shunt field current } I_{sh} = \frac{V}{R_{sh}} = \frac{250}{50} = 5A$$

$$\begin{aligned} \text{Armature current } I_a &= I_L + I_{sh} \\ &= 200 + 5 = 205A \end{aligned}$$

$$\text{current in each conductor } I = \frac{205}{2}$$

$$\text{No of commutator segment} = \frac{Z}{A} \quad A = 2 \text{ wave winding}$$

$$\text{No of segment} = \frac{400}{2} = 200$$

$$\therefore \theta = \frac{4}{200} \times 360 = \frac{36}{5} \text{ degrees}$$

$$\frac{AT_d}{\text{pole}} = ZI \cdot \frac{\theta_m}{360}$$

$$= 400 \left( \frac{205}{2} \right) \times \frac{36}{5 \times 360} = 820AT$$
$$\text{Extra shunt } \frac{\text{turns}}{\text{poles}} = \frac{AT_d}{I_{sh}} = \frac{820}{5} = 164$$

**UNIT 5**  
**PART-A**

**1. List the different methods of speed control in DC motor.[April May-2010]**

- Field method
- Armature control
- Voltage control

**2. What is the precaution to be taken during starting of a DC series motor? Why? [Nov/Dec-2010]**

DC series motor should always be started with some load on the shaft. In this motor, the  $\phi$  is directly proportional to the armature current. i.e.,  $\phi \propto I_a$ . Under no load conditions, the armature current is very low and flux also is less, if the flux is less speed will be very high [ $N \propto 1/\phi$ ].

**3. Explain why Swinburne's test cannot be performed in DC series motor.[Nov/Dec-2011]**

In the Swinburne's method of testing, the motor is not loaded directly but the losses and efficiency at different loads can be estimated. Since series DC motors cannot be started without loads, this no-load test (Swinburne's Test) cannot be performed on DC. series motors.

**4. How will you change the direction of rotation of a DC motor? (May-2018)**

Either the direction of the main field or the direction of current through the armature conductors is to be reserved.

**5. What is critical speed?(April –2010)**

The critical field resistance at that resistance of the field circuit at a given speed at which generator just excites and starts voltage buildings while beyond this value the critical speed.

**6. List the merits and demerits of Swinburne's test. (Nov/Dec-2013)**

**Merits:-**

1. This method is convenient and economical.
2. Since constant losses are known, the efficiency can be estimated at any load.

**Demerits:-**

1. It is difficult to know whether there will be satisfactory commutation at full load.
2. Due to armature reaction at full load there will be distortion in flux which will increase the iron loss.

**7. Enumerate the factors on which the speed of a DC motor depends.**

$$N = K \frac{(V - I_a R_a)}{\Phi}$$

Where

N-Speed of the motor in rpm,            K-Constant

Speed of DC motor depends on three factors.

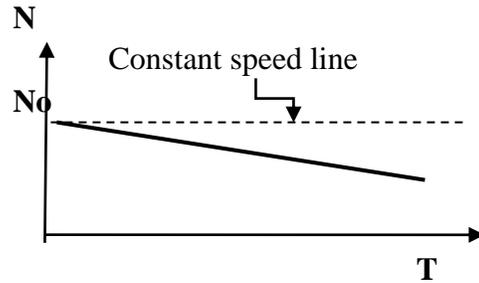
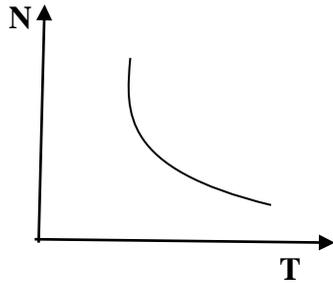
- Flux in the air gap ( $\Phi$ )
- Resistance of the armature circuit( $R_a$ ) and Voltage applied to the armature( $v$ )

**8. What is the function of a no-voltage release coil provided in a DC motor starter? [May-2011]**

The NVR coil produce enough magnetic force of attraction and retain the starter handle in the ON position against spring force.

When the supply voltage fails or becomes lower than a prescribed value the electromagnet may not have enough force and the handle will come back to OFF position due to spring force automatically.

9. Draw the speed –torque characteristics of a series and shunt motor. (May 2016)



10. What are the different types of starters? or List various method of starting DC motor. Dec-2016

- Two point starters – Series motor
- Three point starter – Shunt motor
- Four point starter–

11. State the advantage and disadvantage of Swinburne’s and Hopkinson’s test.[Nov/Dec-2009]

Swinburne’s Test	Hopkinson’s Test
<p><b>Advantages:</b> 1. This method is convenient and economical. 2. Since constant losses are known, the efficiency can be estimated at any load.</p>	<p><b>Advantages:</b> 1. The power required for conducting the test is small hence this method of testing is economical. 2. Since the machines are operated at full load conditions, change in iron loss due to distortion in flux will be included in calculation.</p>
<p><b>Disadvantages:</b> 1. It is difficult to know whether there will be satisfactory commutation at full load. 2. Due to armature reaction at full load there will be distortion in flux which will increase the iron loss.</p>	<p><b>Disadvantages:</b> 1. There is difficulty in availability of two identical machines. 2. The iron losses are different in both the machines because of different excitations.</p>

12. What are the losses in DC machines?[May/June-2009]

The various losses in a DC machine whether it is a motor or a generator are classified into three groups as:

- Copper losses
- Iron or core losses
- Mechanical losses.

13. Name the various methods of testing a DC machines.

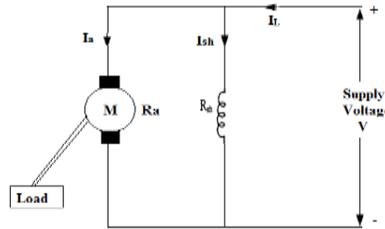
There are different methods of testing DC machines. These methods are broadly classified as:

- Brake Test
- Swinburne’s Test
- Hopkinson’s Test
- Retardation Test or Running down Test.

**14. Name the Protective devices in a starter.**

- No Volt Coil (NVC)
- Over Load Release(OLR)

**15. Draw the circuit model of DC shunt motor.[April/May-2011]**



**16. Why DC series motor is not suitable for belt driven loads?[May/ June-2012]**

Series motors acquire very high speed at no load or at very light load. That is why they should not be used for belt driven loads.

**17. State the methods of speed control in DC series motor?[Nov/ Dec-2012]**

1. Variable resistance in series with motor
2. Flux control method
3. Field diverter
4. Armature diverter
5. Tapped field control.

**18. What is the relation between speed, back emf and flux?**

$$N \propto E_b / \phi$$

Speed of the motor is directly proportional to back emf  $E_b$  and inversely proportional to the flux  $\phi$ .

**19. Specify the technique used to control the speed of DC shunt motor for below and above the rated speed? (APR/MAY 2015)**

By varying the field current as well as armature voltage, the speed of a DC motor can be controlled. The speed of a DC motor can be controlled below the rated speed by armature control method.

The speed of a DC motor can be controlled above the rated speed by field control method, because the flux per pole can be decreased to any value by decreasing the field current.

**20. Why series motor is suited for traction applications? (APR/MAY 2015)(Nov-Dec 2015)**

Torque is directionally proportional to flux and armature current, particularly in DC series motor torque is directly proportional to flux and square of armature current, so starting torque is high. Hence DC series motor is used in traction purpose.

**21. State Fleming's Left hand rule? (Nov-Dec 2015)**

For a current-carrying wire in a magnetic field, such as a wire on the armature of a motor, the rule that if the thumb, first, and second fingers of the left hand are extended at right angles to one another, with the first finger representing the direction of magnetic lines of force and these

middle finger representing the direction of current flow, the thumb will be pointing in the direction of force on the wire. Also known as Fleming's rule.

**22. What is meant by plugging? (May 2016)**

**The reversing of armature connections of the motor for electrical braking is known as plugging.** Or The motor can be stopped immediately by just interchanging any two of the stator leads by doing this, it reverses the direction of the revolving flux, which produces a torque in the reverse direction, thus causing a braking effect on the rotor. This braking period is called the plugging.

**23. What is meant by dynamic braking in DC motor? Dec - 2016**

The armature of the running motor is disconnected from the supply and is connected across a variable resistance R. However, the field winding is left connected to the supply. This type of electrical braking is known as **dynamic braking or Rheostatic braking.**

**24. What is back emf in DC motors?**

As the motor armature rotates, the system of conductor come across alternate North and South Pole magnetic fields causing an emf induced in the conductors. The direction of the emf induced in the direction opposite to the current. As this emf always opposes the flow of current in motor operation it is called back emf

**25. Why a starter is necessary for a DC motor? May 2017**

When a DC motor is directly switched on, at the time of starting, the motor back emf is zero. Due to this, the armature current is very high (nearly 25 times the rated current). Due to the very high current, the motor gets damaged. **To reduce/limit the starting current** of the motor a starter is used.

**26. What are the application of DC motor? May 2017**

**DC Shunt motors**– Light machine tools, centrifugal pumps, wood working machines, Lathes etc.,

**DC Series motors**– Electric train, Cranes, Hoists, Lifts, Blowers, Conveyers etc.,

**DC Compound motors** - Used for driving heavy machine tools for intermittent loads shears, punches machines etc.,

**27. What will happen to the speed of a D.C motor when its flux approaches zero?(Dec-2017)**

The speed starts to decrease to zero

**28. Mention the effect of differential compounding and cumulatively compound on the performance of D.C compound motor (Dec-2017)**

(i) *Cumulative-compound motors* in which series field aids the shunt field.

(ii) *Differential-compound motors* in which series field opposes the shunt field.

Differential compound motors are rarely used due to their poor torque characteristics at heavy loads.

**29. Why commutator employed in d.c.machine (May-2018)**

In case of DC generator, commutator is used to convert generated AC in armature into DC.

It collects the current from the armature conductors,

To produce unidirectional torque in the case of motor.

## PART-B

### 1. Draw and explain the characteristic of DC. motor. [Nov/Dec-2011], May-June 2015

#### DC. Motor Characteristics

There are three principal types of DC. motors viz., **shunt motors, series motors and compound motors**. Both shunt and series types have only one field winding wound on the core of each pole of the motor. The compound type has two separate field windings wound on the core of each pole. The performance of a DC. motor can be judged from its characteristic curves known as motor characteristics, following are the three important characteristics of a DC. motor:

(i) **Torque and Armature current characteristic ( $T_a/I_a$ )**

It is the curve between armature torque  $T_a$  and armature current  $I_a$  of a DC. motor. It is also known as electrical characteristic of the motor.

(ii) **Speed and armature current characteristic ( $N/i_a$ )**

It is the curve between speed  $N$  and armature current  $I_a$  of a DC. motor. It is very important characteristic as it is often the deciding factor in the selection of the motor for a particular application.

(iii) **Speed and torque characteristic ( $N/T_a$ )**

It is the curve between speed  $N$  and armature torque  $T_a$  of a DC. motor. It is also known as mechanical characteristic.

#### Characteristics of Shunt Motors:

- Fig. (1) shows the connections of a DC. shunt motor.
- The field current  $I_{sh}$  is constant since the field winding is directly connected to the supply voltage  $V$  which is assumed to be constant. Hence, the flux in a shunt motor is approximately constant.

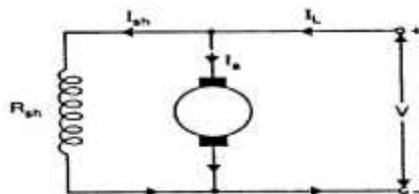


Figure 1. DC motor

#### **$T_a/I_a$ Characteristic.**

We know that in a DC. motor,

$$T_a \propto \phi I_a$$

Since the motor is operating from a constant supply voltage, flux  $\lambda$  is constant (neglecting armature reaction).

$$\therefore T_a \propto I_a$$

Hence  $T_a/I_a$  characteristic is a straight line passing through the origin as shown in Fig. (2). The shaft torque ( $T_{sh}$ ) is less than  $T_a$  and is shown by a dotted line. It is clear from the curve that a very large current is required to start a heavy load. Therefore, a shunt motor should not be started on heavy load.

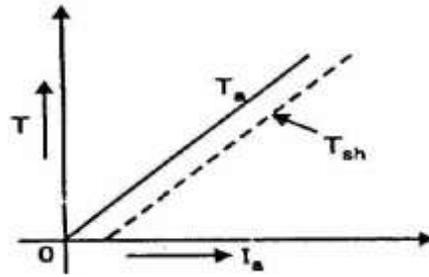


Figure 2.  $T_a/I_a$  characteristic

**(ii)  $N/I_a$  Characteristic.**

The speed  $N$  of a DC. motor is given by;

$$N \propto \frac{E_b}{\phi}$$

The flux  $\phi$  and back e.m.f.  $E_b$  in a shunt motor are almost constant under normal conditions. Therefore, speed of a shunt motor will remain constant as the armature current varies (dotted line AB in Fig.3). When load is increased,  $E_b (= V - I_a R_a)$  and  $\phi$  decrease due to the armature resistance drop and armature reaction respectively. However,  $E_b$  decreases slightly more than  $\phi$  so that the speed of the motor decreases lightly with load (line AC).

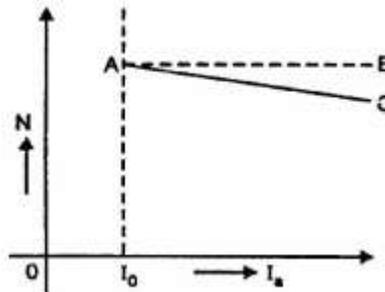


Figure 3.  $N/I_a$  Characteristic

**(iii)  $N/T_a$  Characteristic.**

The curve is obtained by plotting the values of  $N$  and  $T_a$  for various armature currents (See Fig.4). It may be seen that speed falls somewhat as the load torque increases.

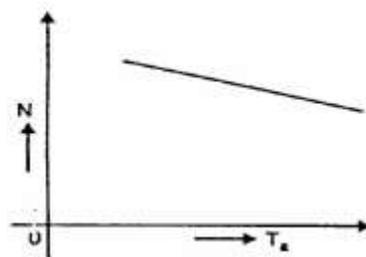


Figure 4.  $N/T_a$  Characteristic

**Applications:**

- ✓ Blowers and Fans
- ✓ Lathe Machines
- ✓ Milling and Drilling
- ✓ Centrifugal and Reciprocate Pumps

**(b) Characteristics of Series Motors**

Fig. (5) shows the connections of a series motor. Note that current passing through the field winding is the same as that in the armature. If the mechanical load on the motor increases, the armature current also increases. Hence, the flux in a series motor increases with the increase in armature current and vice-versa.

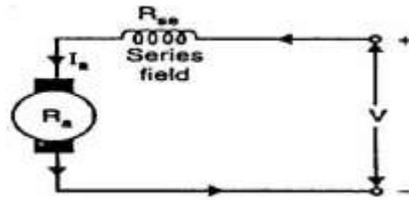


Figure 5. series motor

**Ta/Ia Characteristic.**

We know that;  $T_a \propto \phi I_a$

Up to magnetic saturation,  $\phi \propto I_a$ ; so that  $T_a \propto I_a^2$

After magnetic saturation,  $\phi$  is constant so that  $T_a \propto I_a$

Thus up to magnetic saturation, the armature torque is directly proportional to the square of armature current. If  $I_a$  is doubled,  $T_a$  is almost quadrupled.

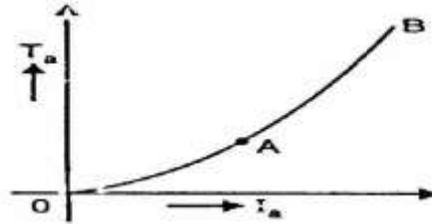


Figure 6. Ta/Ia Characteristic

Therefore,  $T_a/I_a$  curve upto magnetic saturation is a parabola (portion OA of the curve in Fig. 6). However, after magnetic saturation, torque is directly proportional to the armature current. Therefore,  $T_a/I_a$  curve after magnetic saturation is a straight line (portion AB of the curve). It may be seen that in the initial portion of the curve (i.e. up to magnetic saturation),  $T_a \propto I_a^2$ . This means that starting torque of a DC series motor will be very high as compared to a shunt motor (where that  $T_a \propto I_a$ ).

**(ii) N/Ia Characteristic.**

The speed N of a series motor is given by;

$$N \propto E_b / \phi$$

where

$$E = V - I_a R_a + R_{se}$$

When the armature current increases, the back e.m.f.  $E_b$  decreases due to  $I_a (R_a + R_{se})$  drop while the flux  $\phi$  increases. However,  $I_a (R_a + R_{se})$  drop is quite small under normal conditions and may be neglected.

$$\therefore N \propto 1/\phi$$

$$\propto 1/I_a \text{ upto magnetic saturation}$$

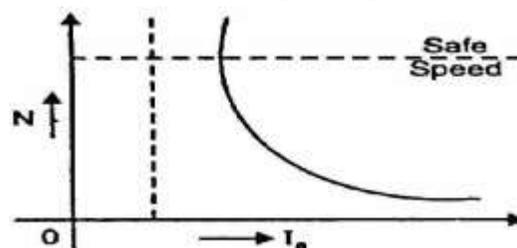


Fig. 7.N/Ia Characteristic

Thus, up to magnetic saturation, the N/Ia curve follows the hyperbolic path as shown in Fig. (7). After saturation, the flux becomes constant and so does the speed.

### N/Ta Characteristic.

The N/Ta characteristic of a series motor is shown in Fig. (8). It is clear that series motor develops high torque at low speed and vice-versa. It is because an increase in torque requires an increase in armature current, which is also the field current. The result is that flux is strengthened and hence the speed drops ( $\because N \propto 1/\phi$ ). Reverse happens should the torque be low.

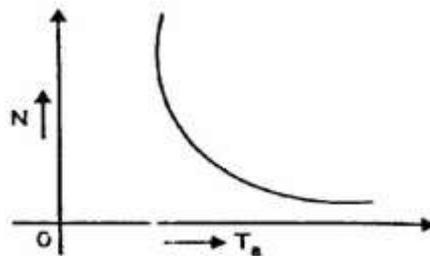


Figure (8) N/Ta Characteristic

### Applications:

- ✓ Cranes
- ✓ Hoists and elevators
- ✓ Conveyors
- ✓ Electric Locomotives.

### 3) Compound Motors

A compound motor has both series field and shunt field. The shunt field is always stronger than the series field. Compound motors are of two types i) **Cumulative-compound motors** in which series field aids the shunt field.(ii) **Differential-compound motors** in which series field opposes the shunt field. Differential compound motors are rarely used due to their poor torque characteristics at heavy loads.

### Characteristics of Cumulative Compound Motors

#### (i)Ta/Ia Characteristic.

As the load increases, the series field increases but shunt field strength remains constant. Consequently, total flux is increased and hence the armature torque ( $\because T_a \propto \phi I_a$ ).It may be noted that torque of a cumulative-compound motor is greater than that of shunt motor for a given armature current due to series field [Fig. 10].

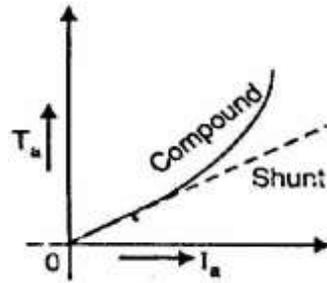


Figure 10.  $T_a/I_a$  Characteristic

**(ii)  $N/I_a$  Characteristic.**

As explained above, as the load increases, the flux per pole also increases. Consequently, the speed ( $N \propto 1/\phi$ ) of the motor falls as load increases (Fig.11). It may be noted that as the load is added, the increased amount of flux causes the speed to decrease more than does the speed of a shunt motor. Thus the speed regulation of a cumulative compound motor is poorer than that of a shunt motor.

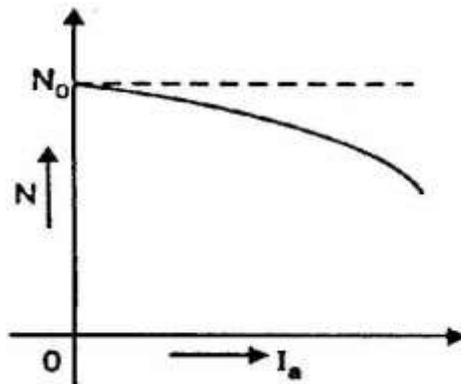


Figure 11.  $N/I_a$  Characteristic

**(iii)  $N/T_a$  Characteristic.**

Fig. (12) shows  $N/T_a$  characteristic of a cumulative compound motor. For a given armature current, the torque of a cumulative compound motor is more than that of a shunt motor but less than that of a series motor.

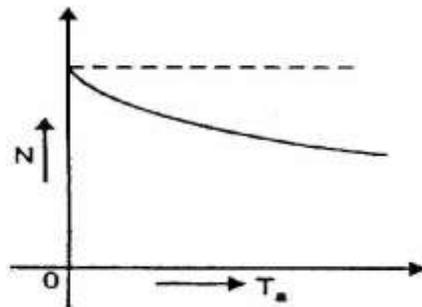
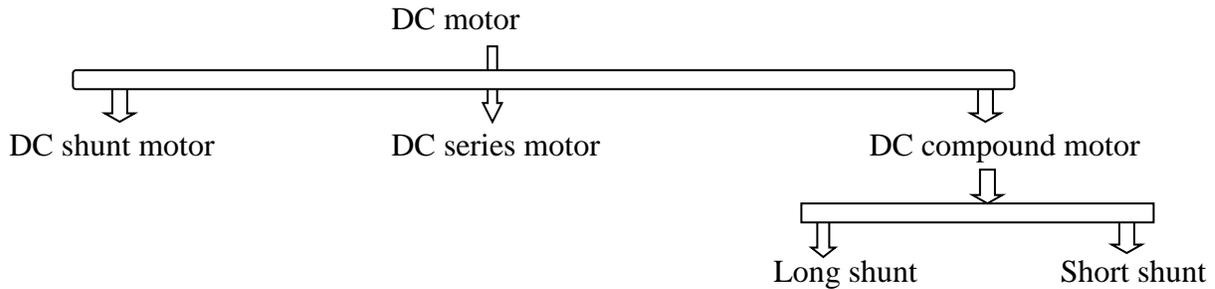


Figure 12.  $N/T_a$  Characteristic

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**2. Explain the types of DC motor and write the voltage equation for the same.**

The DC motors are classified depending upon the way of connecting the field winding with the armature winding. The figure 1 shows the various types of DC motors.



**DC SHUNT MOTOR**

In this type, the field winding is connected across the armature winding and the combination is connected across the supply. DC shunt motor is shown in the figure 2 below.

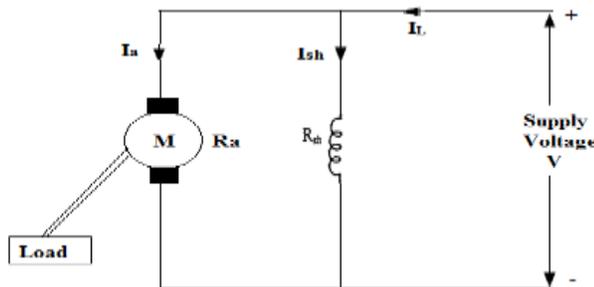


Figure 2 DC shunt motor

- Let  $R_{sh}$  be the resistance of shunt field winding,  $R_a$  is the resistance of armature winding.
- The value of  $R_a$  is very small while  $R_{sh}$  is large.
- Hence shunt field winding has more number of turns with less cross-sectional area.

**Voltage Equation:**

The voltage across the armature and field winding is equal to the supply voltage. It is given by,

$$V = E_b + I_a R_a + V_{brush}$$

Where,

$V_{brush}$  is generally neglected.

**DC SERIES MOTOR:**

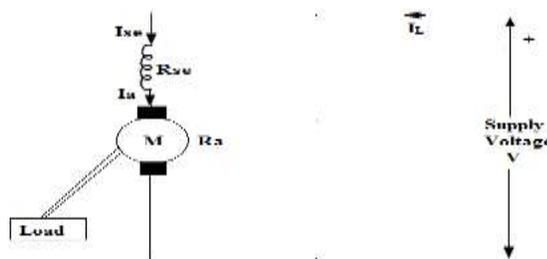


Figure 3. DC series motor

In this type of motor, the series field winding is connected in series with the armature and the supply, as shown in the figure 3 below.

- Let  $R_{se}$  be the resistance of the series field winding.
- The value of  $R_{se}$  is very much small and it is made up of small number of turns having large cross-sectional area.

**Voltage Equation:**

$$V = E_b + I_a(R_a + R_{se}) + V_{brush}$$

**DC COMPOUND MOTOR:**

The compound motor consists of part of the field winding connected in series and part of the field winding connected in parallel with armature.

**a) Long Shunt Compound Motor:**

In this type, the shunt field winding is connected across the combination of armature and series field winding as shown in figure 4 below.

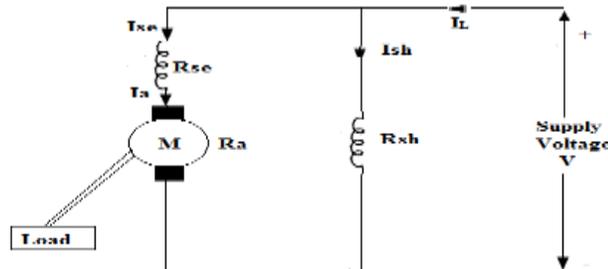


Figure 4. Long shunt compound motor

**Voltage Equation:**

$$V = E_b + I_a R_a + I_{se} R_{se} + V_{brush}$$

But  $I_{se} = I_a$

Hence  $V = E_b + I_a (R_a + R_{se}) + V_{brush}$

**(b) Short shunt compound motor:**

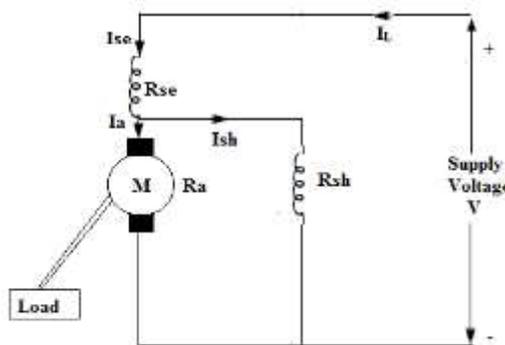


Figure 5 Short shunt motor

In this type, the shunt field is connected purely in parallel with armature and the series field is connected in series with this combination as shown in figure 5.

$$I_L = I_{se}$$

The entire line current is passing through the series winding.

And  $I_L = I_a + I_{sh}$

Now the drop across the shunt field winding is to be calculated from the voltage equation.

So  $V = E_b + I_{se}R_{se} + I_aR_a + V_{brush}$

But  $I_{se} = I_L$

Therefore  $V = E_b + I_LR_{se} + I_aR_a + V_{brush}$

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### 3. Explain the various starting methods of DC motor? [April/May-2010]

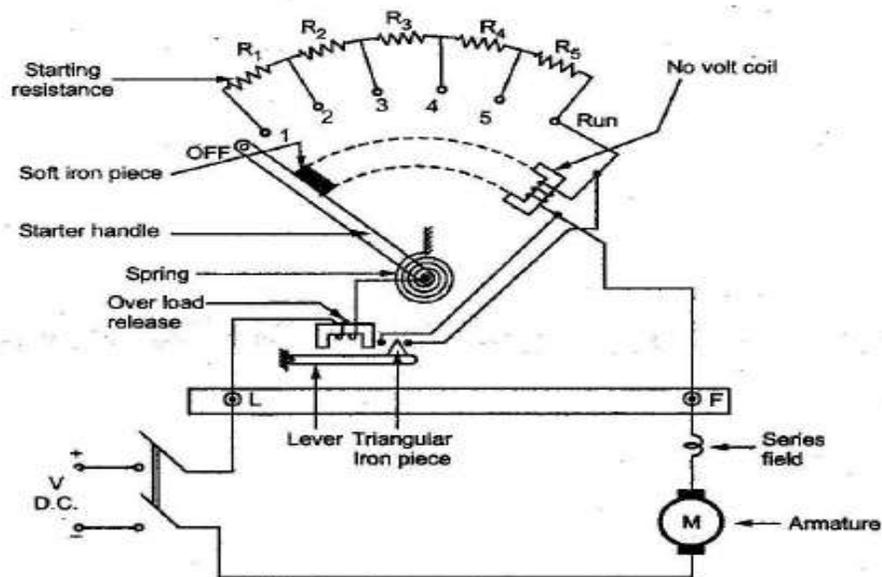
Or

**Why Starting current is high at the moment of starting a DC motor? Explain the method of limiting the starting current in DC motors?(May-June 2015)(Nov-Dec 2015,Dec-2017)**

- All the DC. motors are basically self - starting motors. Whenever the armature and the field winding of a DC motor receives supply, motoring action takes place. So starter is not required to start a DC motor but it enables us to start the motor in a desired, safe way.
- When a DC motor is directly switched on, at the time of starting, the motor back emf is zero. Due to this, the armature current is very high (25 times the rated current). Due to the very high current, the motor gets damaged. To avoid this, a resistance is introduced in series with the armature (for the duration of starting period only), which limits the starting current to a safe value. The starting resistance is gradually cut as the motor gains speed and develops the back emf, which then regulates the speed of the motor.
- Types of DC. motor starters
  - (i) Two point starter
  - (ii) Three point starter
  - (iii) Four point starter

#### **TWO POINT STARTER:**

- 3 point and 4 point starter are used for DC. Shunt motor. In case of series motor, field and armature are in series and hence starting resistance is inserted in series with the field and armature. Such a starter used to limit the starting current in case of DC series motor is called two point starter.
- The basic construction of two point starter is similar to that of three point starter except the fact that it has only two terminals namely Line (L) and Field (F). The F terminal is one end of the series combination of field and the armature winding. The starter is shown in fig.

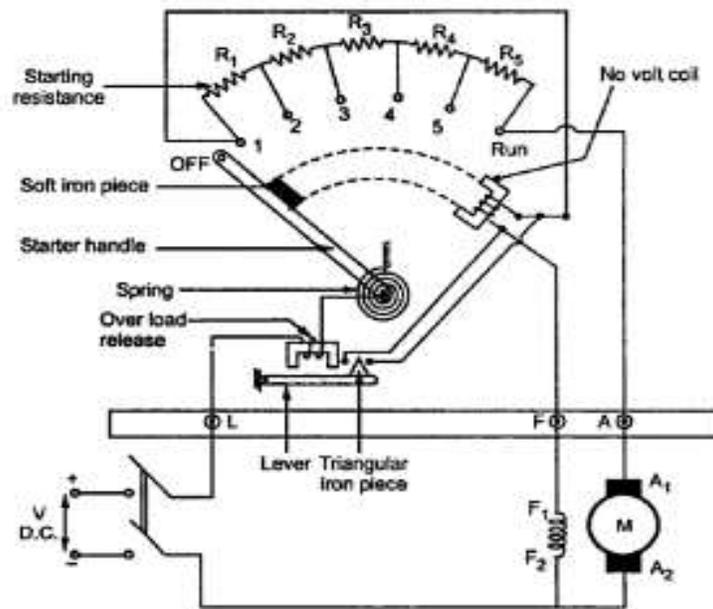


- The handle of the starter is in OFF position. When it is moved to ON, motor gets the supply (Similar to 3 point starter) and the entire starting resistance is in series with the armature and field. It limits the starting current.
- The current through no volt coil energizes it and when handle reaches to RUN position, the no volt coil holds the handle by attracting the soft iron piece on the handle. Hence no volt coil is also called hold on coil.
- The main problem in case of DC series motor is its over speeding action when the load is less. This can be prevented using two point starter. The no volt coil is designed in such a way that it hold the handle in RUN position only when it carries sufficient current, for which motor can run safely. If there is loss of load then current drawn by the motor decreases, due to this no volt coil loses its required magnetism and release the handle.
- Under spring force, handle comes back to OFF position, protecting the motor from over speeding. Similarly if there is any supply problem such that voltage decreases suddenly then also no volt coil releases the handle and protects the motor from difficult supply conditions.
- The over load condition can be prevented using overload release. When motor draws excessively high current due to overload, then current through overload magnet increases. This energizes the magnet upto such an extent that it attracts the lever below it.
- When lever is lifted upwards, the triangular piece attached to it touches the two point, which are the two ends of no volt coil. Thus no volt coil gets shorted, losing its magnetism and releasing the handle back to OFF position. This protects the motor from overloading conditions.

### **THREE POINT STARTER**

- The Starter is basically a variable resistance, divided into number of sections. The contact points of these sections are called studs and brought out separately shown as OFF, 1, 2 ... up to RUN. There are three main points of this starter:

1. `L` -- line terminal to be connected to positive of supply.
  2. `A` -- To be connected to the armature winding.
  3. `F` -- To be connected to the field winding.
- Point `L` is further connected to an electromagnet called **Over Load Release (OLR)**. The second end of `OLR` is connected to a point where handle of the starter is pivoted. This handle is free to move from its other side against the force of the spring. This spring brings back the handle to the OFF position under the influence of its own force.
  - Another parallel path is derived from the stud `1`, given to another electromagnet called the **No Volt Coil (NVC)**. The NVC is further connected to the terminal `F`. The starting resistance is entirely series with the armature. The OLR and NVC are the protecting devices of the starter.



### Operation:

- Initially the handle is in the OFF position. The DC supply to the motor is switched on. Then handle is slowly moved against the spring force to make a contact with stud no.1. At this point, field winding gets supply through the parallel path provided to starting resistance, through NVC. While entire starting resistance comes in series with the armature and armature current which is high at start, gets limited.
- As the handle is moved further, it goes on making contact with the studs 2, 3, 4 etc., cutting out the starting resistance gradually from the armature circuit. Finally when the starter handle is in `RUN` position, the entire starting resistance gets removed from the armature circuit and motor starts operating with normal speed. The handle is moved manually.

### No Volt Coil:

- The NVC consists of thin wire of many turns and is connected in series with the field winding of the motor. This coil is magnetized when the current flows through the shunt field winding. Its function is to attract the handle of the starter and keep it in the ON position. It releases the handle and goes to the OFF position by the spring tension due to the failure of supply and this way disconnects the motor from the supply.

### **Overload Release Coil:**

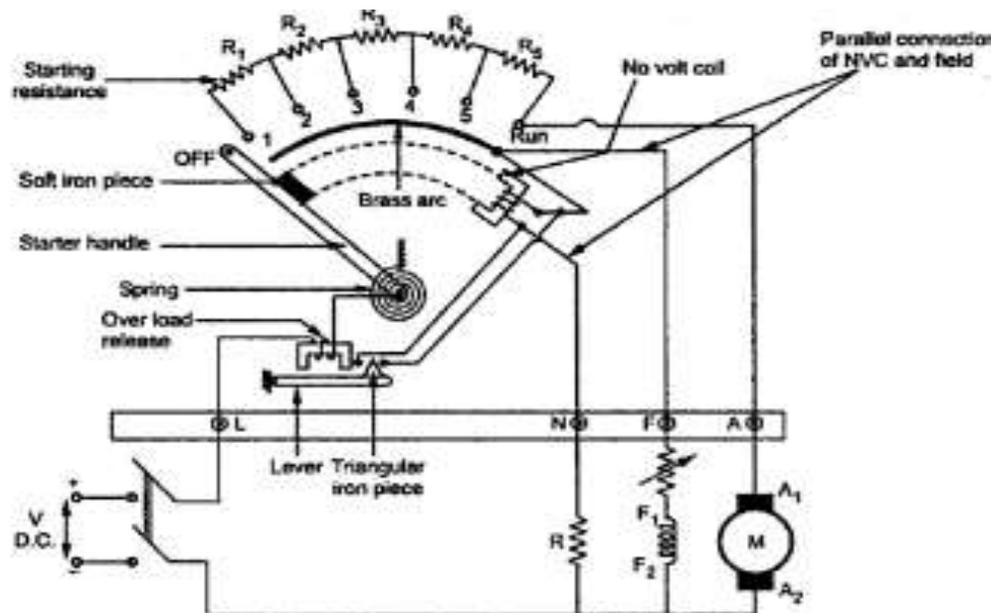
- Its function is to demagnetize the NVC in the case of fault or overload of the machine. It consists of few turns of thick wire and is connected in series to the armature. This coil will only be sufficiently magnetized when excessive current flows through the armature due to overload or some fault.
- Now the coil attracts the tripping plunger which short circuited the terminal of the NVC. Then the NVC will be demagnetized and release the starting handle which will come to the OFF position at once due to the spring tension. Thus the motor stops.

### **Disadvantages:**

- In this starter, the NVC and the field winding are in series. So while Controlling the speed of the motor above rated, field current is reduced by adding the extra resistance in series with the field winding. Due to this, the current through NVC also reduces. Due to this, magnetism produced by NVC also reduces.
- This may release the handle from its RUN position switching off the motor. To avoid the dependency of NVC and the field winding, Four Point starter is used, in which NVC and the field winding are connected in parallel.

### **FOUR POINT STARTER:**

- The basic difference between three point and four point starter is the connection of NVC. In three point, NVC is in series with the field winding while in four point starter NVC is connected independently across the supply through the fourth terminal called 'N' in addition to the 'L', 'F' and 'A'.



- Hence any change in the field current does not affect the performance of the NVC. Thus it is ensured that NVC always produce a force which is enough to hold the handle in 'RUN' position, against force of the spring, under all the operating conditions. Such a current is adjusted through

NVC with the help of fixed resistance R connected in series with the NVC using fourth point 'N' as shown.

**Disadvantage:**

- The only limitation of the four point starter is, it does not provide high speed protection to the motor. If under running condition, field gets opened, the field current reduces to zero. But there is some residual flux present and  $N\alpha 1/\phi$  the motor tries to run with dangerously high speed. This is called **high speeding action**.
- In three point starter as NVC is in series with the field, under such field failure, NVC releases handle to the OFF position. But in four point starter NVC is connected directly across the supply and its current is maintained irrespective of the current through the field winding.
- Hence it always maintains handle in the RUN position, as long as supply is there. And thus it does not protect the motor from field failure conditions which result into the high speeding of the motor.

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**4. Explain in details the various methods of speed control in DC motor [May/June-2012, 2017,dec-2017]**

**Speed Control of DC Motors:**

The speed of a motor is given by the where  $R_a =$  Armature Circuit Resistance.

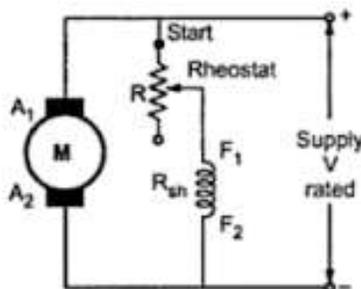
It is obvious that the speed can be controlled by varying

- i. Flux/pole,  $\Phi$  (**Flux Control**)
- ii. Resistance  $R_a$  of armature circuit (**Rheostatic Control**) and
- iii. Applied voltage  $V$  (**Voltage Control**).

**Speed Control of Shunt motor:**

Variation of Flux or Flux Control Method:

- By decreasing the flux, the speed can be increased and vice versa.
- The flux of a DC motor can be changed by changing  $I_{sh}$  with help of a shunt field rheostat.
- Since  $I_{sh}$  is relatively small, shunt field rheostat has to carry only a small current, which means  $I_{sh}^2 R_{loss}$  is small, so that rheostat is small in size.



(ii) Armature or Rheostatic Control Method:

- This method is used when speeds below the no-load speed are required.
- As the supply voltage is normally constant, the voltage across the armature is varied by inserting a variable rheostat in series with the armature circuit.



- The voltage of the generator can be varied from zero up to its maximum value by means of its field regulator.
- By reversing the direction of the field current of G by means of the reversing switch RS, generated voltage can be reversed and hence the direction of rotation of  $M_1$ .
- It should be remembered that motor generator set always runs in the same direction.

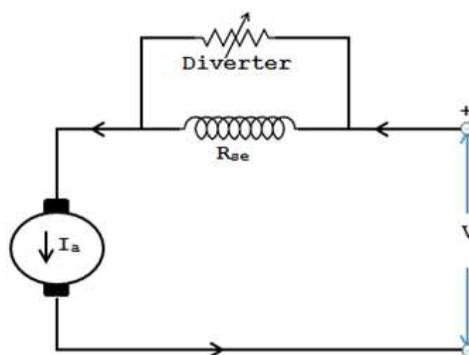
### Speed Control of Series Motors:

#### Flux Control Method:

Variations in the flux of a series motor can be brought about in any one of the following ways:

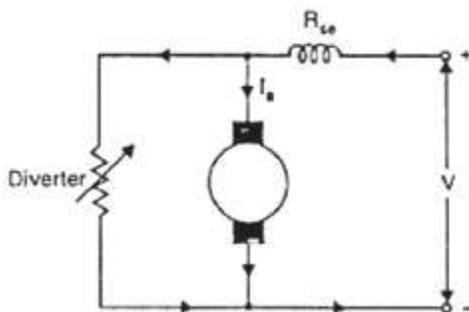
##### (a) Field Diverters:

- The series winding are shunted by a variable resistance known as field diverter.
- Any desired amount of current can be passed through the diverter by adjusting its resistance.
- Hence the flux can be decreased and consequently, the speed of the motor increased.



##### (b) Armature Diverter:

- A diverter across the armature can be used for giving speeds lower than the normal speed.
- For a given constant load torque, if  $I_a$  is reduced due to armature diverter, the  $\Phi$  must increase ( $\because T_a \propto \Phi I_a$ ). This results in an increase in current taken from the supply (which increases the flux and a fall in speed ( $N \propto I/\Phi$ )).
- The variation in speed can be controlled by varying the diverter resistance.

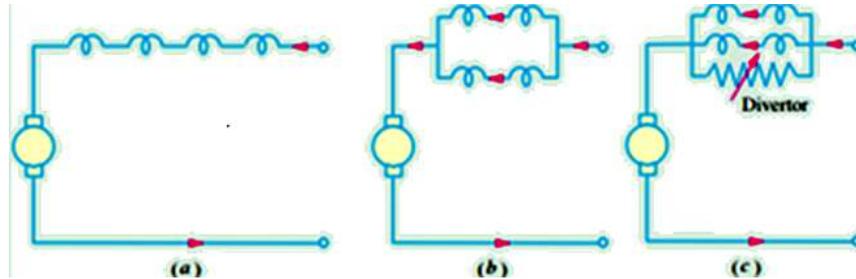


#### I Trapped Field Control Field:

- This method is often used in electric traction.
- The number of series field turns in the circuit can be changed.
- With full field, the motor runs at its minimum speed which can be raised in steps by cutting out some of the series turns.

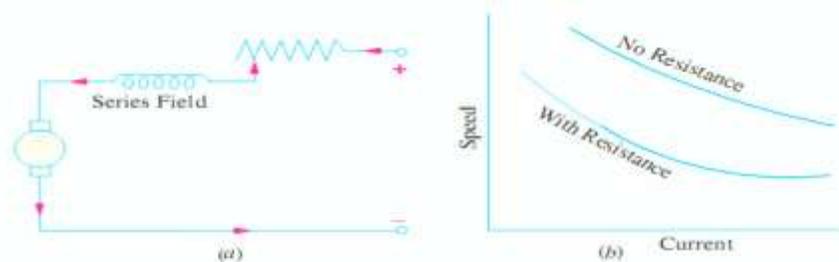
(d) Paralleling Field coils:

- This method used for fan motors, several speeds can be obtained by regrouping the field coils.
- It is seen that for a 4-pole motor, three speeds can be obtained easily.



Variable Resistance in Series with Motor:

- By increasing the resistance in series with the armature the voltage applied across the armature terminals can be decreased.
- With reduced voltage across the armature, the speed is reduced.
- However, it will be noted that since full motor current passes through this resistance, there is a considerable loss of power in it.



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**5. Derive the Torque and Speed Equations of the DC Motor.**

Torque is turning or twisting moment of a force about an axis. It is measured by the product of the force and the radius at which this force acts.

Consider a pulley of radius  $r$  metre acted upon by a circumferential force of  $F$  newton which causes it to rotate at speed  $N$  rpm.

Torque  $T = F \times r$  Newton – meter(Nm)

Work done by this force in one revolution = Force  $\times$  distance  
 $= F \times 2\pi r$  Joules

And

$$P = \text{Power developed} = \frac{\text{Work done}}{\text{Time}}$$

$$= F \times \frac{2\pi r}{\text{Time for one revolution}}$$

$$= F \times \frac{2\pi r}{60N} = F \times r \left( \frac{2\pi N}{60} \right)$$

$$P = T \times \omega$$

Where T=Torque in Nm

$$\omega = \text{Angular speed in rad/sec} \frac{2\pi N}{60} = \frac{2\pi NT}{60} \text{ watts}$$

**Armature Torque of a motor:**

Let  $T_a$  be the gross torque developed by the armature of motor running at N r.p.m. It is also called armature torque.

$$\text{Power developed in armature} = T_a \left( \frac{2\pi N}{60} \right) \dots \dots \dots (i)$$

$$\text{Electrical power converted into mechanical power in the Armature} = E_b I_a \text{ Watts} \dots \dots \dots (ii)$$

Equating (i) and (ii) we get

$$T_a \left( \frac{2\pi N}{60} \right) = E_b I_a$$

But  $E_b$  in a motor is given by

$$E_b = (\Phi ZN)/60 (P/A)$$

$$T_a \left( \frac{2\pi N}{60} \right) = \frac{\Phi ZN}{60} \left( \frac{P}{A} \right) I_a$$

$$T_a = \frac{1}{2\pi} \times \Phi Z I_a \left( \frac{P}{A} \right) \text{ NM}$$

$$T_a = 0.159 \Phi Z I_a \left( \frac{P}{A} \right) \text{ NM}$$

**Speed and Torque Equation:**

For a DC motor, the speed equation is obtained as follows:

We know  $E_b = V - I_a R_a = (\Phi ZN)/60 (P/A)$

$$V - I_a R_a = (\Phi ZN)/60 (P/A)$$

$$N = \frac{V - I_a R_a}{\Phi Z} \times \left( \frac{60A}{P} \right)$$

Since for a given machine, z, A and P constants.

$$N = K \left( \frac{V - I_a R_a}{\Phi} \right)$$

Where „K is constant.

Speed equation becomes  $N \propto (V - I_a R_a)/\Phi$

**6. Explain various types of braking of DC motor**

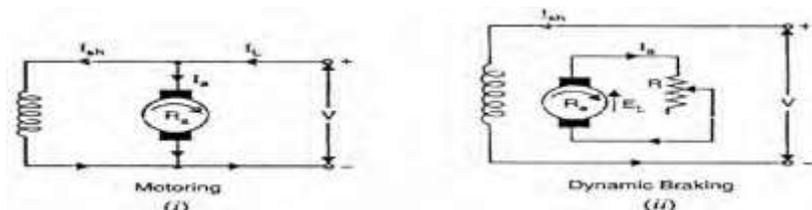
**Electric braking**

The electric braking of a DC motor is of three types, (i) Rheostatic or dynamic braking, (ii) Plugging or reverse current braking and (iii) Regenerative braking.

The main advantage of using electric braking is that it reduces the wear and tear of mechanical brakes and cuts down the stopping time considerably due to high braking retardation.

**1. Rheostatic or Dynamic braking**

- ✓ In this method, the armature of the running motor is disconnected from the supply and is connected across a variable resistance R.
- ✓ However, the field winding is left connected to the supply. The armature, while slowing down, rotates in a strong magnetic field and, therefore, operates as a generator, sending a large current



through resistance R.

- ✓ This causes the energy possessed by the rotating armature to be dissipated quickly as heat in the resistance. As a result, the motor is brought to standstill quickly.
- ✓ It shows dynamic braking of a shunt motor. The braking torque can be controlled by varying the resistance R. If the value of R is decreased as the motor speed decreases, the braking torque may be maintained at a high value.
- ✓ At a low value of speed, the braking torque becomes small and the final stopping of the motor is

$$\text{Armature current, } I_a = \frac{E_b}{R + R_a} = \frac{k_1 N \phi}{R + R_a} \quad (\because E_b \propto \phi N)$$

$$\text{Braking torque, } T_B = k_2 I_a \phi = k_2 \phi \left( \frac{k_1 N \phi}{R + R_a} \right) = k_3 N \phi^2$$

where  $k_2$  and  $k_3$  are constants

For a shunt motor,  $\phi$  is constant.

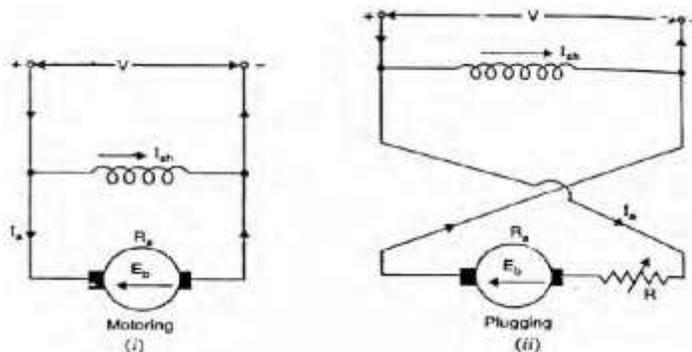
$$\therefore \text{ Braking torque, } T_B \propto N$$

Therefore, braking torque decreases as the motor speed decreases.

- ✓ due to friction.

## 2. Plugging:

- ✓ In this method, connections to the armature are reversed so that motor tends to rotate in the opposite direction, thus providing the necessary braking effect.
- ✓ When the motor comes to rest, the supply must be cut off otherwise the motor will start rotating in the opposite direction.



- ✓ Armature connections are reversed while the connections of the field winding are kept the same.
- ✓ As a result the current in the armature reverses. During the normal running of the motor, the back e.m.f.,  $E_b$  opposes the applied voltage  $V$ .
- ✓ However, when armature connections are reversed, back e.m.f.  $E_b$  and  $V$  are in the same direction around the circuit.
- ✓ Therefore, a voltage equal to  $V + E_b$  is impressed across the armature circuit. Since  $E_b \sim V$ , the impressed voltage is approximately  $2V$ .

- ✓ In order to limit the current to safe value, a variable resistance R is inserted in the circuit at the time of changing armature connections.

$$\text{Armature current, } I_a = \frac{V + E_b}{R + R_a} = \frac{V}{R + R_a} + \frac{k_1 N \phi}{R + R_a} \quad (\because E_b \propto \phi N)$$

$$\text{Braking torque, } T_B = k_2 I_a \phi = k_2 \phi \left( \frac{V}{R + R_a} + \frac{k_1 N \phi}{R + R_a} \right) = k_3 \phi + k_4 N \phi^2$$

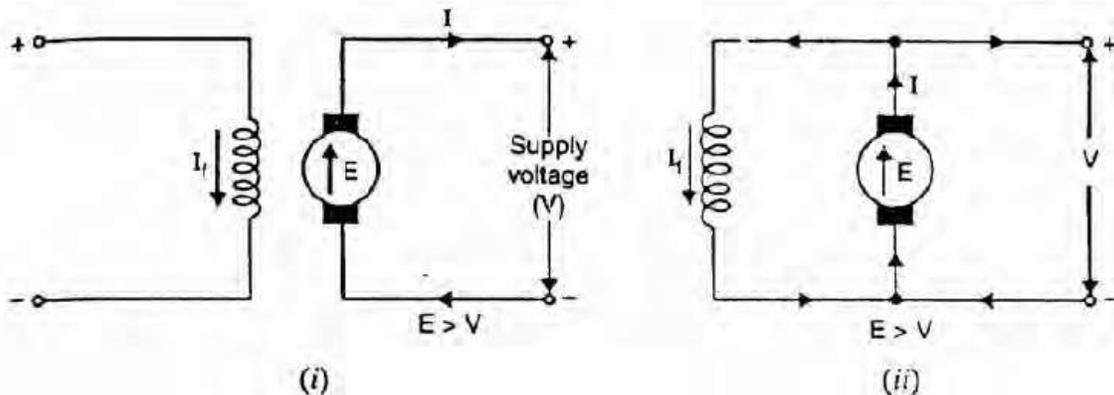
For a shunt motor,  $\phi$  is constant.

$$\therefore \text{Braking torque, } T_B = k_5 + k_6 N$$

Thus braking torque decreases as the motor slows down. Note that there is some braking torque ( $T_B = k_5$ ) even when the motor speed is zero.

### 3. Regenerative braking

- ✓ The motor is run as a generator. As a result, the kinetic energy of the motor is converted into electrical energy and returned to the supply.
- ✓ In one method, field winding is disconnected from the supply and field current is increased by exciting it from another source.



- ✓ As a result, induced e.m.f. E exceeds the supply voltage V and the machine feeds energy into the supply.
- ✓ Thus braking torque is provided up to the speed at which induced e.m.f. and supply voltage are equal.
- ✓ As the machine slows down, it is not possible to maintain induced e.m.f. at a higher value than the supply voltage.
- ✓ Therefore, this method is possible only for a limited range of speed.
- ✓ In a second method, the field excitation does not change but the load causes the motor to run above the normal speed .
- ✓ As a result, the induced e.m.f. E becomes greater than the supply voltage V.
- ✓ The direction of armature current I, therefore, reverses but the direction of shunt field current  $I_f$  remains unaltered.
- ✓ Hence the torque is reversed and the speed falls until E becomes less than V.

## 7. Explain testing of DC machine?

Testing of DC machines can be broadly classified as

- i) Direct method of Testing
- ii) Indirect method of testing

### Direct method of testing:

In this method, the DC machine is loaded directly by means of a brake applied to a water cooled pulley coupled to the shaft of the machine. The input and output are measured and efficiency is determined by

$$\eta = \frac{\text{output}}{\text{input}}$$

It is not practically possible to arrange loads for machines of large capacity.

### Indirect method of testing:

In this method, the losses are determined without actual loading the machine. If the losses are known, then efficiency can be determined. Swinburne's test and Hopkinson's test are commonly used on shunt motors. But, as series motor cannot be started on No-load, these tests cannot be conducted on DC series motor.

- (i) **Brake Test:** is a direct method of testing. In this method of testing motor shaft is coupled to a Water cooled pulley which is loaded by means of weight as shown in fig.

$W_1 =$  Suspended Weight in kg

$W_2 =$  Reading in spring balance in kg

$R =$  Radius of the pulley

$N =$  Speed in rps

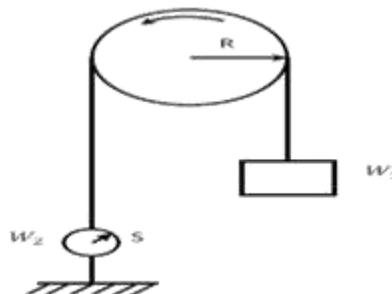
$V =$  Supply voltage

$I =$  Full load current

Net pull due to Friction =  $(W_1 - W_2)$  kg

=  $9.81(W_1 - W_2)$  Newton

Shaft Torque  $T_{sh} = (W_1 - W_2)R$  kg - mt



## 8. Explain in detail about the working Principle of DC motor?

### DC. Motor Principle

A DC MOTOR is a machine that converts Electrical energy into mechanical energy.

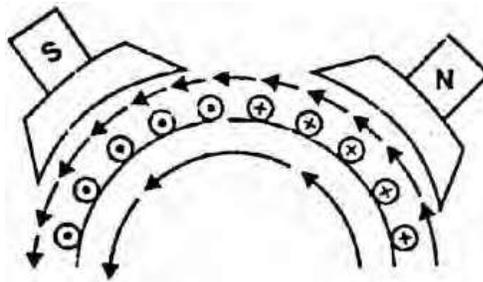
A machine that converts DC. power into mechanical power is known as a DC. motor. Its operation is based on the principle that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force. The direction of this force is given by Fleming's left hand rule and magnitude is given by  $F=BI\ell$  Newton.

$B$ =Magnetic field intensity in  $\text{wb/m}^2$

$I$ =Current in Amperes

$\ell$  =Length of the conductor in meter.

Basically, there is no constructional difference between a DC. motor and a DC. generator. The same DC. machine can be run as a generator or motor.



### Working of DC. Motor

Consider a part of a multi polar DC motor as shown in Fig. When the terminals of the motor are connected to an external source of DC. supply:(i) the field magnets are excited developing alternate N and S poles;(ii) the armature conductors carry ^currents. All conductors under N-pole carry currents in one direction while all the conductors under S-pole carry currents in the opposite direction. Suppose the conductors under N-pole carry currents into the plane of the paper and those under S-pole carry currents out of the plane of the paper as shown in Fig. Since each armature conductor is carrying current and is placed in the magnetic field, mechanical force acts on it. Referring to Fig. (4.1) and applying Fleming's left hand rule, it is clear that force on each conductor is tending to rotate the armature in anticlockwise direction. All these forces add together to produce a driving torque which sets the armature rotating. When the conductor moves from one side of a brush to the other, the current in that conductor is reversed and at the same time it comes under the influence of next pole which is of opposite polarity. Consequently, the direction of force on the conductor remains the same

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## 9. Mention the various Application of DC motor. May 2017

### DC shunt motor

1. For driving constant speed line shafting.
2. Lathe
3. Centrifugal pumps

4. Machine tools
5. Blowers and fans
6. Reciprocating pumps

#### **DC series motor**

1. Electric locomotives
2. Rapid transit systems
  3. Trolley cars
  4. Cranes and hoists
5. Conveyors

#### **DC compound motor**

1. Elevators.
2. Heavy planers
3. Rolling mills
4. Air compressor

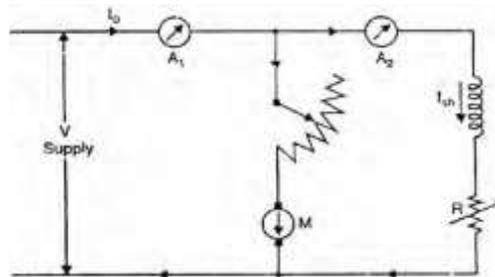
### **10. Swinburne's Method for Determining Efficiency. Dec – 2016**

**With the help of neat circuit diagram, explain Swinburne's test and derive the relations for efficiency (Both for generator and Motor). May 2017**

In this method, the DC machine (generator or motor) is run as a motor at no load and losses of the machine are determined. Once the losses of the machine are known, its efficiency at any desired load can be determined in advance. It may be noted that this method is applicable to those machines in which flux is practically constant at all loads e.g., shunt and compound machines. Let us see how the efficiency of a DC shunt machine (generator or motor) is determined by this method.

#### **Determination of hot resistances of windings**

The armature resistance and shunt field resistance are measured using a battery, voltmeter and ammeter. Since these resistances are measured when the machine is cold, they must be converted to values corresponding to the temperature at which the machine would work on full-load. Generally, these values are measured for a temperature rise of 40°C above the room temperature. Let the hot resistances of armature and shunt field be  $R_a$  and  $R_{sh}$  respectively



#### **Determination of constant losses**

The machine is run as a motor on no-load with supply voltage adjusted to the rated voltage. The speed of the motor is adjusted to the rated speed with the help of field regulator  $R$  as shown in Fig.

Let  $V =$  Supply voltage

$I_0 =$  No-load current read by ammeter  $A_1$

$I_{sh} =$  Shunt-field current read by ammeter  $A_2$

No-load armature current,  $I_{a0} = I_0 - I_{sh}$

No-load input power to motor  $= V I_0$

No-load power input to armature  $= V I_{a0} = V(I_0 - I_{sh})$

Since the output of the motor is zero, the no-load input power to the armature supplies (a) iron losses in the core (b) friction loss (c) windage loss (d) armature Cu loss [ $I_{a0}^2 R_a$  or  $(I_0 - I_{sh})^2 R_a$ ].

Constant losses,  $WC =$  Input to motor  $-$  Armature Cu loss

$$WC = V I_0 - (I_0 - I_{sh})^2 R_a$$

Since constant losses are known, the efficiency of the machine at any other load can be determined.

Suppose it is desired to determine the efficiency of the machine at load current  $I$ . Then,

Armature current,  $I_a = I - I_{sh}$  ... if the machine is motoring

$= I + I_{sh}$  ... if the machine is generating

### **Efficiency when running as a motor**

Input power to motor  $= VI$

Armature Cu loss  $= I_a^2 R_a = (I - I_{sh})^2 R_a$

Constant losses  $= WC$

Total losses  $= (I - I_{sh})^2 R_a + WC$

$$\text{Motor efficiency, } \eta_m = \frac{\text{Input} - \text{Losses}}{\text{Input}} = \frac{VI - (I - I_{sh})^2 R_a + WC}{VI}$$

### **Efficiency when running as a generator**

Output of generator  $= VI$

Armature Cu loss  $= I_a^2 R_a = (I + I_{sh})^2 R_a$

Constant losses  $= WC$

Total losses  $= (I + I_{sh})^2 R_a + WC$

$$\text{Generator efficiency, } \eta_g = \frac{\text{Output}}{\text{Output} + \text{Losses}} = \frac{VI}{VI + (I + I_{sh})^2 R_a + WC}$$

### **Advantages of Swinburne's test**

(i) The power required to carry out the test is small because it is a no-load test. Therefore, this method is quite economical.

The efficiency can be determined at any load because constant losses are known.

This test is very convenient.

### **Disadvantages of Swinburne's test**

1. It does not take into account the stray load losses that occur when the machine is loaded.

2. This test does not enable us to check the performance of the machine on full-load. For example, it does not indicate whether commutation on full load is satisfactory and whether the temperature rise is within the specified limits.

3. This test does not give quite accurate efficiency of the machine. It is because iron losses under actual load are greater than those measured. This is mainly due to armature reaction distorting the field.

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### 11. Regenerative or Hopkinson's-Test (May 2016)

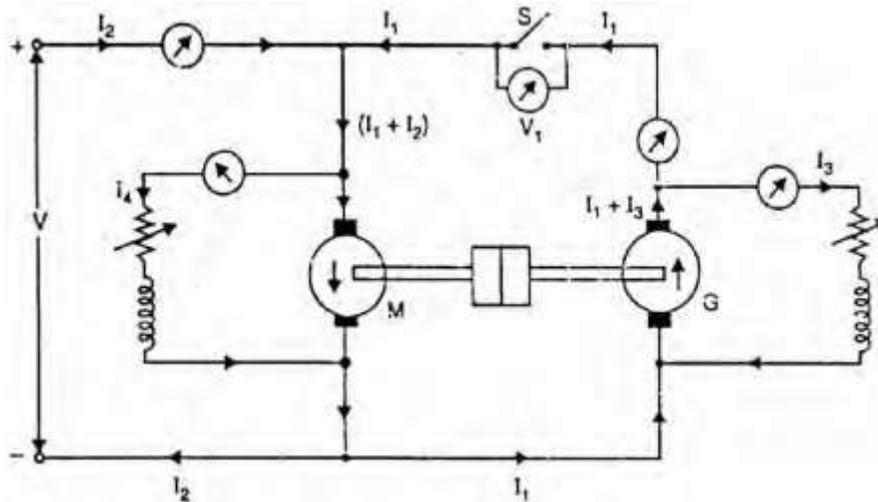
This method of determining the efficiency of a DC machine saves power and gives more accurate results. In order to carry out this test, we require two identical DC machines and a source of electrical power.

#### Principle

Two identical DC shunt machines are mechanically coupled and connected in parallel across the DC supply. By adjusting the field excitations of the machines, one is run as a motor and the other as a generator. The electric power from the generator and electrical power from the DC supply are fed to the motor. The electric power given to the motor is mostly converted into mechanical power, the rest going to the various motor losses. This mechanical power is given to the generator. The electrical power of the generator is given to the motor except that which is wasted as generator losses. Thus the electrical power taken from the DC supply is the sum of motor and generator losses and this can be measured directly by a voltmeter and an ammeter. Since the power input from the DC supply is equal to the power required to supply the losses of the two machines, this test can be carried out with a small amount of power. By adjusting the field strengths of the machines, any load can be put on the machines. Therefore, we can measure the total loss of the machines at any load. Since the machines can be tested under full-load conditions (of course at the expense of power equal I losses in the two machines), the temperatures rise and commutation qualities of the machines can be observed.

#### Circuit

The essential connections for Hopkinson's test. Two identical DC shunt machines are mechanically coupled and are connected in parallel across the DC supply. By adjusting the field strengths of the two machines, the machine M is made to run as a motor and machine G as a generator. The motor M draws current  $I_1$  from the generator G and current  $I_2$  from the DC supply so that input current to motor M is  $(I_1 + I_2)$ . Power taken from the DC supply is  $VI_2$  and is equal to the total motor and generator losses. The field current of motor M is  $I_4$  and that of generator G is  $I_3$ .



#### Calculations

If  $V$  be the supply voltage, then,

$$\text{Motor input} = V(I_1 + I_2)$$

$$\text{Generator output} = VI_1$$

We shall find the efficiencies of the machines considering two cases viz. assuming that both machines have the same efficiency  $\eta$  (ii) assuming iron, friction and windage losses are the same in both machines.

**(i) Assuming that both machines have the same efficiency  $\eta$**

$$\text{Motor output} = \eta \times \text{motor input} = \eta V(I_1 + I_2) = \text{Generator input}$$

$$\text{Generator output} = \eta \times \text{generator input} = \eta \times \eta V(I_1 + I_2) = \eta^2 V(I_1 + I_2)$$

But generator output is  $VI_1$

$$\eta^2 V(I_1 + I_2) = VI_1$$

$$\eta = \sqrt{\frac{I_1}{I_1 + I_2}}$$

This expression gives the value of efficiency sufficiently accurate for a rough test. However, if accuracy is required, the efficiencies of the two machines should be calculated separately as below.

**(ii) Assuming that iron, friction and windage losses are same in both machines.**

It is not to assume that the two machines have the same efficiency. I because armature and field in the two machines are not the same. However, iron, friction and windage losses in the two machines will be the same because the machines are identical. On this assumption, we can find the of each machine as under:

Let  $R_a$  = armature resistance of each machine

$I_3$  = field current of generator G

$I_4$  = field current of motor M

Armature Cu loss in generator =  $(I_1 + I_3)^2 R_a$

Armature Cu loss in motor =  $(I_1 + I_2 - I_4)^2 R_a$

Shunt Cu loss in generator =  $V I_3$

Shunt Cu loss in motor =  $V I_4$

Power drawn from the DC supply is  $VI_2$  and is equal to the total losses of the motor and generator

$$VI_2 = \text{Total losses of motor and generator}$$

If we subtract armature and shunt Cu losses of the two machines from  $VI_2$ , we get iron, friction windage losses of the two machines.

Iron, friction and windage losses of two machines (M and G)

$$= VI_2 - [(I_1 + I_3)^2 R_a + (I_1 + I_2 - I_4)^2 R_a + VI_3 + VI_4] = W(\text{Say})$$

Iron, friction and windage losses of each machine =  $W/2$

**For generator**

$$\text{out put of generator} = VI_1$$

$$\text{Total losses} = \frac{W}{2} + (I_1 + I_3)^2 R_a + VI_3 = W_g$$

$$\text{Generator efficiency} = \eta_g = \frac{VI_1}{VI_1 + W_g}$$

**For motor**

$$\text{input to motor} = V(I_1 + I_2)$$

$$\text{Total losses} = (I_1 + I_2 - I_4)^2 R_a + VI_4 + \frac{W}{2} = W_m$$

$$\text{Motor efficiency } \eta_m = \frac{\text{Input} - \text{Losses}}{\text{Input}} = \frac{V(I_1 + I_2) - W_m}{V(I_1 + I_2)}$$

12. Explain in detail the construction and working operation of retardation test on DC motor?(Nov-Dec 2015)

This method is generally employed to shunt generators and shunt motors. From this method we can get stray losses. Thus if armature and shunt copper losses at any given load current are known then efficiency of a machine can be easily estimated.

The connections required for conducting this test are shown in the Fig. 1.

The machine whose test is to be taken is run at a speed which is slightly above its normal speed. The supply to the motor is cut off while the field is kept excited. The armature consequently slows down and its kinetic energy is used in supplying the rotational or stray losses which includes iron, friction and winding loss.

If  $I$  is the amount of inertia of the armature and  $\omega$  is the angular velocity.

$$\text{Kinetic energy of armature} = 0.5 I \omega^2$$

∴ Rotational losses,  $W = \text{Rate of change of kinetic energy}$

$$= \frac{d}{dt} \left( \frac{1}{2} I \omega^2 \right) = I \omega \frac{d\omega}{dt}$$

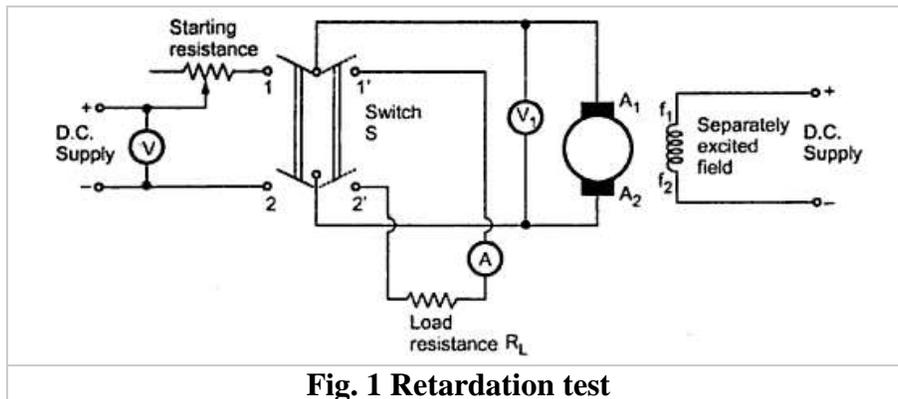


Fig. 1 Retardation test

Angular velocity,  $\omega = (2\pi N)/60$

$$W = I \left( \frac{2\pi N}{60} \right) \frac{d}{dt} \left( \frac{2\pi N}{60} \right) = I \left( \frac{2\pi N}{60} \right) \left( \frac{2\pi}{60} \right) \frac{dN}{dt}$$

$$W = \left( \frac{2\pi}{60} \right)^2 I N \frac{dN}{dt}$$

Thus if we if to find the rotational losses, the moment of inertia  $I$  and  $dN/dt$  must be known.

These quantities can be found as follows ;-

**Determination of  $dN/dt$ :**

The voltmeter  $V_1$  which is connected across the armature will read the back e.m.f. of the motor. We know that back e.m.f. is proportional to speed so that voltmeter is calibrated to read the speed directly.

When motor is cut off from the supply, the speed decrease in speed is noted with the help of stop watch. A curve showing variation between time and speed which is obtained from voltmeter which is suitably calibrated is shown in the Fig. 2.

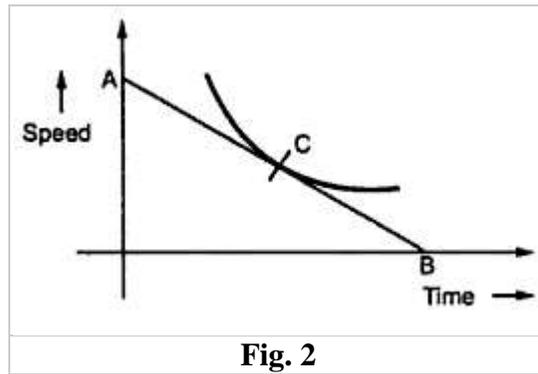


Fig. 2

At any point C corresponding to normal speed, a tangent AB is drawn. Then

$$\frac{dN}{dt} = \frac{OA(\text{in rpm})}{OB(\text{in seconds})}$$

The value obtained from above can be substituted in the expression for W which can give the rotational losses.

### Determination of moment of inertia (I)

#### Method (a) Using Flywheel

The armature supply is cut off and time required for definite change in speed is noted to draw the corresponding curve as we have drawn in previous case. This curve is drawn considering only armature of the machine. Now a flywheel with known moment of the inertia say is  $I_1$  keyed onto the shaft and the same curve is drawn again. The slowing down time will be extended as combined moment of inertia of the two is increased.

For any given speed  $(dN/dt_1)$  and  $(dN/dt_2)$  are determined same as previous case. It can be seen that the losses in both the cases are almost same as addition of flywheel will not make much difference to the losses.

In the first case where flywheel is not there then,

$$W = \left(\frac{2\pi}{60}\right)^2 IN \frac{dN}{dt_1}$$

Adding the flywheel to the motor armature in second case we get,

$$W = \left(\frac{2\pi}{60}\right)^2 (I + I_1)N \frac{dN}{dt_1}$$

$$IN \frac{dN}{dt_1} = (I + I_1)N \frac{dN}{dt_2}$$

$$\frac{(I + I_1)}{I} = \frac{\frac{dN}{dt_1}}{\frac{dN}{dt_2}}$$

$$I = \frac{(I + I_1) \frac{dN}{dt_2}}{\frac{dN}{dt_1}}$$

$$I = I_1 \times \frac{\frac{dN}{dt_2}}{\frac{dN}{dt_1} - \frac{dN}{dt_2}} = I_1 \times \frac{dt_1}{dt_2 - dt_1}$$

$$I = I_1 \times \frac{t_1}{t_2 - t_1}$$

### Method (b) Without using Flywheel

In this method time is noted for the machine to slow down by say 5 % considering the armature alone. The a retarding torque either mechanical or electrical is applied. Preferably electrical retarding torque is applied and time required to slow down by 5% is noted again. The method by which electrical torque can be provided is shown in the Fig. 1 in which the switch S after disconnecting from the supply is thrown to terminals 1'2'. The machine then gets connected to a non-inductive load resistance  $R_L$ . The power drawn by this resistance will acts as a retarding torque on the armature which will make it slow more quickly.

The additional loss in the resistance will be equal to product of ammeter reading and the average reading of the voltmeter (for a fall of 5% of voltmeter reading, the time is noted.) The ammeter reading is also changing so its average reading is taken. Thus the additional losses is  $I_a^2 (R_a + R)$ . Let  $t_1$  be the time when armature is considered alone and  $t_2$  be the time when armature is connected across a load resistance,  $V$  be average voltage across  $R$  and  $I_a$  be the average current and  $W'$  is additional retarding electrical torque supplied by motor.

$$W = \left(\frac{2\pi}{60}\right)^2 IN \frac{dN}{dt_1}$$

$$W + W' = \left(\frac{2\pi}{60}\right)^2 IN \frac{dN}{dt_2}$$

If  $dN$  i.e. change in speed is same in two cases then

$$\frac{W + W'}{W} = \frac{\frac{1}{dt_2}}{\frac{1}{dt_1}} = \frac{dt_1}{dt_2} = \frac{t_1}{t_2}$$

$$(W + W')t_2 = Wt_1$$

$$W(t_1 - t_2) = W't_2$$

$$W = W' \frac{t_2}{t_1 - t_2}$$

Here  $dN/dt_1$  is rate of change in speed without extra load whereas  $dN/dt_2$  is rate change in speed with extra electrical load which provides retarding torque.

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### 13. Derive in detail the condition for maximum efficiency of DC Machines?(Nov-Dec 2015)

For a DC. machine, its overall efficiency is given by,

$$\% \eta = \frac{\text{Total output}}{\text{Total input}} \times 100$$

Let  $P_{out}$  = total output of a machine

$P_{in}$  = total input of a machine

$P_{cu}$  = variable losses

$P_i$  = constant losses

then  $P_{in} = P_{out} + P_{cu} + P_i$

$$\% \eta = \frac{P_{out}}{P_{in}} \times 100 = \frac{P_{out}}{P_{out} + \text{losses}} \times 100$$

$$\% \eta = \frac{P_{out}}{P_{in} + P_{cu} + P_i} \times 100$$

### Condition for Maximum Efficiency

In case of a DC. generator the output is given by,

$$P_{out} = VI$$

$$P_{cu} = \text{variable losses} = I_a^2 R_a = I^2 R_a$$

$$I_a = I$$

$$\therefore \% \eta = \frac{VI}{VI + I^2 R_a + P_i} \times 100 = \frac{1}{1 + \left( \frac{IR_a}{V} + \frac{P_i}{VI} \right)} \times 100$$

..... neglecting shunt field current

The efficiency is maximum, when the denominator is minimum. According to maxima-minima theorem,

$$\begin{aligned} \frac{d}{dt} \left[ 1 + \left( \frac{IR_a}{V} + \frac{P_i}{VI} \right) \right] &= 0 \\ \frac{IR_a}{V} + \frac{P_i}{VI^2} &= 0 \\ I_a^2 R_a - P_i &= 0 \\ I_a^2 R_a &= P_i = P_{cu} \end{aligned}$$

Thus for the maximum efficiency, the condition is,

$$\text{Variable losses} = \text{Constant losses}$$

### Current at Maximum Efficiency

From the condition of maximum efficiency, the current through the DC. machine at the time of maximum efficiency can be obtained.

**For shunt machines :** The  $I_{sh}$  is constant and the loss  $V I_{sh}$  is treated to be the part of constant losses. The variable losses are  $I_a^2 R_a$ .

At maximum efficiency,

$$I_a^2 R_a = P_i = (\text{Stray} + \text{shunt field losses})$$

$$I_a = \sqrt{\frac{P_i}{R_a}} = \sqrt{\frac{\text{Constant losses}}{\text{Armature resistance}}}$$

This is the armature current at maximum efficiency. Neglecting  $I_{sh}$ ,  $I_a = I_L$  is the line current of the machine.

**For series machines :** The current through series field is same as armature current which is same as line current. Hence the constant losses are only mechanical losses while the variable losses are the copper losses in armature as well as series field winding due to the armature current.

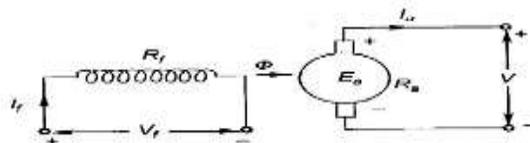
At maximum efficiency,

$$I_a^2 (R_a + R_{se}) = P_i = \text{Mechanical losses}$$

$$I_a = \sqrt{\frac{P_i}{R_a + R_{se}}}$$

### 14. Draw the circuit model of DC machine?

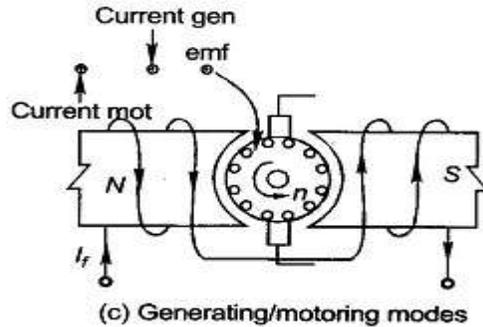
The parallel path of DC machine armature is symmetrical and each has an induced emf  $E_a$  and resistance  $R_p$ . The armature can be represented by the circuit model with voltage  $E_a$  and series resistance.  $R_a = R_p/A$ . The armature resistance is quite small so as to limit the copper loss in acceptable value. The field coil placed 90 ° to the brush axis as per the actual arrangement in the machine. The voltage drop at brush commutator contact is fixed, independent of armature current as the conduction process is mainly through numerous short arcs. However this voltage being small is modeled as linear resistance and lumped with  $R_a$



### Generating mode:

If the machine operates in generating mode when  $I_a$  is in the direction of induced emf  $E_a$ . For the armature circuit

$$V = E_a - I_a R_a; E_a > V$$



**Fig. 7.6**

The mechanical power converted to electrical form is

$$P_{\text{mech(in)}/\text{net}} = E_a I_a = P_{\text{elec(out)}/\text{gross}}$$

Net electrical output is

$$P_0 = V I_a - E_a I_a$$

$$V I_a - E_a I_a = I_a^2 R_a = \text{armature copper loss}$$

$$P_{\text{mech(in)}/\text{gross}} = \text{shaft power} = P_{\text{mech(in)}/\text{net}} + \text{rotational loss}$$

Torque  $T$  of electromagnetic origin is opposite to the direction of rotation of armature. i.e. mechanical power is absorbed and a prime mover is needed to run the machine.

The conductor emf and current are also in the same direction for generating mode.

### Motoring mode:

$$V(\text{terminal voltage}) = E_a + I_a R_a; V > E_a$$

The electrical power converted to mechanical form is

$$P_{\text{elec(in)}/\text{net}} = E_a I_a = P_{\text{mech(out)}/\text{gross}}$$

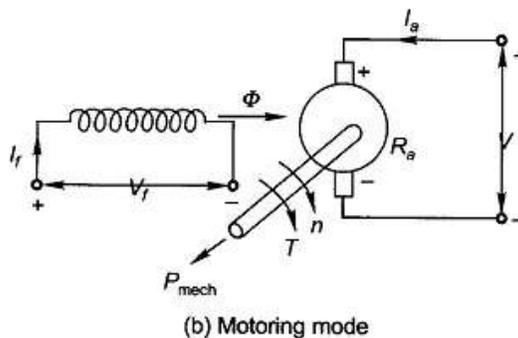
The electrical power input is  $P_i = V I_a$

$$V I_a - E_a I_a = I_a^2 R_a = \text{armature copper loss}$$

$$P_{\text{mech(out)}/\text{net}} = \text{shaft power} = P_{\text{mech(out)}/\text{gross}} - \text{rotational loss}$$

Torque  $T$  of electromagnetic origin is in the direction of rotation of armature. i.e. mechanical power is put out and is absorbed by the load.

The conductor emf and current are also in the opposite direction for motoring mode.



### Problem

15. A 250V DC shunt motor has  $R_f=150\Omega$  and  $R_a=0.6\Omega$ . The motor operates on no-load with a full field flux at its base speed of 100rpm with  $I_a=5A$ . If the machine drives a load requiring a torque of 100Nm, cal. Armature current and speed of motor. If the motor is required to develop 10 KW at 1200rpm what is the required value of the external series resistance in the field circuit? Assume linear magnetization saturation and armature reaction. (May-2011)

Given:  $R_f=150\Omega$ ,  $R_a=0.6\Omega$ ,  $V=250V$ ,  $I_{a0}=5A$

Formula used:  $E_{b0} = V - I_{a0}R_a$ ,  $T_{a0} = \frac{E_{b0}I_{a0}}{\frac{2\pi N_0}{60}}$ ,

Soln:

$$R_f=150\Omega, R_a=0.6\Omega, V=250V, I_{a0}=5A$$

$$I_{sh} = \frac{V}{R_f} = \frac{250}{150} = 1.667$$

$$E_{b0} = V - I_{a0}R_a = 250 - 5 \times 0.6 = 247V$$

$$\therefore T_{a0} = \frac{E_{b0}I_{a0}}{\frac{2\pi N_0}{60}} = 1000\text{rpm}$$
$$= \frac{247 \times 5}{\frac{2\pi \times 1000}{60}}$$

$$T_{a0} = 11.793\text{Nm}$$

Now  $T \propto \phi I_a$

$T \propto I_a$  ( $\phi$  constant for shunt motor)

$$\frac{T_{a0}}{T_{a1}} = \frac{I_{a0}}{I_{a1}} \text{ where } T_{a1} \rightarrow \text{New torque} = 100\text{Nm}$$

$$\frac{11.793}{100} = \frac{5}{I_{a1}}$$

$$I_{a1} = 42.396A \quad (\text{New armature current})$$

$$E_{b1} = V - I_{a1}R_a$$
$$= 250 - 42.396(0.6) = 224.56V$$

Now  $N \propto \frac{E_b}{\phi}$

$N \propto E_b$

$$\frac{N_0}{N_1} = \frac{E_{b0}}{E_{b1}}$$

$$\frac{1000}{N_1} = \frac{247}{224.56}$$

$$N_1 = 909.16 \text{ rpm.}$$

Now motor is required to develop 12 kw at 1200rpm with an external resistance  $R_x$  in the field circuit.

$$N_2 = 1200 \text{ rpm}$$

Power developed by the motor is

$$P_m = 12 \text{ kw.}$$

$$P_m = E_{b2} I_{a2} = \text{power developed}$$

$$12 \times 10^3 = [V - I_{a2} R_a] I_{a2}$$

$$12 \times 10^3 = [250 I_{a2} - I_{a2}^2 (0.6)]$$

$$250 I_{a2} - I_{a2}^2 (0.6) - 12 \times 10^3 = 0$$

$$0.6 I_{a2}^2 - 250 I_{a2} + 12 \times 10^3 = 0$$

$$I_{a2} = 55.35 \text{ A (Neglect higher value)}$$

$$N \propto \frac{E_b}{\phi}$$

$$N \propto \frac{E_b}{I_{sh}}$$

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}} \times \frac{I_{sh2}}{I_{sh1}} I_{sh} = I_{sh1}$$

$$\frac{909.16}{1200} = \frac{224.56}{216.79} \times \frac{I_{sh2}}{1.67} E_{b2} = V - I_{a2} R_a = 216.79$$

$$I_{sh2} = 1.219 \text{ A}$$

$$\text{But } I_{sh2} = \frac{V}{R_{sh} + R_x}$$

$$1.219 = \frac{250}{150 + R_x}$$

$$R_x = 55.05 \Omega$$

-----&&&&-----

16. A 220V, 7.5Kw series motor is mechanically coupled to a fan. When running at 400 rpm the motor draws 30A from the mains (220V). The torque required by the fan is proportional to the square of speed  $R_a=0.6\Omega$ ,  $R_{se}=0.4\Omega$ . Neglect armature and rotational loss. Also assume the magnetization characteristics of the motor to be linear

i) Determine the power delivered to the fan and torque developed by the motor.

ii) Cal. The external resistance to be added in series to the armature circuit to reduce the fan speed to 200 rpm. (April/May-2011)

Given: 220V, 7.5Kw,  $N=400$ ,  $I=30\text{A}$ ,  $R_a=0.6\Omega$ ,  $R_{se}=0.4\Omega$

Formula used:

$$P_{dev} = P(\text{no rotational loss})$$

$$P = E_a I_a$$

$$T_{dev} = K'_a K_f I_{se} I_a = K'_a K_f I_a^2 I_{sc} = I_a \text{ for linear magnetization}$$

$$T_{fin} = K_{fn}^2$$

$$T_{dev} = T_{fin} = T \text{ \& } I_a \propto n$$

Solution:

Operation at 400 rpm ( $R_{ext} = 0$ )

$$E_a = 220 - (0.6 + 0.4) \times 30$$

$$E_a = 190V$$

$$P = 190 \times 30$$

$$P=5.7Kw$$

$$T_w = E_a I_a \quad \text{or} \quad P = \frac{2\pi NT}{60}$$

$$T = \frac{P \times 60}{2\pi N}$$

$$T = \frac{5.7 \times 10^3}{2\pi \times 400} = 136 \text{ Nm}$$

ii) Operation at 200rpm ( $R_{ext} = ?$ )

$$T = 136 \times \left(\frac{200}{400}\right)^2 = 34 \text{ Nm}$$

$$I_a = 30 \times \frac{200}{400} = 15A$$

$$P = \frac{2\pi NT}{60} = E_a I_a$$

$$34 \left(\frac{2\pi \times 200}{60}\right) = [220 - (0.6 + 0.4 + R_{ext})]15$$

$$R_{ext} = 10.5\Omega$$

$$P = T_w = 34 \left(\frac{2\pi \times 200}{60}\right)$$

$$P=0.721Kw$$

-----&&&&-----

**17. A series motor of resistance  $1 \Omega$  between terminals runs at 800rpm at 200V with a current of 15A. Find the speed at which it will run when connected in series with a  $5\Omega$  resistance and taking the same current at the same supply voltage.(Nov/Dec-2010)**

**Soln.**

$$\frac{N_2}{N_1} = \frac{E_{a2}}{E_{a1}} \times \frac{I_{a1}}{I_{a2}}$$

$$N_1 = 800\text{rpm}$$

$$\frac{N_2}{800} = \frac{110}{185} \times \frac{15}{15} = 476 \text{ rpm} \quad E_{a2} = V - I_{a2}R_a = 200 - 15 \times 6 = 110 \quad (5 \Omega \text{ connected in series})$$

$$E_{a1} = V - I_{a1}R_a = 200 - 15 \times 1 = 185$$

-----&&&&-----

18. Two shunt generator running in parallel supply a total load of 300A. The generators have armature resistances 0.5 and 0.03Ω, field resistance 20 and 25Ω and induced emf's 400 and 380V respectively. Determine of o/p of each generator in Kw Make a comment on load sharing?(May2005).

Given:

I=300, armature resistances 0.5 and 0.03Ω, field resistance 20 and 25Ω, induced emf's 400 and 380V

Formula used:

$$E_{g1} = V + I_{a1}R_{a1}$$

$$E_{g2} = V + I_{a2}R_{a2}$$

Solution:

$$I_1 + I_2 = 300: I_2 = 300 - I_1$$

$$I_{sh1} = \frac{V}{R_{sh1}} = \frac{V}{30}$$

$$I_{sh2} = \frac{V}{R_{sh2}} = \frac{V}{25}$$

$$I_{a1} = I_1 + \frac{V}{30} : I_{a2} = I_2 + \frac{V}{25} = (300 - I_1) + \frac{V}{25}$$

For each generator,  $E_g = V + I_a R_a$

$$E_{g1} = V + I_{a1}R_{a1}$$

$$E_{g2} = V + I_{a2}R_{a2}$$

$$400 = V + \left[ I_1 + \frac{V}{30} \right] (0.5) \text{----- 1}$$

$$380 = V + \left[ I_2 + \frac{V}{25} \right] (0.03) \text{----- 2}$$

$$400 = 0.5I_1 + \frac{0.5V}{30} + V$$

$$0.5I_1 = 400 - \frac{0.5V}{30} - V = 400 - 1.0616V \text{----- 3}$$

$$380 = V + \left[ I_2 + \frac{V}{25} \right] (0.03) = V + 0.03I_2 + \frac{0.03V}{25}$$

$$380 = 1.0012V + 0.03(300 - I_1) = 1.0012V + 9 - 0.03I_1$$

$$0.03I_1 = -371 + 1.0012V$$

$$0.3I_1 = -1.0012V - 371 \text{----- 4}$$

Sub 3 and 4

$$I_1 = 43.911A$$

$$V=371.871V$$

$$I_2 = 256.089A$$

$$VI_1 = 16.3292kw, VI_2 = 95.232kw$$

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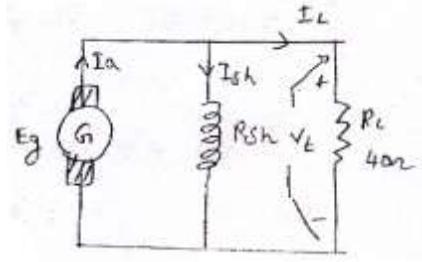
19. A 4 pole lap connected DC machine has 540 armature conductors. If the flux per pole is 0.03 wb and runs at 1500 rpm, determine the emf generated. If this machine is driven as a shunt generator with the same flux and speed, calculate the terminal voltage when it 39 identical load resistance of 40Ω, Given armature resistance is 2Ω and shunt field circuit resistance of 450Ω, Also find the load current?(May/June2007)

**Given:** P=4, lap i.e A=P, Z=540

$$\Phi=0.03\text{wb}, N=1500\text{rpm}, R_a=2\Omega, R_{sh}=450\Omega$$

**Formula used:**  $E_g = \frac{\Phi P N Z}{60 A}$ ,  $V_t = E_g - I_a R_a$ ,  $I_a = I_L + I_{sh}$

**Solution:**



$$P=4, \text{ lap i.e } A=P, Z=540$$

$$\Phi=0.03\text{wb}, N=1500\text{rpm}, R_a=2\Omega, R_{sh}=450\Omega$$

$$E_g = \frac{\Phi P N Z}{60 A} = \frac{0.03 \times 4 \times 1500 \times 540}{60 \times 4} = 405\text{V}$$

$$V_t = E_g - I_a R_a \text{-----1}$$

$$I_L = \frac{V_t}{R_L} = \frac{V_t}{40} \text{-----2}$$

$$I_{sh} = \frac{V_t}{R_{sh}} = \frac{V_t}{450} \text{-----3}$$

$$I_a = I_L + I_{sh} = \frac{V_t}{40} + \frac{V_t}{450}$$

$$I_a = 0.027222 V_t \text{-----4}$$

Substituting in equation 1

$$V_t = E_g - 0.027222 V_t \times R_a$$

$$(1 + 0.027222 \times 2) V_t = 405$$

$$V_t = 384.088\text{V}$$

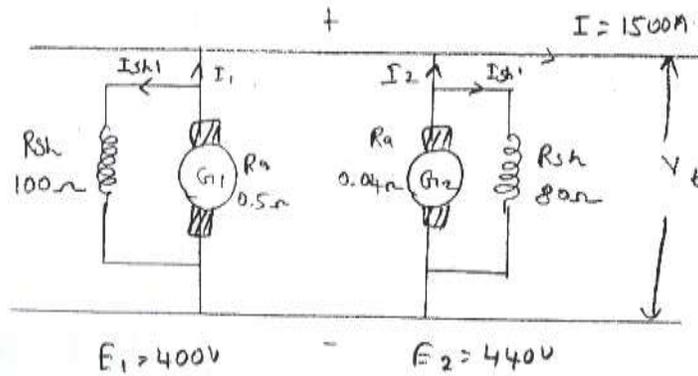
$$I_L = \frac{V_t}{40} = \frac{384.088}{40}$$

$$I_L = 9.6022\text{A}$$

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20. Two DC generator are connected in parallel to supply a load of 1500A. one generator has an armature resistance of 0.5Ω and an emf of 400V while the other has an armature resistance of 0.04Ω and an emf of 440V. The resistance of shunt field are 100Ω and 80Ω respectively. Calculate the currents supplied by the individual generator and the terminal voltage. (Nov/Dec-2008)

**Solution:**



$$I = I_1 + I_2 = 1500 \text{ ----- } 1$$

$$I_2 = 1500 - I_1$$

$$I_{sh1} = \frac{V_t}{100} ; I_{sh2} = \frac{V_t}{80}$$

$$I_{a1} = I_1 + I_{sh1} = I_1 + \frac{V_t}{100}$$

$$I_{a2} = I_2 + I_{sh2} = I_2 + \frac{V_t}{80} = 1500 - I_1 + \frac{V_t}{80}$$

For each machine,  $E = V + I_a R_a$

$$E_1 = V_t + I_{a1} R_a$$

$$E_2 = V_t + I_{a2} R_a$$

$$400 = V_t + (I_1 + \frac{V_t}{100}) \times 0.5$$

$$440 = V_t + (1500 - I_1 + \frac{V_t}{80}) \times 0.04$$

$$1.005V_t + 0.5I_1 = 400$$

$$1.0005V_t - 0.04I_1 = 380$$

Solving the equation we get

$$V_t = 381.163V, I_2 = 33.861A$$

Current supplied by generator 1 = 33.861A

Current supplied by generator 2 = 1466.139A

$$V_t = 381.163V$$

Output of generator 1 =  $I_1 \times V_1 = 12.9Kw$

Output of generator 2 =  $I_2 \times V_t = 558.84Kw$

-----&&&-----

21. A 12 pole DC generator has a simple wave wound armature containing 144 coils of 10 turns each. The resistance of each turn is  $0.011\Omega$ . Its flux per pole is  $0.05\text{wb}$  and it is running at a speed of 200rpm calculate the individual armature voltage and the effective armature resistance. Nov/Dec-2010.

**Given:**

$P=12, 144$  coils, wave i.e  $A=2, 10$  turns/coil

Resistance of each turn= $0.011\Omega, \phi=0.05\text{wb}$ .

**Formula used:**  $E_g = \frac{\phi PNZ}{60A}$

**Solution:**

$P=12, 144$  coils, wave i.e  $A=2, 10$  turns/coil

Resistance of each turn= $0.011\Omega, \phi=0.05\text{wb}$ .

Total turns= $144 \times 10=1440$

$Z=\text{total conductors}=1440 \times 2=2880$

$$E_g = \frac{\phi PNZ}{60A} = \frac{0.05 \times 12 \times 200 \times 2880}{60 \times 2} = 2880V$$

For  $A=2$  parallel plates, there are  $2880/2=1440$  conductors per parallel path

i.e 720 turns per parallel path

Resistance/parallel path= $720 \times 0.011=7.92\Omega$

Such two paths in parallel, hence  $R_a=7.92/2=3.96\Omega$

$$R_a=3.96\Omega$$

-----&&&-----

22. A 100Kw DC shunt generator driven by a belt from an engine runs at 750 rpm and is connected to 230v DC mains. When the belt breaks, it continues to run as a motor drawing 9 kw from the mains. At what speed would it run? Given: Armature resistance =  $0.018\Omega$  and field resistance =  $115\Omega$ [Nov/Dec-2013].

**Given:**  $P_{\text{out}}=100\text{Kw}=100 \times 10^3$  Watts

$N_1 = 750$  rpm,  $V=230V$

**Formula used:**  $I_{\text{sh}} = \frac{V}{R_{\text{sh}}}, I_a = I_L + I_{\text{sh}}, E_g = V + I_a R_a$

When DC machine runs as generator

$P_{\text{out}}=100\text{Kw}=100 \times 10^3$  Watts

$N_1 = 750$  rpm,  $V=230V$

Load current  $I_L = \frac{P_{\text{out}}}{V} = \frac{100 \times 1000}{230} = 434.78A$

$I_{\text{sh}} = \frac{V}{R_{\text{sh}}} = \frac{230}{115} = 2A$

$I_a = I_L + I_{\text{sh}}$   
 $= 434.78 + 2 = 436.78A$

$$E_g = V + I_a R_a$$

$$=230+(436.78 \times 0.018)$$

$$=237.86V$$

When DC machines runs as motor

$$P_{out}=9Kw=9 \times 10^3W$$

$$N_2 = ?$$

$$\text{Load current } I_L = \frac{P_{out}}{V} = \frac{9 \times 10^3}{230} = 39.13A$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{230}{115} = 2A$$

$$I_a = I_L - I_{sh} \\ = 39.13 - 2 = 37.13A$$

$$E_b = V - I_a R_a \\ = 230 + (436.78 \times 0.018) \\ = 229.33V$$

$N \propto E$  (For shunt motor)

$$\frac{N_2}{N_1} = \frac{E_b}{E_g} = \frac{N_2}{750} = \frac{229.33}{237.86} = 723.10 \text{ r.p.m}$$

$$N_2 = 723.10 \text{ r.p.m}$$

-----&&&-----

**23. A 400 Volts DC shunt motor has a no load speed of 1450 RPM, the line current being 9 amperes. At full loaded conditions, the line current is 75 A. If the shunt field resistance is 200 ohms and armature resistance is 0.5 ohm. Calculate the full load speed.(May-June 2015)**

**Given data:**

$$V=400 \text{ V} ; N=1450 \text{ Rpm} ; I_l=9 \text{ A} ; I_l=75 \text{ A (At full load)} ; R_{sh}=200 \text{ ohm} ; R_a=0.5 \text{ ohm}$$

**Solutions:**

$$I_a = I_L - I_{sh}$$

$$I_a = 7 \text{ A}$$

$$V - I_a R_a = K \cdot N_1$$

$$400 - 3.5 = K \cdot 1450$$

$$K = 0.27344$$

**At full Load:**

$$I_a = 75 - 2 = 73 \text{ A}$$

$$V - I_a R_a = K \cdot N_2$$

$$400 - (73 \times 0.5) = N_2 (0.27344)$$

$$N_2 = 1329 \text{ Rpm.}$$

-----

**24. A 230 V DC shunt motor on no-load runs at a speed of 1200 RPM and draws a current of 4.5 Amperes. The armature and shunt Field resistance are 0.3 ohm and 230 ohm respectively. Calculate the back EMF induced and speed, When loaded and drawing a current of 36 A.(May-June 2015)**

**Given data:**

$$V=230 \text{ V} ; N=1200 \text{ Rpm} ; I_l=4.5 \text{ A} ; R_{sh}=230 \text{ ohm} ; R_a=0.3 \text{ ohm}$$

**Find:**

i. **Back EMF**

ii. **Speed**

**Solutions:**

$$I_a = I_L - I_{Sh}$$

$$I_a = 3.5A$$

$$V - I_a R_a = K.N$$

$$230 - 1.05 = K.1200$$

$$K = 0.1907$$

**In Load:**

$$I_a = 36 A$$

$$V - I_a R_a = K.N$$

$$230 - 10.8 = 0.1907N$$

$$N = \frac{230 - 10.8}{0.1907}$$

$$N = 1149.4 \text{ Rpm.}$$

$$E_b = V - I_a R_a$$

$$E_b = 230 - (10.8)$$

$$E_b = 219.92 V.$$

**25. The no – load test of a 44.76 Kw, 220V, DC. shunt motor gave the following figure:**

**Input current = 13.25A, field current = 2.55A, Resistance of the armature at 75°C = 0.032Ω and brush drop = 2V. Estimate the full load current and efficiency. (May 2016)**

**No load condition:**

$$\text{No load input} = 220 \times 13.25 = 2915w$$

$$\text{Armature current} = 13.25 - 2.55 = 10.7A$$

$$\text{Armature cu loss} = 10.7 \times 0.032 = 3.6w$$

$$\text{Loss due to brush drop} = 2 \times 10.7 = 21.4w$$

$$\text{Variable loss} = 21.4 + 3.6 = 25w$$

$$\text{Constant loss } W_c = 2915 - 25 = 2890w$$

**Full load condition:**

If  $I_a$  is the full load armature current, the full load motor input current is  $(I_a + 2.55)A$ .

$$\text{Full load motor input power} = 220 (I_a + 2.55)w$$

This input must be equal to the sum of

(i) Output = 44.76Kw

(ii)  $W_c = 2890w$

(iii) Brush loss =  $2I_a$

(iv) Armature cu loss =  $0.032 I_a^2$

$$220 (I_a + 2.55) = 44.76 \times 10^3 + 2890 + 2I_a + 0.032I_a^2$$

$$0.032 I_a^2 - 218 I_a + 47.09 = 0$$

$$I_a = 223.5 \text{ A}$$

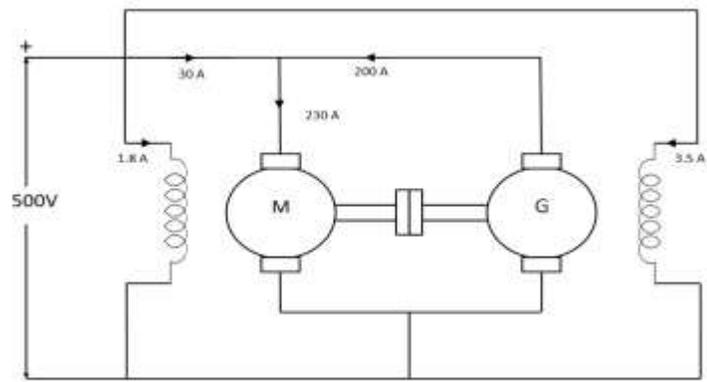
$$\text{Line current } I = I_a + I_{sh} = 223.5 + 2.55 = 226 \text{ A}$$

$$\text{Full load input power} = 226 \times 220 = 49720 \text{ W}$$

$$\text{Full load efficiency} = 44.760 \times 10^3 / 49720$$

$$= 90\%$$

**26. In a Hopkinson's test on a pair of 500V, 100kW shunt generator. The following data was obtained: Auxiliary supply 30A at 500V; Generator output current 200A; Field current 3.5A and 1.8 A;  $r_a = 0.075 \Omega$  for each machine; voltage drop at brushes = 2 V/machine; calculate the efficiency of the machine as a generator. Dec - 2016**



### Solution

$$\text{Motor armature copper loss} = I_a^2 R_a + \text{Power loss due to brush drop}$$

$$= (230)^2 \times 0.075 + 230 \times 2 = 4428 \text{ W}$$

$$\text{Motor field copper loss} = I_{sh}^2 R_{sh} = V I_{sh} = 500 \times 1.8 = 900 \text{ W}$$

$$\text{Generator armature copper loss} = (200)^2 \times 0.075 + 200 \times 2 = 3400 \text{ W}$$

$$\text{Generator field copper loss} = 500 \times 3.5 = 1750 \text{ W.}$$

$$\text{Total copper loss for 2 machines} = 4428 + 900 + 3400 + 1750 = 10478 \text{ W}$$

$$\text{Power drawn} = 500 \times 30 = 15000 \text{ W}$$

$$\text{Therefore stray loss for the two machines} = 15000 - 10478 = 4522 \text{ W}$$

$$\text{Stray loss / machine} = \frac{4522}{2} = 2261 \text{ W}$$

$$\text{Therefore total losses in generator} = 3400 + 1750 + 2261 = 7411 \text{ W}$$

$$\text{Generator output} = 500 \times 200 = 100000 \text{ W}$$

$$\text{Therefore } \eta_{\text{generator}} = \frac{100000}{100000 + 7411} = 93.09 \%$$

**27.A440VD.C.shunt motor takes 4A at no load. Its armature and field resistance are 0.4 ohms and 220ohms respectively .Estimate the KW output and efficiency when the motor takes 60A on full load(May-2018)**

**Given:**  $V=440V, I=4A, R_a = 0.4\&220, I$

**To find:** KW output

Formula used:  $I_{sh} = \frac{V}{R_{sh}}, \eta_{motor} = \frac{\text{output power}}{\text{Input power}}$

**Solution:**

$$\begin{aligned}\text{No load input power} &= VI_0 \\ &= 440 \times 4 = 1760W\end{aligned}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{440}{220} = 2A$$

$$\begin{aligned}\text{No load armature current } I_{a0} &= I_0 - I_{sh} \\ &= 4 - 2 = 2A\end{aligned}$$

$$\text{No load armature CU losses} = I_{a0}^2 R_a = 2^2 \times 0.4 = 1.6W$$

$$\begin{aligned}\text{Constant losses} &= \text{No load input power} - \text{No load armature CU losses} \\ &= 1760 - 1.6 = 1758.4W\end{aligned}$$

Efficiency when machine working as a motor

$$\text{Line current } I_l = 60A$$

$$\begin{aligned}\text{Armature current } I_a &= I_l - I_{sh} \\ &= 60 - 2 = 58A\end{aligned}$$

$$\text{Armature CU losses} = I_a^2 R_a = (58)^2 \times 0.4 = 1345.6W$$

$$\begin{aligned}\text{Total losses} &= \text{Armature CU losses} + \text{Constant loss} \\ &= 1345.6 + 1758.4 = 3104W\end{aligned}$$

$$\begin{aligned}\text{Input power} &= VI_L \\ &= 440 \times 60 = 26400W\end{aligned}$$

$$\begin{aligned}\text{output power} &= \text{Input power} - \text{losses} \\ &= 26400 - 3104 = 23296W\end{aligned}$$

$$\eta_{motor} = \frac{\text{output power}}{\text{Input power}} = \frac{23296}{26400} \times 100 = 88.24\%$$

Efficiency when machine working as a generator

$$\text{Input power} = VI_L = 26400W$$

$$\begin{aligned}\text{Armature current } I_a &= I_l + I_{sh} \\ &= 60 + 2 = 62A\end{aligned}$$

$$\text{Armature CU losses} = I_a^2 R_a = (62)^2 \times 0.4 = 1537.6W$$

$$\begin{aligned}\text{Total losses} &= \text{Armature CU losses} + \text{Constant loss} \\ &= 1537.6 + 1758.4 = 3296W\end{aligned}$$

$$\begin{aligned}\text{Input power} &= \text{output power} + \text{losses} \\ &= 26400 + 3296 \\ &= 29696\end{aligned}$$

$$\eta_{generator} = \frac{\text{output power}}{\text{Input power}} = \frac{26400}{29696} \times 100 = 88.90\%$$

**28.Determine developed torque and shaft torque of 220V, 4pole series motor with 800 conductors wave-connected supplying a load of 8.2 kW by taking 45A from the mains. The flux per pole is 25mwb and its armature circuit resistance is 0.6Ω(May-2018)**

**Given:**  $V=220v, Z=800, P=8.2KW, \Phi = 25mwb, R_a = 0.6$

Formula used:  $T_a = 0.159\Phi Z I_a \left(\frac{P}{A}\right), E_b = \frac{\Phi Z N}{60} \left(\frac{P}{A}\right), T_{sh} = \frac{\text{out put in Watts}}{\frac{2\pi N}{60}}$

Solution:

$$\begin{aligned}
 \text{Armature Torque } T_a &= 0.159 \Phi Z I_a \left( \frac{P}{A} \right) \\
 &= 0.159 \times 25 \times 10^{-3} \times 800 \times 45 \left( \frac{4}{2} \right) = 286.2 \text{ NM} \\
 E_b &= V - I_a R_a = 220 - 45 \times 0.6 = 193 \text{ V} \\
 E_b &= \frac{\Phi Z N}{60} \left( \frac{P}{A} \right) \\
 E_b &= \frac{25 \times 10^{-3} \times 800 \times N}{60} \left( \frac{4}{2} \right) \\
 11580 &= 40N \\
 N &= \frac{11580}{40} = 289.5 \text{ rpm} \\
 \therefore T_{sh} &= \frac{\text{out put in Watts}}{\frac{2\pi N}{60}} \\
 \frac{8.2 \times 10^{-3}}{\frac{2\pi \times 289.5}{60}} &= 270.618 \text{ NM}
 \end{aligned}$$

**PART – C – Question (May 2017)**

1. A shunt motor runs at 600 rpm from 250V supply and takes a line current of 50A. Its armature and field resistances are 0.4 and 125 respectively. Neglecting the effects of armature reaction and allowing 2V brush drop. Calculate (i) The no – load speed if the no-load line current is 5A (ii) The percentage reduction in flux per pole in order that the speed may be 800 rpm when the armature current is 40A.

**Given data:**  $N_2 = 600 \text{ rpm}$ ,  $V = 250 \text{ V}$ ,  $I_L = 50 \text{ A}$ ,  $R_a = 0.4 \Omega$   
 $R_{sh} = 125 \Omega$ , brush drop = 2V

**Formula used:**  $I_{Sh} = \frac{V}{R_{Sh}}$ ,  $I_{a0} = I_o - I_{sh}$ ,  $E_{b1} = V - I_{a0} R_a$ ,  $I_a = I_L - I_{sh}$

**Solution:**

Shunt field current  $I_{Sh} = \frac{V}{R_{Sh}} = 250/125 = 2 \text{ A}$

No – load armature current  $I_{a0} = I_o - I_{sh} = 5 - 2 = 3 \text{ A}$

Back emf at no-load  $E_{b1} = V - I_{a0} R_a = 250 - 3 \times 0.4 = 248.8 \text{ V}$

Armature current  $I_a = I_L - I_{sh} = 50 - 2 = 48 \text{ A}$

Back emf at full load  $E_{b2} = V - I_a R_a = 250 - 48 \times 0.4 = 230.8 \text{ V}$

Here, flux is constant, then speed equation becomes

$$\frac{N_1}{N_2} = \frac{E_{b1}}{E_{b2}}$$

$$\frac{N_1}{600} = \frac{248.8}{230.8}$$

$N_1 = 646.8 \text{ rpm}$

(ii) **Back emf at 40A,**

$$E_{b2} = V - I_{a2} R_a = 250 - 40 \times 0.4 = 234 \text{ V}$$

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}} \times \frac{\Phi_1}{\Phi_2}$$

$$\frac{800}{646.8} = \frac{234}{248.8} \times \frac{\phi_1}{\phi_2}$$

$$\phi_2 = 0.76 \phi_1$$

$$\text{Reduction in field flux} = \frac{\phi_1 - \phi_2}{\phi_1} = \frac{\phi_1 - 0.76\phi_1}{\phi_1} \times 100$$

$$= 23.96\%$$

**2.A 220V,22A,1000rpm DC shunt motor has armature circuit resistance of 0.1Ω and field resistance of 100Ω.calculate the value of additional resistance to be inserted in the armature circuit in order to reduce the speed to 800rpm.Assume the load torque to be (i)proportional to the speed and(ii)proportional to square of the speed.(May-2018)**

**Given:** V=220V, I=22A, N=1000, R<sub>a</sub>=0.1, R<sub>sh</sub>=100, N<sub>2</sub>=800

Formula used:  $I_{sh} = \frac{V}{R_{sh}}$ ,  $I_{a1} = I_L - I_{sh}$ ,  $E_{b1} = V - I_{a1}R_a$ ,  $\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1}$ ,  $E_{b2} = V - I_{a2}(R_a + R)$

Solution:

(i)

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{100} = 2.2A$$

$$I_{a1} = I_L - I_{sh} = 22 - 2.2 = 19.8A$$

$$\frac{I_{a2}}{I_{a1}} = \frac{N_2}{N_1}$$

$$I_{a2} = \frac{19.8 \times 800}{1000} = 15.84A$$

$$E_{b1} = V - I_{a1}R_a = 220 - 19.8 \times 0.1 = 218.02V$$

$$E_{b2} = V - I_{a2}(R_a + R) = 220 - 15.8 \times (0.1 + R) = 218.02V$$

$$= 220 - 1.58 - 15.8R$$

$$E_{b2} = 218.42 - 15.8R$$

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1} \Rightarrow \frac{218.42 - 15.8R}{218.02} = \frac{800}{1000}$$

$$R = 2.785\Omega$$

(ii)  $T \propto N^2$

$$I_{a2} = 0.64I_{a1} = 0.64 \times 19.8 = 12.672A$$

$$E_{b2} = V - I_{a2}(R_a + R) = 220 - 12.672 \times (0.1 + R)$$

$$= 220 - 1.2672 - 12.672R$$

$$\frac{E_{b2}}{E_{b1}} = \frac{N_2}{N_1} \Rightarrow \frac{218.732 - 12.672R}{218.02} = \frac{800}{1000}$$

$$R = 3.497\Omega$$