MAILAM ENGINEERING COLLEGE

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(Approved by AICTE, New Delhi, Affiliated to Anna University Chennai& Accredited by TCS)

Department of Electrical & Electronics Engineering

SUB CODE / NAME: EE6302/ ELECTROMAGNETIC THEORY

YEAR / SEC: II / A & B

SYLLABUS

UNIT I ELECTROSTATICS – I: Sources and effects of electromagnetic fields – Coordinate Systems – Vector fields –Gradient, Divergence, Curl – theorems and applications - Coulomb's Law – Electric field intensity – Field due to discrete and continuous charges – Gauss's law and applications

UNIT II ELECTROSTATICS - II: Electric potential – Electric field and equipotential plots, Uniform and Non-Uniform field, Utilization factor – Electric field in free space, conductors, dielectrics - Dielectric polarization - Dielectric strength - Electric field in multiple dielectrics – Boundary conditions, Poisson's and Laplace's equations, Capacitance, Energy density, Applications.

UNIT III MAGNETOSTATICS: Lorentz force, magnetic field intensity (H) – Biot–Savart's Law - Ampere's Circuit Law – H due to straight conductors, circular loop, infinite sheet of current, Magnetic flux density (B) – B in free space, conductor, magnetic materials – Magnetization, Magnetic field in multiple media – Boundary conditions, scalar and vector potential, Poisson's Equation, Magnetic force, Torque, Inductance, Energy density, Applications.

UNIT IV ELECTRODYNAMIC FIELDS: Magnetic Circuits - Faraday's law – Transformer and motional EMF – Displacement current - Maxwell's equations (differential and integral form) – Relation between field theory and circuit theory – Applications.

UNIT V ELECTROMAGNETIC WAVES: Electromagnetic wave generation and equations – Wave parameters; velocity, intrinsic impedance, propagation constant – Waves in free space, lossy and lossless dielectrics, conductors- skin depth - Poynting vector – Plane wave reflection and refraction – Standing Wave – Applications.

TEXT BOOKS:

1. Mathew N. O. Sadiku, 'Principles of Electromagnetics', 4 th Edition ,Oxford University Press Inc. First India edition, 2009.
2. Ashutosh Pramanik, 'Electromagnetism – Theory and Applications', PHI Learning Private Limited, New Delhi, Second Edition-2009.

REFERENCES:

1. Joseph. A.Edminister, 'Schaum's Outline of Electromagnetics, Third Edition (Schaum's Outline Series), Tata McGraw Hill, 2010
2. William H. Hayt and John A. Buck, 'Engineering Electromagnetics', Tata McGraw Hill 8th Revised edition, 2011.

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EE8391 ELECTROMAGNETIC THEORY

UNIT I ELECTROSTATICS – I

Sources and effects of electromagnetic fields – Coordinate Systems – Vector fields –Gradient, Divergence, Curl – theorems and applications - Coulomb's Law – Electric field intensity – Field due to discrete and continuous charges – Gauss's law and applications.

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Electric potential – Electric field and equipotential plots, Uniform and Non-Uniform field, Utilization factor – Electric field in free space, conductors, dielectrics - Dielectric polarization – Dielectric strength - Electric field in multiple dielectrics – Boundary conditions, Poisson's and Laplace's equations, Capacitance, Energy density, Applications.

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Lorentz force, magnetic field intensity (H) – Biot–Savart's Law - Ampere's Circuit Law – H due to straight conductors, circular loop, infinite sheet of current, Magnetic flux density (B) – B in free space, conductor, magnetic materials – Magnetization, Magnetic field in multiple media – Boundary conditions, scalar and vector potential, Poisson's Equation, Magnetic force, Torque, Inductance, Energy density, Applications.

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TEXT BOOKS:

1. Mathew N. O. Sadiku, 'Principles of Electromagnetics', 6th Edition, Oxford University Press Inc. Asian edition, 2015.
2. William H. Hayt and John A. Buck, 'Engineering Electromagnetics', McGraw Hill Special Indian edition, 2014.
3. Kraus and Fleish, 'Electromagnetics with Applications', McGraw Hill International Editions, Fifth Edition, 2010.

Part A - Q & A

1. **Write some of the sources of electric and magnetic fields. Or what are the practical applications of electromagnetic fields(Nov 2008, May 2013, and Dec 2015)**
   - Current carrying conductors
   - Mobile phones
   - Microwave oven
   - Computer and Television screens
   - High voltage Power lines

2. **Mention any two sources of electromagnetic fields.(May 2008)(April/May 2017)**

   Moving charges produce current and current carrying conductor produce a magnetic field. In such case, electric and magnetic fields are related to each other and are called electromagnetic fields.

3. **What are the different co-ordinate systems? (May 2010)**

   (i) Cartesian (or) rectangular Co-ordinate system.
   (ii) Cylindrical Co-ordinate system.
   (iii) Spherical Co-ordinate system.

4. **Define a unit vector and its value in Cartesianco-ordinate axis?(Apr 2008)**

   Unit vector is a vector having unit magnitude and directed along the co-ordinate axes.

   For Cartesian co-ordinates, $\vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$ co- ordinate where $\vec{a}_x, \vec{a}_y$ and $\vec{a}_z$ are unit vectors in the direction of x,y and z respectively.

5. **How the unit vectors are defined in cylindrical co-ordinate systems? (May 2008)**

   Unit vector is a vector having unit magnitude and directed along the co-ordinate axes.ForCylindrical co-ordinates, $\vec{P} = P_r\vec{a}_r + P_\phi\vec{a}_\phi + P_z\vec{a}_z$, where $\vec{a}_r, \vec{a}_\phi$ and a_z are unit vectors in the direction of radius(r), phase plane(φ) and height(z)respectively.

6. **State stokes theorem. (Apr 2004, 2007)(May/June 2014)(Nov/Dec 2016)(April/May 2017)**

   The line integral of a vector around a closed path is equal to the surface integral of the normal component of its curl over any surface bounded by the path.

   $$\oint \vec{H}.\vec{dl} = \iint (\nabla \times \vec{H}).\vec{dS}_n$$

7. **State divergence theorem.(Nov 2008, 2009)**

   Theintegral of the normal component of any vector field over a closed surface is equal to the integral of the divergence of that vector field over a volume v enclosed by the closed surface

   $$\oiint_s \vec{D}.\vec{dS}_n = \iiint \nabla.\vec{D}dv$$

8. Explain the termsirrotational and solenoidal as applied to vector $\vec{F}$. (Nov 2004)

If curl of vector is Zero $\vec{F}$ is irrotationalandif divergence of a vector is Zero, $\vec{F}$ is said to be solenoidal.

(i)$\nabla \times \vec{F} = 0$ means irrotational field and (ii) $\nabla . \vec{F} = 0$ means solenoidal field

9. Give practical example for diverging and curling fields. (May 2008)

The air velocity going out from a punctured tube and velocity going out from a hole in a gas balloon are examples of diverging fields.

The water velocity in a river and velocity of a rigid body rotating about a fixed axisareexamples of curling fields.

10. What is the physical significance of divergence of a vector field? (June 2006)

The divergence of the vector flux density $\vec{D}$ is the outflow from a small closed surface per unit volume as the volume shrinks to zero. Its significance is that,it tells us about the diverging nature of the field and the rate of change of field strength in the direction of the field.

$$\nabla . \vec{D} = \frac{Lt}{\Delta v \to 0} \oint \frac{\vec{D}.\vec{ds}}{\Delta v}$$

11. What are the conditions for two vectors are parallel and perpendicular.(May 2005)

- If $\vec{A} \times \vec{B}$ =0 then the two vectors A and B are said to be parallel.
- If $\vec{A} . \vec{B}$ =0 then the two vectors A and B are said to be perpendicular.

12. Define Gradient of a scalar. (May 2004) (Nov/Dec 2017)

The gradient of any scalar function is the maximum space rate of change of the function. If the scalar V represents electrical potential and ∇V represent potential gradient, then

$$\nabla V = \frac{dV}{dx} \vec{a}_x + \frac{dV}{dy} \vec{a}_y + \frac{dV}{dz} \vec{a}_z$$

This operation is called gradient.∇V= grad V. Gradient of a scalar is always a vector.

13. Define divergence of a vector.(May 2006)

The divergence of a vector 'A' at any point is defined as the limit of its surface integral per unit volume as the volume enclosed by the surface shrinks to zero.

$$\nabla . \vec{A} = \frac{Lt}{\Delta v \to 0} \oint \frac{\vec{A}.\vec{ds}}{\Delta v}$$

It can be expressed as $\nabla . \vec{A} = \frac{\partial Ax}{\partial x} + \frac{\partial Ay}{\partial y} + \frac{\partial Az}{\partial z}$. Divergence of a vector is a scalar quantity.

14. Define Curl of a vector. What is its physical significance? (May 2013)

The curl of a vector 'A' at any point is defined as limit of its surface integral of its cross product with normal over a closed surface, per unit volume as the volume shrinks to zero. Its significance is that, it tells us about the curling nature of a field and rate of change of field strength in the direction at right angles to the field.

$$\text{CurlA} = \nabla \times \vec{A} = \lim_{v \to 0} \frac{1}{v} \oiint \vec{n} \times \vec{A}.dS$$

It is given as

$$\nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A_x & A_y & A_z \end{vmatrix}$$

15. Write down the formula for conversion of a point from Cartesian to cylindrical system.

The Cartesian Co-ordinate (x, y, z) canbe converted into cylindrical Co-ordinates (ρ ,ϕ,z) as,

Given	Transform
x	$\rho = \sqrt{x^2 + y^2}$
y	ϕ= $\tan^{-1}$ (y/x)
z	z= z

16. Write down the formula for conversion of a point from Cylindrical to Cartesian system.

The cylindrical Co-ordinates (ρ , ϕ, z) can be converted into Cartesian Co-ordinates (x,y,z) as,

Given	Transform
ρ	x= ρ cosθ
ϕ	y= ρ sinϕ
z	z=Z

17. Write down the formula for conversion of a point from Cartesian to Spherical system.

The Cartesian Co-ordinates (x,y,z) can be transformed into spherical Co-ordinates (r,θ,ϕ) as,

Given	Transform
X	r= $\sqrt{x^2 + y^2 + z^2}$
y	$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right) = \cos^{-1}\left(\frac{z}{r}\right)$
z	ϕ= $\tan^{-1}$ (y/x)

18. Write down the formula for conversion of a point from Spherical to Cartesian Co-ordinate system.

The spherical Co-ordinates (r,θ,ϕ) can be transformed into Cartesian Co-ordinates (x, y,z) as,

Given	Transform
r	x=r sinθcosϕ
θ	y= r sinθsinϕ
ϕ	z=r cosθ

19. Give the Cartesian co-ordinate of a point whose cylindrical co-coordinates are ρ=1, ϕ=45°,z=2.

Given: ρ=1, ϕ = 45°, z = 2.The Cartesian co-ordinates are,

$$X=\rho\cos\phi=1\cos 45=0.707$$

$$Y=\rho\sin\phi=1\sin 45=0.707$$

$$Z=z$$

Ans: Cartesian co-ordinates are (0.707, 0.707, 2)

20. Express the divergence of a vector in the three system of orthogonal co-ordinates.

For rectangular co-ordinate system

$$\nabla.\vec{B}=\frac{\partial Bx}{\partial x}+\frac{\partial By}{\partial y}+\frac{\partial Bz}{\partial z}$$

For cylindrical co- ordinate system

$$\nabla.\vec{B}=\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho B\rho)+\frac{1}{\rho}\frac{\partial B\phi}{\partial\phi}+\frac{\partial Bz}{\partial z}$$

For spherical co-ordinate system

$$\nabla.\vec{B}=\frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}(r^2\sin\theta B_r)+\frac{\partial}{\partial\theta}(r\sin\theta B_\theta)+\frac{\partial}{\partial\phi}(rB_\phi)\right]$$

21. Write the expression for differential volume element in terms of co-ordinate systems (Dec 2015)

For rectangular co-ordinates **dv = dxdydz**

For cylindrical co-ordinates **dv = ρdρdθdz**

For spherical co-ordinates **dv = r^2 sinθdr dθdϕ**

22. Write down the expression for differential length in all three co-ordinate systems.

For rectangular co-ordinate dl = $\sqrt{(dx)^2+(dy)^2+(dz)^2}$

For cylindrical co-ordinates dl= $\sqrt{(d\rho)^2+(\rho d\phi)^2+(dz)^2}$

For spherical co-ordinates dl= $\sqrt{(dr)^2+(rd\theta)^2+(r\sin\theta d\phi)^2}$

23. Write down the dot products of unit vectors in co-ordinate systems.

For Cartesian co-ordinate system $\vec{a}_x.\vec{a}_y = \vec{a}_y.\vec{a}_z = \vec{a}_z.\vec{a}_x = 0$

For Cylindrical co-ordinatesystem $\vec{a}_\rho.\vec{a}_\phi = \vec{a}_\phi.\vec{a}_z = \vec{a}_z.\vec{a}_\rho = 0$

For Spherical co-ordinatesystem $\vec{a}_r.\vec{a}_\theta = \vec{a}_\theta.\vec{a}_\phi = \vec{a}_\phi.\vec{a}_r = 0$

24. Show that the vector $\vec{H} = 3y^4z^2\vec{a}_x + 4x^3z^2\vec{a}_y + 3x^2y^2\vec{a}_z$ is solenoidal.

$$\nabla.\vec{H} = (\vec{a}\,x\frac{\partial}{\partial x} + \vec{a}\,y\frac{\partial}{\partial y} + \vec{a}\,z\frac{\partial}{\partial z}).(3y^4z^2\vec{a}_x + 4x^3z^2\vec{a}_y + 3x^2y^2\vec{a}_z)$$

$$= \frac{\partial}{\partial x}(3y4z2) + \frac{\partial}{\partial y}(4x3z2) + \frac{\partial}{\partial z}(3x2y2)$$

$$= 0 + 0 + 0 = 0$$

Thus the given vector is solenoidal.

25. Show that the vector $2y\bar{a}_x + (x^2 + 2yz)\bar{a}_y + (y^2 + 1)\bar{a}_z$ is irrotational.

Let $\bar{F} = 2yx\vec{a}_x + (x^2 + 2yz)\vec{a}_y + (y^2 + 1)\vec{a}_z$

$$\nabla\times\bar{F} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy & x^2+2yz & y^2+1 \end{vmatrix}$$

$$= \vec{a}_x\left[\partial/\partial y(y^2+1) - \partial/\partial z(x^2+2yz)\right] - \vec{a}_y\left[\partial/\partial x(y^2+1) - \partial/\partial z(2xy)\right] + \vec{a}_z\left[\partial/\partial x(x^2+2yz) - \partial/\partial y(2xy)\right] = 0$$

Hence $\vec{F}$ is irrotational.

26. Prove that ∇×∇V =0

We know that, $\nabla V = \frac{dV}{dx}\vec{a}_x + \frac{dV}{dy}\vec{a}_y + \frac{dV}{dz}\vec{a}_z$ where V is a scalar.

Taking curl

$$\nabla\times\nabla V = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \partial V/\partial x & \partial V/\partial y & \partial V/\partial z \end{vmatrix}$$

$$= \vec{a}_x\left(\frac{\partial^2 V}{\partial y\partial z} - \frac{\partial^2 V}{\partial y\partial z}\right) - \vec{a}_y\left(\frac{\partial^2 V}{\partial x\partial z} - \frac{\partial^2 V}{\partial x\partial z}\right) + \vec{a}_z\left(\frac{\partial^2 V}{\partial x\partial y} - \frac{\partial^2 V}{\partial x\partial y}\right) = \mathbf{0}$$

27. Prove that$\nabla.\nabla\times \vec{H}$ =0

Let the vector $\vec{H} = H_x\vec{a}_x + H_y\vec{a}_y + H_z\vec{a}_z$

$$\nabla\times \vec{H} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ H_x & H_y & H_z \end{vmatrix}$$

$$= \vec{a}_x\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - \vec{a}_y\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \vec{a}_z\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

Taking divergence to $\nabla\times \vec{H}$ **,** we get

$$\nabla.\nabla\times \vec{H} = \frac{\partial}{\partial x}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \frac{\partial}{\partial z}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)$$

$$= \frac{\partial^2 H_z}{\partial x\partial y} - \frac{\partial^2 H_y}{\partial x\partial z} - \frac{\partial^2 H_z}{\partial y\partial x} + \frac{\partial^2 H_x}{\partial y\partial z} + \frac{\partial^2 H_y}{\partial z\partial x} - \frac{\partial^2 H_x}{\partial z\partial y} = \mathbf{0}$$

28. State coulomb's law and write the mathematical expression. (April 2010)(Nov/Dec 2017)

Coulomb's law states that, the force between any two point charges Q_1 and Q_2, acts along the line joining the two point charges and the magnitude is.

- Directly proportional to the product (Q_1, Q_2) of the two charges and
- Inversely proportional to the square of the distance between them.

$$\vec{F}_{12} = k\frac{Q_1Q_2}{R}\vec{a}_{12}$$

Where R-distance between Q_1 and Q_2 ; $K = \frac{1}{4\pi\varepsilon_0}$ constant of proportionality

29. State the electric field intensity due to infinite line charge. (Nov 2007)

The electric field intensity due to infinite line charge is given as

$$\vec{E}_p = \frac{\rho_l}{2\pi\varepsilon_0 r}\vec{a}_r \text{ V/m}$$

Where r- perpendicular distance of point p from the line charge

$\vec{a}_r$ - Unit vector in the direction of the $\perp_r$ distance of point P from the line charge.

30. Define Electric filed intensity. (May 2010,2016)

The electric field (or) Electric field intensity is defined as the electric force per unit charge.

$$E = \frac{F}{q} \text{ N/C, } E = \frac{Q}{4\pi\varepsilon r^2} \text{ V/m}$$

31. State Gauss's law. Give some applications of Gauss's law. (May 2016)

Gauss's law state that, the total flux passing through a closed surface is the total charge enclosed by that surface.

$$\psi = \oiint_s \vec{D}_s \vec{d}_s = \mathbf{Q}$$

Gauss's law is used to find onto the electric field intensity or electric flux density only for symmetrical charge distribution.

32. Obtain in the cylindrical co-ordinate system the gradient of the function $f(r, \emptyset, z) = 5r^4z^3sin\emptyset + cos\emptyset + z^2$ (May/June 2014)

In cylindrical co-ordinates, gradient is

$$\nabla f = \frac{\partial f}{\partial r}\overrightarrow{a_r} + \frac{1}{r}\frac{\partial f}{\partial \emptyset}\overrightarrow{a_\emptyset} + \frac{\partial f}{\partial z}\overrightarrow{a_z}$$

$$\frac{\partial f}{\partial r} = 20r^3z^3sin\emptyset, \qquad \frac{1}{r}\frac{\partial f}{\partial \emptyset} = \frac{1}{r}[5r^4z^3cos\emptyset - sin\emptyset], \qquad \frac{\partial f}{\partial z} = 15r^4z^2sin\emptyset + 2z$$

$$\mathbf{\nabla f = 20\, r^3z^3sin\emptyset\overrightarrow{a_r} + \left[5r^3z^3cos\emptyset - \frac{sin\emptyset}{r}\right]\overrightarrow{a_\emptyset} + [15r^4z^2sin\emptyset + 2z]\overrightarrow{a_z}}$$

33. Points P and Q are located at (0,2,4) and (-3,1,5). Calculate the distance vector from P to Q. (Nov/Dec 2014)

Distance vector

$$\overrightarrow{PQ} = \vec{r}_Q - \vec{r}_P$$
$$= \left(\overrightarrow{-3a_x} + \overrightarrow{1a_y} + \overrightarrow{5a_z}\right) - \left(\overrightarrow{0a_x} + \overrightarrow{2a_y} + \overrightarrow{4a_z}\right)$$
$$= \mathbf{\overrightarrow{-3a_x} - \overrightarrow{1a_y} + \overrightarrow{1a_z}}$$

34. Determine the electric flux density at a distance of 20cm due to an infinite sheet of uniform charge 20μC/m^2 lying on the z = 0 plane.(Nov/Dec 2014,15)

$$\vec{E} = \frac{\rho_s}{2\epsilon_0}\vec{a}_N = \frac{20 \times 10^{-6}}{2 \times 8.854 \times 10^{-12}}\vec{a}_z = 1.129 \times 10^6\, V/m$$

35. Given $\vec{A} = 4\vec{a}_x + 6\vec{a}_y - 2\vec{a}_z$ and $\vec{B} = -2\vec{a}_x + 4\vec{a}_y + 8\vec{a}_z$.Show that the vectors are orthogonal. (Apr/May 2015)

Answer:

Orthogonal means perpendicular. If $\theta = 90°$ then the vectors are orthogonal.

$$\vec{A}.\vec{B} = |\vec{A}||\vec{B}|cos\theta$$

$$|\vec{A}| = \sqrt{4^2 + 6^2 + 2^2} = \sqrt{56} \quad and \quad |\vec{B}| = \sqrt{2^2 + 4^2 + 8^2} = \sqrt{84}$$

$$cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{(4\vec{a}_x + 6\vec{a}_y - 2\vec{a}_z).(-2\vec{a}_x + 4\vec{a}_y + 8\vec{a}_z)}{\sqrt{56 \times 84}} = 0$$

$$\theta = cos^{-1}(0) = 90°$$

36. Determine the angle between A= $2\vec{a}_x + 4\vec{a}_y$ and B= $6\vec{a}_y - 4\vec{a}_z$(Nov/Dec 2016)

$$cos\theta = \frac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|} = \frac{(2\vec{a}_x + 4\vec{a}_y).(6\vec{a}_y - 4\vec{a}_z)}{\sqrt{20 \times 52}} = 0.744$$

$$\theta = 41.9°$$

37. Express in matrix form the unit vector transformation from the rectangular to cylindrical co-ordinate system.(Apr/May 2015)

Unit vector transformation from the rectangular to cylindrical co-ordinate system is given as below:

$$\begin{bmatrix} a_\rho \\ a_\emptyset \\ a_z \end{bmatrix} = \begin{vmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} & 0 \\ \frac{-y}{\sqrt{x^2+y^2}} & \frac{x}{\sqrt{x^2+y^2}} & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} a_x \\ a_y \\ a_z \end{vmatrix}$$

38. Find the unit vector extending from the origin towards the point P(3,-1,-2) (Apr/May 2018)

Unit vector $a_{OP} = \frac{\overrightarrow{OP}}{|OP|}$

$$|OP| = \sqrt{(3-0)^2 + (-1-0)^2 + (-2-0)^2} = \sqrt{14}$$

$$\overrightarrow{OP} = (3\vec{a}_x - 1\vec{a}_y - 2\vec{a}_z)$$

$$a_{OP} = \frac{\overrightarrow{OP}}{|OP|} = \frac{(3\vec{a}_x - 1\vec{a}_y - 2\vec{a}_z)}{\sqrt{14}}$$

39. Determine the electric field intensity in free space if $\vec{D} = 30\overrightarrow{a_x} C/m^2$(Apr/May 2018)

$$\vec{D} = \varepsilon_0 \vec{E}$$

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0} = \frac{30\overrightarrow{a_x}}{4\pi \times 10^{-7}}$$

Coordinate Systems

1. Write briefly about Cartesian Co-ordinate System?

This is also called rectangular co-ordinate system. This system has three co-ordinate axes represented as x, y and z which are mutually at right angles to each other. These three axes intersect at a common point called origin of the system.

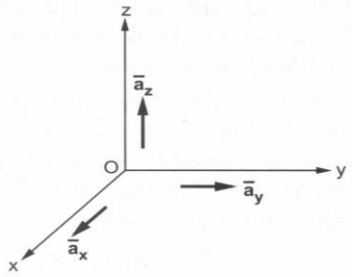


Fig. 1.6.4 Unit vectors in cartesian system

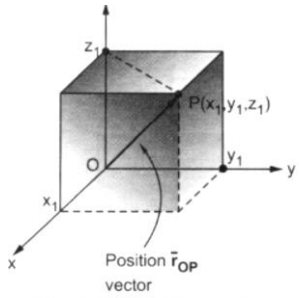


Fig. 1.6.5 Position vector

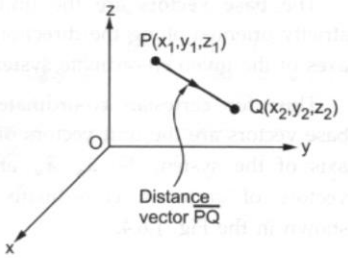


Fig. 1.6.6 Distance vector

$$\bar{r}_{OP} = x_1 \bar{a}_x + y_1 \bar{a}_y + z_1 \bar{a}_z$$

$$|\bar{r}_{OP}| = \sqrt{(x_1)^2 + (y_1)^2 + (z_1)^2}$$

$$\bar{P} = x_1 \bar{a}_x + y_1 \bar{a}_y + z_1 \bar{a}_z$$

$$\bar{Q} = x_2 \bar{a}_x + y_2 \bar{a}_y + z_2 \bar{a}_z$$

Then the distance or the displacement from P to Q is represented by a distance vector PQ and is given by

$$\overline{PQ} = \bar{Q} - \bar{P} = [x_2 - x_1]\bar{a}_x + [y_2 - y_1]\bar{a}_y + [z_2 - z_1]\bar{a}_z$$

This is also called separation vector.
The magnitude of this vector is given by,

$$|\overline{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\bar{a}_{PQ} = \text{Unit vector along PQ} = \frac{\overline{PQ}}{|\overline{PQ}|}$$

Differential Elements in Cartesian Co-ordinate System

$$\overline{dl} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$$

$$|\overline{dl}| = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$$

$$dv = dx\,dy\,dz$$

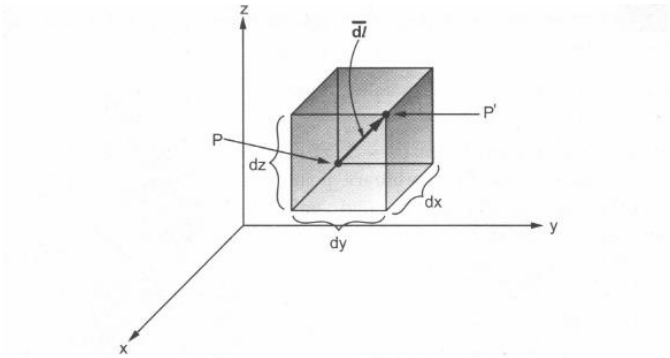


Fig. 1.6.7 Differential elements and different length in cartesian system

The differential surface element $\overline{dS}$ is represented as,

$$\boxed{d\bar{S} = dS\,\bar{a}_n}$$

where dS = Differential surface area of the element

$\bar{a}_n$ = Unit vector normal to the surface dS

$d\bar{S}_x$ = Differential vector surface area normal to x direction

= $dydz\,\bar{a}_x$

$d\bar{S}_y$ = Differential vector surface area normal to y direction

= $dxdz\,\bar{a}_y$

$d\bar{S}_z$ = Differential vector surface area normal to z direction

= $dxdy\,\bar{a}_z$

2. Write briefly about Cylindrical Co-ordinate System?

The circular cylindrical co-ordinate system is the three dimensional version of polar co-ordinate system. The surfaces used to define the cylindrical co-ordinate system are,

- Plane of constant z which is parallel to xy plane.
- A cylinder of radius r with z axis as the axis of the cylinder.
- A half plane perpendicular to xy plane and at an angle ∅with respect to xz plane.

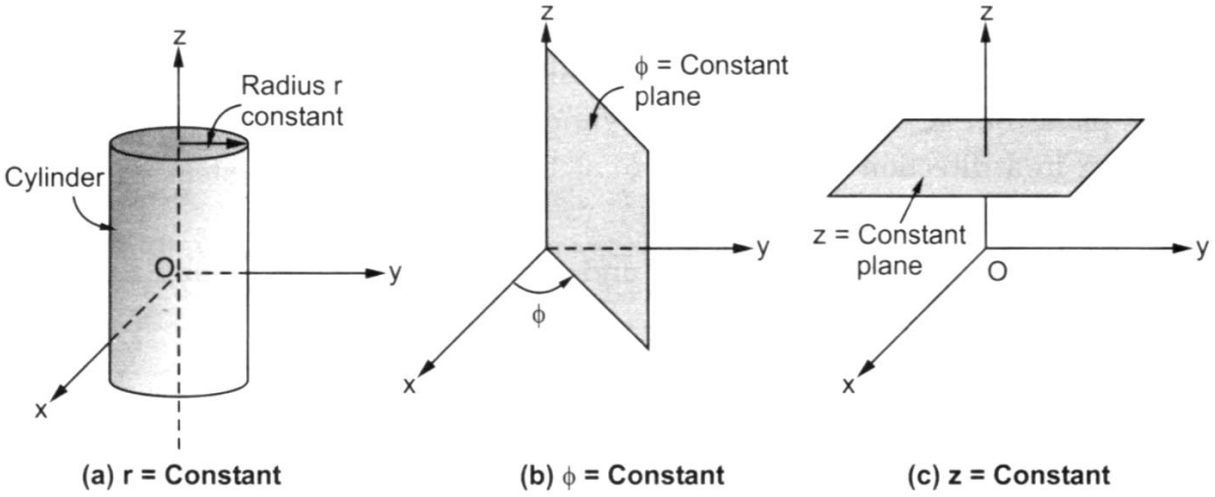
The angle ∅is called azimuthal angle.
The ranges of the variables are,

$$0 \le r \le \infty$$

$$0 \le \phi \le 2\pi$$

$$-\infty < z \le \infty$$

The point P can be defined as the intersection of three surfaces in cylindrical co-ordinate system. These three surfaces are,



(a) r = Constant (b) ϕ = Constant (c) z = Constant

$$\bar{P} = P_r \bar{a}_r + P_\phi \bar{a}_\phi + P_z \bar{a}_z$$

Hence the differential vector length in cylindrical co-ordinate system is given by,

$$\overline{dl} = dr\ \bar{a}_r + r\ d\phi\ \bar{a}_\phi + dz\ \bar{a}_z$$

The magnitude of the differential length vector is given by,

$$|\overline{dl}| = \sqrt{(dr)^2 + (r\,d\phi)^2 + (dz)^2}$$

Hence the differential volume of the differential element formed is given by,

$$dv = r\,dr\,d\phi\,dz$$

The differential surface areas in the three directions are

$d\bar{S}_r$ = Differential vector surface area normal to r direction

$= r\,d\phi\,dz\,\bar{a}_r$

$d\bar{S}_\phi$ = Differential vector surface area normal to ϕ direction

$= dr\,dz\,\bar{a}_\phi$

$d\bar{S}_z$ = Differential vector surface area normal to z direction

$= r\,dr\,d\phi\,\bar{a}_z$

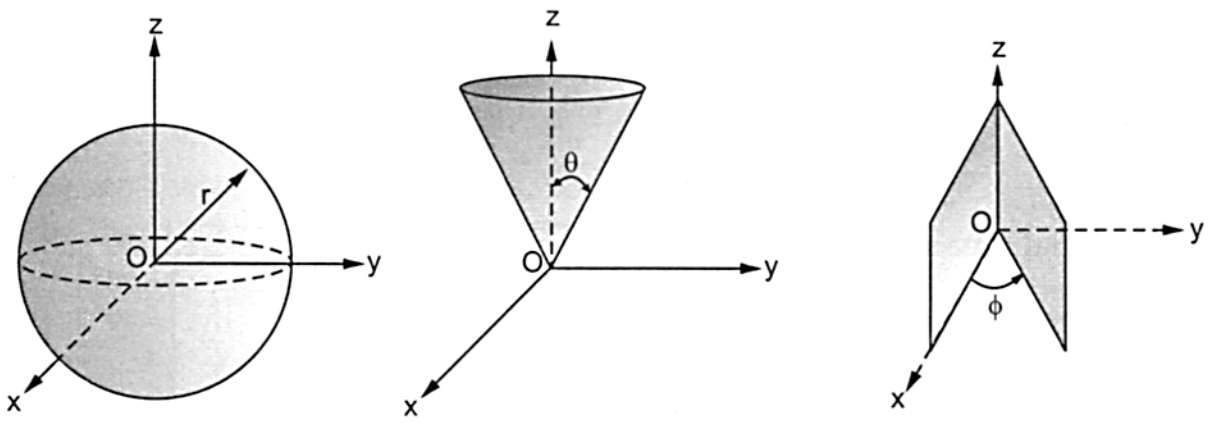
3. With neat diagram explain the spherical system with coordinates $(r, \theta, \emptyset)$ (Apr/May 2018)

Spherical Co-ordinate System

The surfaces which are used to define the spherical co-ordinate system on the three cartesian axes are,

- Sphere of radius r, origin as the centre of the sphere.
- A right circular cone with its apex at the origin and its axis as z axis. Its half angle is θ. It rotates about z axis and e varies from 0 to 180°.
- A half plane perpendicular to xy plane containing z axis, making an angle ϕ with the xz plane.

Thus the three co-ordinates of a point P in the spherical co-ordinate system are (r, θ,ϕ). These surfaces are shown in the Fig.



(a) Sphere of radius r with centre as origin

(b) Right circular cone with apex at origin

(c) Half plane perpendicular to xy plane

The ranges of the variables are,

$$0 \le r < \infty \quad \ldots (1.8.1)$$

$$0 \le \phi \le 2\pi \quad \ldots (1.8.2)$$

$$0 \le \theta \le \pi \text{ as half angle} \quad \ldots (1.8.3)$$

$r = Constant\ which\ is\ a\ sphere\ with\ centre\ as\ origin.$

$\theta = Constant\ which\ is\ right\ circular\ cone\ with\ apex\ as\ origin\ and\ axis\ as\ z-axis$

$\phi = Constant\ is\ a\ plane\ perpendicular\ to\ xyplane$

$$\bar{P} = P_r \bar{a}_r + P_\theta \bar{a}_\theta + P_\phi \bar{a}_\phi$$

Differential Elements in Spherical Co-ordinate System

dr = Differential length in r direction ... (1.8.5)

$r\,d\theta$ = Differential length in θ direction ... (1.8.6)

$r\sin\theta\,d\phi$ = Differential length in ϕ direction ... (1.8.7)

Hence the **differential vector length** in spherical co-ordinate system is given by,

$$\boxed{\bar{dl} = dr\,\bar{a}_r + r\,d\theta\,\bar{a}_\theta + r\sin\theta\,d\phi\,\bar{a}_\phi} \quad \text{... (1.8.8)}$$

the magnitude of the differential length vector is given by,

$$\boxed{\left|\bar{dl}\right| = \sqrt{(dr)^2 + (r\,d\theta)^2 + (r\sin\theta\,d\phi)^2}} \quad \text{... (1.8.9)}$$

Hence the **differential volume** of the differential element formed, in spherical co-ordinate system is given by,

$$\boxed{dv = r^2\sin\theta\,dr\,d\theta\,d\phi} \quad \text{... (1.8.10)}$$

The **differential surface areas** in the three directions are shown in the Fig. 1.8.7.

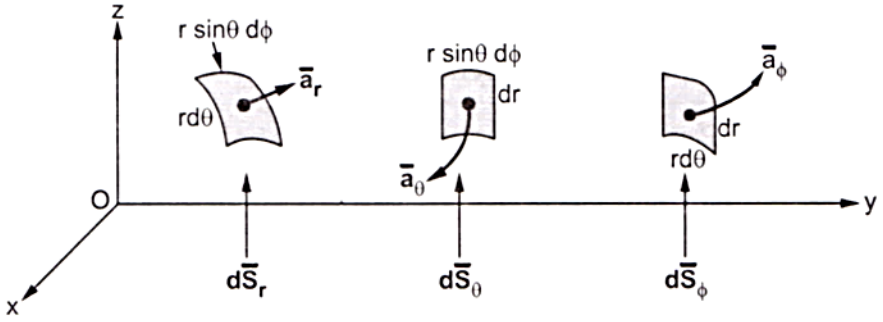


Fig. 1.8.7 Differential surface elements in spherical co-ordinate system

The vector representation of these differential surface areas are given by,

$d\bar{S}_r$ = Differential vector surface area normal to r direction

$= r^2\sin\theta\,d\theta\,d\phi$ ... (1.8.11)

$d\bar{S}_\theta$ = Differential vector surface area normal to θ direction

$= r\sin\theta\,dr\,d\phi$ ... (1.8.12)

$d\bar{S}_\phi$ = Differential vector surface area normal to ϕ direction

$= r\,dr\,d\theta$ ... (1.8.13)

4. Vector field is given by the expression

(a) $\vec{F} = \frac{1}{r}\vec{U}_r$ **(in cylindrical form)**

(b) $\vec{F} = \frac{1}{R}\vec{U}_R$ **(in spherical co-ordinate).**

Determine F in each ease in Cartesian form at a point (1, 1, 1). (Nov 2012, Apr 2010)

Solution:

(a) $\vec{F} = \frac{1}{r}\vec{U}_r$ (in cylindrical form)

$F = F_rU_r + F_\phi U_\phi + F_zU_z$

Where $F_\phi = F_z = 0$

$\therefore \quad F_r = \frac{1}{r}$

At point (x=1, y=1,z=1)

$$r = \sqrt{(1^2 + 1^2)} = \sqrt{2}$$

$$\Rightarrow F = \frac{1}{\sqrt{2}}$$

We know that $F_x = F_r \frac{x}{\sqrt{x^2 + y^2}} = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

$F_y = F_r \frac{y}{\sqrt{x^2 + y^2}} = \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{2}$

Hence field at (1,1,1)

$\vec{F} = \frac{1}{2}(\vec{U}_x + \vec{U}_y)$

(b) $\vec{F} = \frac{1}{R}\vec{U}_R$ (in spherical co-ordinate)

$F = F_RU_R + F_\theta U_\theta + F_\phi U_\phi$

Where $F_R = \frac{1}{R}$

At point x=1,y =1 and z=1

$R = \sqrt{(1+1+1)} = \sqrt{3} \therefore F_R = \frac{1}{\sqrt{3}}$

$F_x = F_R\left(\frac{x}{R}\right)$ (as $F_\phi = F_\theta = 0$)

$= \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$

$F_y = F_r\left(\frac{y}{R}\right) = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$

$F_z = F_r\left(\frac{z}{R}\right) = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$

Hence at (1,1,1), $\vec{F} = \frac{1}{3}(\vec{U}_x + \vec{U}_y + \vec{U}_z)$

5. **Given point P (-2, 6, 3) and vector** $\vec{A} = y\vec{a}_x + (x+z)\vec{a}_y$**. Evaluate** $\vec{A}$ **at P in the Cartesian, cylindrical and spherical systems.(May 2013, Nov 2012, Apr 2011, Nov 2009)**

(ii) If $\vec{B} = y\vec{a}_x + (x+z)\vec{a}_y$ **and a point Qis located at (-2, 6, 3), express**

- **The point Q in cylindrical and spherical coordinates,**
- $\vec{B}$**in spherical coordinates. (Nov/Dec 2014)**

Solution:

At point P, x=-2, y=6, z = 3

Hence $\rho = \sqrt{x^2 + y^2} = \sqrt{4+36} = 6.32$

$\phi = \tan^{-1}(y/x) = \tan^{-1}(6/-2) = 108.43°$

z =3

$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{4+36+9} = 7$

$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} = \tan^{-1}\frac{\sqrt{40}}{3} = 64.62°$

Thus P (-2,6,3)=P(6.32,108.43°,3)= P (7,64.62°,108.43°)

In the Cartesian system $\vec{A}$ **at P is** $\vec{A} = 6\vec{a}_x + \vec{a}_y$

For vector A, $A_x = y$, $A_y = x+z$, $A_z = 0$

Hence in cylindrical system

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y \\ x+z \\ 0 \end{bmatrix}$$

$\Rightarrow A_\rho = y\cos\phi + (x+z)\sin\phi$

$A_\phi = -y\sin\phi + (x+z)\cos\phi$

$A_z = 0$

Put $x = \rho\cos\phi$, $y = \rho\sin\phi$

$A = (A_\rho, A_\phi, A_z) = [\rho\cos\phi\sin\phi + (\rho\cos\phi + z)\sin\phi]\vec{a}_\rho + [-\rho\sin^2\phi + (\rho\cos\phi + z)\cos\phi]\vec{a}_\phi$

$\therefore$ At point P, substituting $\rho = 6.32$ and $\phi = 108.43°$, we get, $\vec{A} = -0.9487\vec{a}_\rho - 6.008\vec{a}_\phi$

Similarly in spherical co-ordinate system

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} y \\ x+z \\ 0 \end{bmatrix}$$

$\Rightarrow A_r = y\sin\theta\cos\phi + (x+z)\sin\theta\sin\phi$

$A_\theta = y\cos\theta\cos\phi + (x+z)\cos\theta\sin\phi$

$A_\phi = -y\sin\phi + (x+z)\cos\phi$

But $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$

Substituting these yields,

$A = (A_r, A_\theta, A_\phi)$

$= r[\sin^2\theta\cos\phi\sin\phi + (\sin\theta\cos\phi + \cos\theta)\sin\theta\sin\phi]\vec{a}_r + r[\sin\theta\cos\theta\sin\phi\cos\phi + (\sin\theta\cos\phi + \cos\theta)\cos\theta\sin\phi]\vec{a}_\theta + r[-\sin\theta\sin2\phi + (\sin\theta\cos\phi + \cos\theta)\cos\phi]\vec{a}_\phi$

$\therefore$ At point P, substituting $r = 7$, $\theta = 64.62$ and $\phi = 108.43°$, we get

$$\vec{A} = -0.8571\vec{a}_r - 0.4066\vec{a}_\theta - 6.008\vec{a}_\phi$$

$|A(x, y, z)| = |A(\rho, \theta, z)| = |A(r, \theta, \phi)| = 6.083$

---

6. Express the vector $\vec{B} = \frac{10}{r}\vec{a}_r + r\cos\theta\vec{a}_\theta + \vec{a}_\phi$ in Cartesian and Cylindrical co-ordinates. Find $\vec{B}$ at (-3,4,0) and $\vec{B}$ at $(5, \frac{\pi}{2}, -2)$.

Solution :

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} \frac{10}{r} \\ r\cos\theta \\ 1 \end{bmatrix}$$

$B_x = \frac{10}{r}\sin\theta\cos\phi + r\cos^2\theta\cos\phi - \sin\phi$

$B_y = \frac{10}{r}\sin\theta\sin\phi + r\cos^2\theta\sin\phi + \cos\phi$

$B_z = \frac{10}{r}\cos\theta - r\cos\theta\sin\theta$

But $r = \sqrt{x^2 + y^2 + z^2}$

$\theta = \tan^{-1}\frac{\sqrt{x^2 + y^2}}{z}$

$\phi = \tan^{-1} y/x$

Hence, $\sin\theta = \frac{\rho}{r} = \frac{\sqrt{x^2 + y^2}}{\sqrt{x^2 + y^2 + z^2}}$, $\cos\theta = \frac{z}{r} = \frac{z}{\sqrt{x^2 + y^2 + z^2}}$

$\sin\phi = \frac{y}{\rho} = \frac{y}{\sqrt{x^2+y^2}}$,$\cos\phi = \frac{x}{\rho} = \frac{x}{\sqrt{x^2+y^2}}$

Substituting these, we get

$$B_x = \frac{10\sqrt{x^2+y^2}}{(x^2+y^2+z^2)} \frac{x}{\sqrt{x^2+y^2}} + \frac{\sqrt{x^2+y^2+z^2}}{(x^2+y^2+z^2)} \frac{z^2 x}{\sqrt{x^2+y^2}} - \frac{y}{\sqrt{x^2+y^2}}$$

$$= \frac{10x}{x^2+y^2+z^2} + \frac{xz^2}{\sqrt{(x^2+y^2)(x^2+y^2+z^2)}} - \frac{y}{\sqrt{(x^2+y^2)}}$$

$$B_y = \frac{10\sqrt{x^2+y^2}}{(x^2+y^2+z^2)} \frac{y}{\sqrt{x^2+y^2}} + \frac{\sqrt{x^2+y^2+z^2}}{(x^2+y^2+z^2)} \frac{z^2 y}{\sqrt{x^2+y^2}} - \frac{y}{\sqrt{x^2+y^2}}$$

$$= \frac{10y}{x^2+y^2+z^2} + \frac{yz^2}{\sqrt{(x^2+y^2)(x^2+y^2+z^2)}} + \frac{x}{\sqrt{(x^2+y^2)}}$$

$$= \frac{10z}{x^2+y^2+z^2} - \frac{z\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}$$

$\vec{B} = B_x\vec{a}_x + B_y\vec{a}_y + B_z\vec{a}_z$, where B_x, B_y, $B_z$areas given above.

At point (-3,4,0) x =-3, y= 4 and z=0,

$B_x = -\frac{30}{25} + 0 - \frac{4}{5} = -2$ $B_y = \frac{40}{25} + 0 - \frac{3}{5} = 1$

$B_z = 0-0=0$

Thus $\vec{B} = -2\vec{a}_x + \vec{a}_y$

For spherical to cylindrical vector transformation,

$$\begin{bmatrix} B_\rho \\ B_\phi \\ B_z \end{bmatrix} = \begin{vmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{vmatrix} \begin{bmatrix} \frac{10}{r} \\ r\cos\theta \\ 1 \end{bmatrix}$$

$B_\rho = \frac{10}{r}\sin\theta + r\cos^2\theta$, $B_\phi = 1$, $B_z = \frac{10}{r}\cos\theta - r\sin\theta\cos\theta$

But, $r = \sqrt{\rho^2+z^2}$ and$\theta = \tan^{-1}\frac{\rho}{z}$

$\Rightarrow \sin\theta = \frac{\rho}{\sqrt{\rho^2+z^2}}$ and $8\cos\theta = \frac{z}{\sqrt{\rho^2+z^2}}$

$\Rightarrow B_\rho = \frac{10\rho}{\rho^2+z^2} + \sqrt{\rho^2+z^2}\ \frac{z^2}{\rho^2+z^2}$ and $B_z = \frac{10z}{\rho^2+z^2} - \sqrt{\rho^2+z^2} \cdot \frac{\rho z}{\rho^2+z^2}$

Hence $\vec{B} = \left(\frac{10\rho}{\rho^2+z^2} + \frac{z^2}{\sqrt{\rho^2+z^2}}\right)\vec{a}_\rho + \vec{a}_\phi + \left(\frac{10z}{\rho^2+z^2} - \frac{\rho z}{\sqrt{\rho^2+z^2}}\right)\vec{a}_z$

At point $(5,\frac{\pi}{2},2)$, $\rho=5, \phi=\frac{\pi}{2}$ and z=2

$$\Rightarrow \vec{B}=\left(\frac{50}{59}+\frac{4}{\sqrt{29}}\right)\vec{a}_\rho+\vec{a}_\phi+\left(\frac{-20}{29}+\frac{10}{\sqrt{29}}\right)\vec{a}_z$$

$$\Rightarrow \vec{B}=2.467\,\vec{a}_\rho+\vec{a}_\phi+1.167\vec{a}_z$$

Note that, in all three co-ordinates,

$|B(x,y,z)| = |B(\rho,\phi,z)| = |B(r,\theta,\phi)| = 2.907$

---

7. Transform $4\bar{a}_x - 2\bar{a}_y - 4\bar{a}_z$ at (2,3,5)to cylindrical coordinates.(Nov/Dec 2016)

Given

$A_x=4$; $A_y=-2$; $A_z=-4$

x=2 ; y=3; z=5

$$A_\rho = A_x\,cos\emptyset + A_y\,sin\emptyset$$
$$= 4\,cos\emptyset - 2\,sin\emptyset$$
$$A_\emptyset = -A_x\,sin\,\emptyset + A_y\,cos\emptyset$$
$$= 4\,sin\,\emptyset - 2\,cos\emptyset$$
$$A_z = -4$$

$\emptyset = tan^{-1}\left(\frac{y}{x}\right) = tan^{-1}\left(\frac{3}{2}\right) = 56.31^o$

$$A_\rho = 4\,cos\,56.31º - 2\,sin 56.31º = 1.04$$

$$A_\emptyset = 4\sin 56.31\,º - 2\,cos 56.31º = -4.44$$
$$A_z = -4$$

The cylindrical vector

$$\bar{A} = 1.04\bar{a}_\rho - 4.44\bar{a}_\emptyset - 4\bar{a}_z$$

8. State the formulas for divergence, curl and gradient.

Divergence

$$\nabla \cdot \bar{\mathbf{F}} = \text{Divergence of } \bar{\mathbf{F}}$$

where ∇ = Vector operator $= \frac{\partial}{\partial x}\bar{\mathbf{a}}_x + \frac{\partial}{\partial y}\bar{\mathbf{a}}_y + \frac{\partial}{\partial z}\bar{\mathbf{a}}_z$

But $\bar{\mathbf{F}} = F_x\,\bar{\mathbf{a}}_x + F_y\,\bar{\mathbf{a}}_y + F_z\,\bar{\mathbf{a}}_z$

$$\therefore \quad \boxed{\nabla \cdot \bar{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = \text{div } \bar{F}}$$

This is divergence of $\bar{F}$ in **Cartesian system.**

Similarly divergence in other co-ordinate systems are,

$$\therefore \quad \boxed{\begin{aligned} \nabla \cdot \bar{F} &= \frac{1}{r}\frac{\partial}{\partial r}(rF_r) + \frac{1}{r}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z} \quad \text{Cylindrical} \\ \nabla \cdot \bar{F} &= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi} \quad \text{Spherical} \end{aligned}}$$

the vector field having its divergence zero is called solenoidal field.

$$\therefore \quad \boxed{\nabla \cdot \bar{A} = 0 \quad \text{for } \bar{A} \text{ to be solenoidal}}$$

Curl of a Vector

The circulation of a vector field around a closed path is given by curl of a vector.

In various co-ordinate systems, the curl of $\bar{F}$ is given by,

$$\nabla \times \bar{F} = \left[\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right]\bar{a}_x + \left[\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right]\bar{a}_y + \left[\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right]\bar{a}_z$$

i.e. $$\boxed{\nabla \times \bar{F} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \quad \text{Cartesian}} \qquad \ldots (1.18.3)$$

$$\nabla \times \bar{F} = \left[\frac{1}{r}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right]\bar{a}_r + \left[\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right]\bar{a}_\phi + \left[\frac{1}{r}\frac{\partial (rF_\phi)}{\partial r} - \frac{1}{r}\frac{\partial F_r}{\partial \phi}\right]\bar{a}_z$$

i.e. $$\boxed{\nabla \times \bar{F} = \frac{1}{r}\begin{vmatrix} \bar{a}_r & r\bar{a}_\phi & \bar{a}_z \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ F_r & rF_\phi & F_z \end{vmatrix} \quad \text{Cylindrical}} \qquad \ldots (1.18.4)$$

$$\nabla \times \bar{F} = \frac{1}{r \sin\theta}\left[\frac{\partial F_\phi \sin\theta}{\partial\theta} - \frac{\partial F_\theta}{\partial\phi}\right]\bar{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial F_r}{\partial\phi} - \frac{\partial (r F_\phi)}{\partial r}\right]\bar{a}_\theta + \frac{1}{r}\left[\frac{\partial (r F_\theta)}{\partial r} - \frac{\partial F_r}{\partial\theta}\right]\bar{a}_\phi$$

i.e.
$$\nabla \times \bar{F} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \bar{a}_r & r\bar{a}_\theta & r\sin\theta\,\bar{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ F_r & rF_\theta & r\sin\theta\,F_\phi \end{vmatrix} \quad \text{Spherical} \qquad \ldots (1.18.5)$$

Physical Significance of a Curl

- Thus if curl of a vector field exists then the field is called rotational.
- For irrotational vector field, the curl vanishes i.e. curl is zero.

Gradient of a Scalar

This is the scalar function and denoted as W (x, y, z).

$$\nabla \text{ (del)} = \frac{\partial}{\partial x}\bar{a}_x + \frac{\partial}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z$$

$$\text{Grad } W = \nabla W = \left(\frac{\partial}{\partial x}\bar{a}_x + \frac{\partial}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z\right) W$$

$$\therefore \quad \text{Grad } W = \frac{\partial W}{\partial x}\bar{a}_x + \frac{\partial W}{\partial y}\bar{a}_y + \frac{\partial W}{\partial z}\bar{a}_z$$

- *Gradient of a scalar* is *a vector.*

The gradient of a scalar W in various co-ordinate systems are given by,

Sr. No.	Co-ordinate system	Grad W = ∇ W
1.	Cartesian	$\nabla W = \frac{\partial W}{\partial x}\bar{a}_x + \frac{\partial W}{\partial y}\bar{a}_y + \frac{\partial W}{\partial z}\bar{a}_z$
2.	Cylindrical	$\nabla W = \frac{\partial W}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial W}{\partial\phi}\bar{a}_\phi + \frac{\partial W}{\partial z}\bar{a}_z$
3.	Spherical	$\nabla W = \frac{\partial W}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial W}{\partial\theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial W}{\partial\phi}\bar{a}_\phi$

Properties of Gradient of a Scalar

The various properties of a gradient of a scalar field W are,

- The gradient Δ W gives the maximum rate of change of W per unit distance.
- The gradient Δ W always indicates the direction of the maximum rate of change of W.
- The gradient Δ W at any point is perpendicular to the constant W surface, which passes through the point.

9. In cylindrical system of co-ordinate a vector is given by the expression $\vec{F} = 2r\cos^2\phi.\vec{a}_r + 3r^2\sin z\vec{a}_\phi + 4z\sin^2\phi\vec{a}_z$. Find the divergence.

Solution: We know that in cylindrical co-ordinates,

$\text{Div}\vec{F} = \frac{1}{r}\partial/\partial r\,(r\,\vec{F}_r) + \frac{1}{r}\frac{\partial\vec{F}_\phi}{\partial\phi} + \frac{\partial\vec{F}_z}{\partial z}$

$F_r = 2r\cos^2\phi$, $F_\phi = 3r^2\sin z$, $F_z = 4z\sin^2\phi$

$\text{Div}\vec{F} = \frac{1}{r}\partial/\partial r\,(2r^2\cos^2\phi) + \frac{1}{r}\partial/\partial\phi\,(3r^2\sin z) + \frac{\partial}{\partial z}(4z\sin^2\phi)$

$= \frac{1}{r}(4r)\cos^2\phi + \frac{1}{r}(0) + 4\sin^2\phi$

$= 4\cos^2\phi + 4\sin^2\phi$

$\text{Div}\vec{F} = 4$

10. Given $\vec{F} = R^2\sin\theta\vec{U}_R + B\phi\vec{U}_\theta + 2R\vec{U}\phi$ in spherical system of co-ordinates. Find the divergence of the vector F.

Solution: We know that in spherical co-ordinates,

$\nabla.\vec{F} = \frac{1}{R^2}\partial/\partial R\,(R^2R^2\sin\theta) + \frac{1}{R\sin\theta}\partial/\partial\theta\,(B\,\phi\sin\theta) + \frac{1}{R\sin\theta}\partial/\partial\theta\,(2R)$

$= \frac{4R^3}{R^2}\sin\theta + \frac{1}{R\sin\theta}(B\,\phi)(\cos\theta) + 0$

$= 4R\sin\theta + \left(\frac{B\theta}{R}\right)\cos\theta$

11. Prove the identities (Nov 2009)

(i) div(curl F) =0 (or) ∇.∇×F=0 and (ii) curl(grad ϕ) =0 (or) ∇×∇ϕ=0

Solution:

(i) div(curl F) = 0 (or) ∇.∇×F = 0

Let $\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$

Curl $\vec{F}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Fx & Fy & Fz \end{vmatrix}$

$=\left(\frac{\partial Fz}{\partial y}-\frac{\partial Fy}{\partial z}\right)\vec{i}+\left(\frac{\partial Fx}{\partial z}-\frac{\partial Fz}{\partial x}\right)\vec{j}+\left(\frac{\partial Fy}{\partial x}-\frac{\partial Fx}{\partial y}\right)\vec{k}$

Let $\vec{G}$= Curl $\vec{F}$

Then, Div(Curl $\vec{F}$) = Div$\vec{G}$ $=\frac{\partial Gx}{\partial x}+\frac{\partial Gy}{\partial y}+\frac{\partial Gz}{\partial z}$

Where,$G_x=\frac{\partial Fz}{\partial y}-\frac{\partial Fy}{\partial z}$

$G_y=\frac{\partial Fx}{\partial z}-\frac{\partial Fz}{\partial x}$

$G_z=\frac{\partial Fy}{\partial x}-\frac{\partial Fx}{\partial y}$

Div$\vec{G}$ $=\left(\frac{\partial^2 Fz}{\partial x\partial y}-\frac{\partial^2 Fy}{\partial z\partial x}\right)+\left(\frac{\partial^2 Fx}{\partial y\partial z}-\frac{\partial^2 Fz}{\partial x\partial y}\right)+\left(\frac{\partial^2 Fy}{\partial x\partial z}-\frac{\partial^2 Fx}{\partial y\partial z}\right)$

=0

Thus Div(curl $\vec{F}$)= 0 is proved.

(ii) Let $\vec{F}$=grad ϕ= $\frac{\partial\phi}{\partial x}\vec{i}+\frac{\partial\phi}{\partial y}\vec{j}+\frac{\partial\phi}{\partial z}\vec{k}$

Curl $\vec{F}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{\partial\phi}{\partial x} & \frac{\partial\phi}{\partial y} & \frac{\partial\phi}{\partial z} \end{vmatrix}$

$=\left(\frac{\partial^2\phi}{\partial y\partial z}-\frac{\partial^2\phi}{\partial y\partial z}\right)\vec{i}-\left(\frac{\partial^2\phi}{\partial x\partial z}-\frac{\partial^2\phi}{\partial x\partial z}\right)\vec{j}+\left(\frac{\partial^2\phi}{\partial x\partial y}-\frac{\partial^2\phi}{\partial x\partial y}\right)\vec{k}$

$=(0)\vec{i}-(0)\vec{j}+(0)\vec{k}=0$

Thus, curl $\bar{F}$=curl (grad ∅)=0 is proved.

12. If a scalar potential is given by the expression ϕ = xyz, determine the potential gradient and also prove that the vector F= gradϕis irrotational.(Apr 2010)

Solution:

Potential function which is a scalar is ϕ=xyz

$\nabla\phi = \frac{\partial\phi}{\partial x}\vec{i} + \frac{\partial\phi}{\partial y}\vec{j} + \frac{\partial\phi}{\partial z}\vec{k}$

$\frac{\partial\phi}{\partial x}$=yz, $\frac{\partial\phi}{\partial y}$=zx, $\frac{\partial\phi}{\partial z}$=xy

Potential gradient $\nabla\phi = yz\vec{i} + zx\vec{j} + xy\vec{k}$

$\Rightarrow \vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$

Where, F_x=yz, F_y=zx, F_z=xy

The vector F is said to be irrotational if curl F=0

$$\text{Curl F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz & zx & xy \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}(xy) - \frac{\partial}{\partial z}(yz)\right]\vec{i} + \left[\frac{\partial}{\partial z}(yz) - \frac{\partial}{\partial x}(xy)\right]\vec{j} + \left[\frac{\partial}{\partial x}(zx) - \frac{\partial}{\partial y}(yz)\right]\vec{k}$$

$=(x-x)\vec{i}+(y-y)\vec{j}+(z-z)\vec{k}$

$=0$

So curl F=curl(gradϕ)=0 is proved

13. Prove the following identity.Curl Curl$\vec{F}$ =GradDiv F-∇^2F where $\nabla^2 F = \nabla^2 F_x\vec{i} + \nabla^2 F_y\vec{j} + \nabla^2 F_z\vec{k}$ **;**

and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ **.**

Solution:

$$\text{Curl } \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ Fx & Fy & Fz \end{vmatrix}$$

$$= \left(\frac{\partial Fz}{\partial y} - \frac{\partial Fy}{\partial z}\right)\vec{i} + \left(\frac{\partial Fx}{\partial z} - \frac{\partial Fz}{\partial x}\right)\vec{j} + \left(\frac{\partial Fy}{\partial x} - \frac{\partial Fx}{\partial y}\right)\vec{k}$$

Curl $\vec{F}=\vec{G}=G_x\vec{i} + G_y\vec{j} + G_z\vec{k}$

$G_x = \frac{\partial Fz}{\partial y} - \frac{\partial Fy}{\partial z}$, $G_y = \frac{\partial Fx}{\partial z} - \frac{\partial Fz}{\partial x}$, $G_z = \frac{\partial Fy}{\partial x} - \frac{\partial Fx}{\partial y}$

Curl G = Curl Curl$\vec{F}$

Substituting and simplifying,we get

$$\nabla\times\nabla\times\vec{F}=\vec{i}\ \frac{\partial}{\partial x}\left(\frac{\partial Fx}{\partial x}+\frac{\partial Fy}{\partial y}+\frac{\partial Fz}{\partial z}\right)-\vec{i}\ \nabla^2F_x+\vec{j}\ \frac{\partial}{\partial y}\left(\frac{\partial Fx}{\partial x}+\frac{\partial Fy}{\partial y}+\frac{\partial Fz}{\partial z}\right)-\vec{j}\ \nabla^2F_y$$

$$+\vec{k}\ \frac{\partial}{\partial z}\left(\frac{\partial\ Fx}{\partial x}+\frac{\partial Fy}{\partial y}+\frac{\partial Fz}{\partial z}\right)-\vec{k}\ \nabla^2F_z$$

$$=(\vec{i}\ \frac{\partial}{\partial x}+\vec{j}\ \frac{\partial}{\partial y}+\vec{k}\ \frac{\partial}{\partial z})\ (\text{div F})-\nabla^2\ F=\nabla(\nabla.F)-\nabla^2F$$

Thus, $\nabla\times\nabla\times\vec{F}=\nabla(\nabla.F)-\nabla^2F$ is proved.

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14. Determine the curl of the following vectors: (Nov 2012)

a) $\vec{A}$ = yz $\vec{a}_x$ + 4xy $\vec{a}_y$ + y $\vec{a}_z$

b) $\vec{A}$ = ρz sinϕ$\vec{a}_\rho$ + 3ρz^{2}cosϕ$\vec{a}_\phi$

Solution:

a) $\vec{A}$ = yz $\vec{a}_x$ + 4xy $\vec{a}_y$ + y $\vec{a}_z$

A_x = yz, A_y = 4xy, A_z = y

$$\text{Curl }\vec{A}=\begin{vmatrix}\vec{a}_x & \vec{a}_y & \vec{a}_z\\ \partial/\partial x & \partial/\partial y & \partial/\partial z\\ A_x & A_y & A_z\end{vmatrix}$$

$$=\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)\vec{a}_x+\left(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\right)\vec{a}_y+\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)\vec{a}_z$$

$$\Rightarrow\nabla\times\vec{A}=\vec{a}_x(1-0)+\vec{a}_y(y-0)+\vec{a}_z(4y-z)$$

$$=\vec{a}_x+y\vec{a}_y+(4y-z)\vec{a}_z$$

b) $\vec{A}$ = ρz sinϕ$\vec{a}_\rho$ + 3ρz^{2}cosϕ$\vec{a}_\phi$

A_ρ = ρz sinϕ, A_ϕ = 3ρz^{2}cosϕ, A_z = 0

$$\text{Curl }\vec{A}=\frac{1}{\rho}\begin{vmatrix}\vec{a}_\rho & \rho\vec{a}_\phi & \vec{a}_z\\ \partial/\partial\rho & \partial/\partial\phi & \partial/\partial z\\ A_\rho & \rho A_\phi & A_z\end{vmatrix}$$

$$=\frac{1}{\rho}\left\{\left(\frac{\partial A_z}{\partial\phi}-\frac{\rho\partial A_\phi}{\partial z}\right)\vec{a}_\rho\ -\left(\frac{\partial A_z}{\partial\rho}-\frac{\partial A_\rho}{\partial z}\right)\rho\vec{a}_\phi+\left(\frac{\rho\partial A_\phi}{\partial\rho}-\frac{\partial A_\rho}{\partial\phi}\right)\vec{a}_z\right\}$$

$$\Rightarrow\nabla\times\vec{A}=\vec{a}_\rho(0-6\rho z\cos\phi)+\vec{a}_\varphi(0-\rho\sin\phi)+\vec{a}_z(6\rho z^2\cos\phi-\rho z\cos\phi)$$

$$=-6\ \rho z\cos\phi\vec{a}_\rho+\rho\sin\phi\vec{a}_\varphi+(1/\rho)(6\rho z^2\cos\phi-\rho z\cos\phi)\vec{a}_z$$

$$=-6\ \rho z\cos\phi\vec{a}_\rho+\rho\sin\phi\vec{a}_\varphi+(6z-1)z\cos\phi\vec{a}_z$$

---

15. Verify whether the given vector field is both solenoidal and irrotational.

$$\vec{E} = yz\vec{a}_x + xz\vec{a}_y + xy\vec{a}_z$$

Solution:

To find divergence:

$$\nabla.\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$= \frac{\partial}{\partial x}(yz) + \frac{\partial}{\partial y}(xz) + \frac{\partial}{\partial z}(xy)$$

$$= 0$$

⇨ The vector field *E* is solenoidal.

To find curl:

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} = (x-x)\vec{a}_x - (y-y)\vec{a}_y + (z-z)\vec{a}_z = 0$$

⇨ The field is irrotational.

Hence, the given field is both solenoidal and irrotational.

(B) Show that the vector $\vec{E} = (6xy + z^3)\vec{a}_x + (3x^2 - z)\vec{a}_y + (3xz^2 - y)\vec{a}_z$ is irrotational and find its scalar potential. (Apr / May 2015)

Solution:

If $\vec{E}$ is irrotational then curl $\vec{E}$ = 0. So we have to find $\nabla \times \vec{E}$

$$\nabla \times \vec{E} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (6xy + z^3) & (3x^2 - z) & (3xz^2 - y) \end{vmatrix}$$

$$= (-1+1)\vec{a}_x - (3z^2 - 3z^2)\vec{a}_y + (6x - 6x)\vec{a}_z = 0$$

⇨ The field is irrotational.

To find scalar potential: Assuming $\vec{E}$ is in 'x' direction,

$$V = -\int \vec{E}.\overrightarrow{dl}$$

$$= -\int \vec{E}.\overrightarrow{dx}$$

$$= -\int (6xy + z^3)\vec{a}_x.dx\vec{a}_x$$

$$= -(3yx^2 + z^3x)$$

16. Determine the divergence and curl of the following vector fields: (May 2013)

(i) $P = x^2 yz\ \hat{a}_x + xz\ \hat{a}_z$.

(ii) $Q = \rho\sin\phi\,\hat{a}_\rho + \rho^2 z\,\hat{a}_\phi + z\cos\phi\,\hat{a}_z$

(iii) $T = \frac{1}{r^2}\cos\theta\,\hat{a}_r + r\sin\theta\cos\phi\,\hat{a}_\theta + \cos\theta\,\hat{a}_\phi$.

Solution:

Divergence of the given vectors is derived as below:

(i) $P = x^2 yz\,\hat{a}_x + xz\,\hat{a}_z$

$$\nabla \cdot P = \frac{\partial}{\partial x}(x^2 yz) + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(xz)$$

$$= 2xyz + x.$$

(ii) $Q = \rho\sin\phi\,\hat{a}_\rho + \rho^2 z\,\hat{a}_\phi + z\cos\phi\,\hat{a}_z$

$$\nabla \cdot Q = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho Q_\rho) + \frac{1}{\rho}\frac{\partial Q_\phi}{\partial \phi} + \frac{\partial Q_z}{\partial z}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho^2 \sin\phi) + \frac{1}{\rho}\frac{\partial}{\partial \phi}(\rho^2 z) + \frac{\partial}{\partial z}(z\cos\phi)$$

$$= 2\sin\phi + \cos\phi.$$

(iii) $$\nabla \cdot \vec{T} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 T_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(T_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\left(T_\phi\right)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(\cos\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(r\sin^2\theta\cos\phi\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}(\cos\theta)$$

$$= 0 + 2\cos\theta\cos\phi + 0$$

$$= 2\cos\theta\cos\phi.$$

Curl of the given vectors is derived as below:

1) $\bar{P} = x^2yz\,\bar{a}_x + xz\,\bar{a}_z$

$P_x = x^2yz,\ P_y = 0,\ P_z = xz$

$$\therefore \quad \nabla \times \bar{P} = \left[\frac{\partial P_z}{\partial y} - \frac{\partial P_y}{\partial z}\right]\bar{a}_x + \left[\frac{\partial P_x}{\partial z} - \frac{\partial P_z}{\partial x}\right]\bar{a}_y + \left[\frac{\partial P_y}{\partial x} - \frac{\partial P_x}{\partial y}\right]\bar{a}_z$$

$$= [0 - 0]\,\bar{a}_x + [x^2 - z]\,\bar{a}_y + [0 - x^2]\,\bar{a}_z = (x^2 - z)\,\bar{a}_y - x^2\,\bar{a}_z$$

2) $\bar{Q} = \rho \sin\phi\ \bar{a}_\rho + \rho^2 z\ \bar{a}_\phi + z\cos\phi\ \bar{a}_z$

$Q_\rho = \rho\sin\phi,\ Q_\phi = \rho^2 z,\ Q_z = z\cos\phi$

$$\therefore \quad \nabla\times\bar{Q} = \left[\frac{1}{\rho}\frac{\partial Q_z}{\partial\phi} - \frac{\partial Q_\phi}{\partial z}\right]\bar{a}_\rho + \left[\frac{\partial Q_\rho}{\partial z} - \frac{\partial Q_z}{\partial\rho}\right]\bar{a}_\phi + \left[\frac{1}{\rho}\frac{\partial(\rho Q_\phi)}{\partial\rho} - \frac{1}{\rho}\frac{\partial Q_\rho}{\partial\phi}\right]\bar{a}_z$$

$$= \left[\frac{1}{\rho}\times(-z\sin\phi) - \rho^2\right]\bar{a}_\rho + [0-0]\bar{a}_\phi + \left[\frac{1}{\rho}3\rho^2 z - \frac{1}{\rho}\times\rho\cos\phi\right]\bar{a}_z$$

$$= \left(\frac{-z\sin\phi}{\rho} - \rho^2\right)\bar{a}_\rho + (3\rho z - \cos\phi)\bar{a}_z$$

3) $\bar{T} = \frac{1}{r^2}\cos\theta\,\bar{a}_r + r\sin\theta\cos\phi\,\bar{a}_\theta + \cos\theta\ \bar{a}_\phi$

$T_r = \frac{1}{r^2}\cos\theta,\ \ T_\theta = r\sin\theta\cos\phi,\ T_\phi = \cos\theta$

$$\nabla\times\bar{T} = \frac{1}{r\sin\theta}\left[\frac{\partial T_\phi\sin\theta}{\partial\theta} - \frac{\partial T_\theta}{\partial\phi}\right]\bar{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial T_r}{\partial_\phi} - \frac{\partial(rT_\phi)}{\partial r}\right]\bar{a}_\theta + \frac{1}{r}\left[\frac{\partial(rT_\theta)}{\partial r} - \frac{\partial T_r}{\partial\theta}\right]\bar{a}_\phi$$

$$= \frac{1}{r\sin\theta}\left[\frac{\partial(\sin\theta\cos\theta)}{\partial\theta} - r\sin\theta(-\sin\phi)\right]\bar{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}(0) - \frac{\partial(r\cos\theta)}{\partial r}\right]\bar{a}_\theta$$

$$+ \frac{1}{r}\left[\frac{\partial(r^2\sin\theta\cos\phi)}{\partial r} - \frac{1}{r^2}(-\sin\theta)\right]\bar{a}_\phi$$

$$\Rightarrow \frac{\partial(\sin\theta\cos\theta)}{\partial\theta} = \frac{1}{2}\frac{\partial(2\sin\theta\cos\theta)}{\partial\theta} = \frac{1}{2}\frac{\partial(\sin 2\theta)}{\partial\theta} = \frac{1}{2}\times 2\cos 2\theta = \cos 2\theta$$

$$\therefore \nabla\times\bar{T} = \frac{1}{r\sin\theta}[\cos 2\theta + r\sin\theta\sin\phi]\bar{a}_r + \frac{1}{r}[-\cos\theta]\bar{a}_\theta + \frac{1}{r}\left[2r\sin\theta\cos\phi + \frac{\sin\theta}{r^2}\right]\bar{a}_\phi$$

$$= \left[\frac{\cos 2\theta}{r\sin\theta} + \sin\phi\right]\bar{a}_r - \frac{\cos\theta}{r}\bar{a}_\theta + \left[2\sin\theta\cos\phi + \frac{\sin\theta}{r^3}\right]\bar{a}_\phi$$

17. State and prove Divergence theorem.(Nov 2014, Nov 2012, Nov 2009)(Nov/Dec 2016)(April/May 2017)(Nov/Dec 2017)(Apr/May 2018)

Statement:

The volume integral of the divergence of a vector field over a volume is equal to the surface integral of the normal component of this vector over the surface bounding the volume.

$$\iiint \nabla.\vec{A}dv = \oiint_s \vec{A}.d\vec{S}_n$$

Proof:

The divergence of any vector A is given by

$$\nabla.\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Taking the volume integral on both sides,

$$\iiint_v \nabla.\vec{A}\,\mathrm{dv} = \iiint_v \left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right] \mathrm{dx\ dydz} \qquad (\therefore \mathrm{dv} = \mathrm{dx\ dydz})$$

Consider an element volume in x direction,

$$\iiint \frac{\partial A_x}{\partial x}\,\mathrm{dxdydz} = \iint \left[\int \frac{\partial Ax}{dx}.dx\right]\mathrm{dydz}$$

$$\text{But} \int_{x1}^{x2} \frac{\partial A_x}{\partial x}\,\mathrm{dx} = A_{x_2} - A_{x_1} = A_x$$

Then,

$$\iiint_v \frac{\partial A_x}{\partial x}\,\mathrm{dx\ dydz} = \iint_s \mathrm{A_x dydz} = \iint_s \mathrm{A_x dS_x}$$

When dy dz $= \mathrm{dS_x}$ = x component of surface area ds.

Similarly, the following integrands become

$$\iiint_v \frac{\partial A_y}{\partial y}\,\mathrm{dx\ dy\ dz} = \iint_s \mathrm{A_y\,dS_y}$$

$$\iiint_v \frac{\partial A_z}{\partial y}\,\mathrm{dx\ dy\ dz} = \iint_s \mathrm{A_z dS_z}$$

$$\text{Then,} \iiint_v \nabla.\vec{A}\mathrm{dv} = \iiint_v \left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right]\mathrm{dx\ dydz}$$

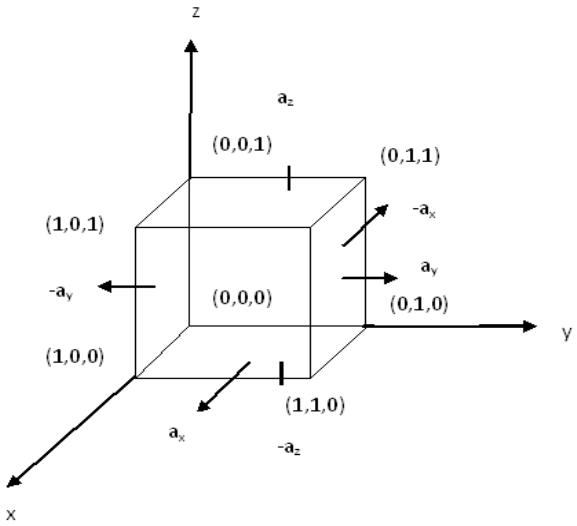
$$= \iint_s (\mathrm{A_x\,dS_x + A_y\,dS_y + A_z\,dS_z}) = \oiint_s \vec{A}.\overrightarrow{ds}$$

Hence Proved.

18. Using divergence theorem, evaluate $\oiint_S \vec{F}.d\vec{S}_n$ **where** $\vec{F}=2xy\,\vec{i}+y^2\,\vec{j}+4yz\,\vec{k}$ **and S is the surface of a cube bounded by x=0, x=1,y=0,y=1 and z=0, z=1.**

Solution: By divergence theorem

$$\oiint_s \vec{F}.d\vec{S}_n = \iiint \nabla.\vec{F}dv$$



R.H.S:

$$\nabla.\vec{F}=\left(\vec{i}\frac{\partial}{\partial x}+\vec{j}\frac{\partial}{\partial y}+\vec{k}\frac{\partial}{\partial z}\right).(2xy\,\vec{i}+y^2\,\vec{j}+4yz\,\vec{k})$$

$$=\frac{\partial}{\partial x}(2xy)+\frac{\partial}{\partial y}(y^2)+\frac{\partial}{\partial z}(4yz)$$

$$=2y+2y+4y=8y$$

$$\iiint \nabla.\vec{F}\,dv=\int_0^1\int_0^1\int_0^1 8y\,dx\,dydz$$

$$=\int_0^1\int_0^1 [8yz]_0^1\,dx\,dy \quad =\int_0^1\int_0^1 8y\,dx\,dy$$

$$=\int_0^1\left[8\frac{y^2}{2}\right]_0^1 dx=4\int_0^1 dx$$

$$=4\,[x]_0^1=4 \rightarrow \textbf{RHS}$$

L.H.S:

$$\oiint_S \vec{F}.d\vec{S}_n=\iint_{Front}\vec{F}.d\vec{S}_x(\vec{a}_x)+\iint_{Back}\vec{F}.d\vec{S}_x(-\vec{a}_x)+\iint_{Right}\vec{F}.d\vec{S}_y(\vec{a}_y)+\iint_{Left}\vec{F}.d\vec{S}_y(-\vec{a}_y)$$

$$+\iint_{Top}\vec{F}.d\vec{S}_z(\vec{a}_z)+\iint_{Bottom}\vec{F}.d\vec{S}_z(-\vec{a}_z)\rightarrow$$ sum of surfaces of all sides of the cube.

We know, $d\vec{S}_x=dydz\vec{a}_x$, $d\vec{S}_y=dx\,dz\vec{a}_y$, $d\vec{S}_z=dx\,dy\vec{a}_z$

$$\Rightarrow \vec{F}.d\vec{S}_x=2xy, \quad \vec{F}.d\vec{S}_y=y^2, \quad \vec{F}.d\vec{S}_z=4yz,$$

$$\iint_{Front}\vec{F}.d\vec{S}_x(\vec{a}_x)=\int_{z=0}^1\int_{y=0}^1 2xy\,dydz=2x\int_{z=0}^1\left[\frac{y^2}{2}\right]_0^1 dydz=2x\int_{z=0}^1\left[\frac{1}{2}\right]dz=x\,[z]_0^1=x=1$$

(Since at front surface x=1)

$$\iint_{Back} \vec{F}\,.d\vec{S}_x(-\vec{a}_x) = \int_{z=0}^{1}\int_{y=0}^{1} 2xy\,dydz = 2x\int_{z=0}^{1}\left[\frac{y^2}{2}\right]_0^1 dz = 2x\int_{z=0}^{1}\left[\frac{1}{2}\right]dz = x\,[z]_0^1 = x = 0$$

(Since at back surface x = 0)

$$\iint_{Right} \vec{F}\,.d\vec{S}_y(\vec{a}_y) = \int_{z=0}^{1}\int_{x=0}^{1} y^2 dxdz = y^2\int_{z=0}^{1}[x]_0^1\,dz = y^2\int_{z=0}^{1} dz = y^2[z]_0^1 = y^2 = 1$$

(Since at right side surface y = 1)

$$\iint_{Left} \vec{F}\,.d\vec{S}_y(-\vec{a}_y) = \int_{z=0}^{1}\int_{x=0}^{1} y^2 dxdz = y^2\int_{z=0}^{1}[x]_0^1\,dz = y^2\int_{z=0}^{1} dz = y^2[z]_0^1 = y^2 = 0$$

(Since at left side surface y = 0)

$$\iint_{Top} \vec{F}\,.d\vec{S}_z(\vec{a}_z) = \int_{y=0}^{1}\int_{x=0}^{1} 4yz\,dxdy = 4yz\int_{y=0}^{1}\left[\frac{x^2}{2}\right]_0^1 dy = 4z\int_{y=0}^{1} ydy = 4z\left[\frac{y^2}{2}\right]_0^1 = 4\left[\frac{1}{2}\right]z = 2$$

(Since at top surface z = 1)

$$\iint_{Bottom} \vec{F}\,.d\vec{S}_z(-\vec{a}_z) = \int_{y=0}^{1}\int_{x=0}^{1} 4yz\,dxdy = 4yz\int_{y=0}^{1}\left[\frac{x^2}{2}\right]_0^1 dy = 4z\int_{y=0}^{1} ydy = 4z\left[\frac{y^2}{2}\right]_0^1 = 4\left[\frac{1}{2}\right]z = 0$$

(Since at bottom surface z = 0)

From the above results, we can see that $\oint_S \vec{F}\,.d\vec{S}_n = 1 + 0 + 1 + 0 + 2 + 0 =$ **4→ LHS**

Thus R.H.S. = L.H.S. Hence Divergence theorem proved.

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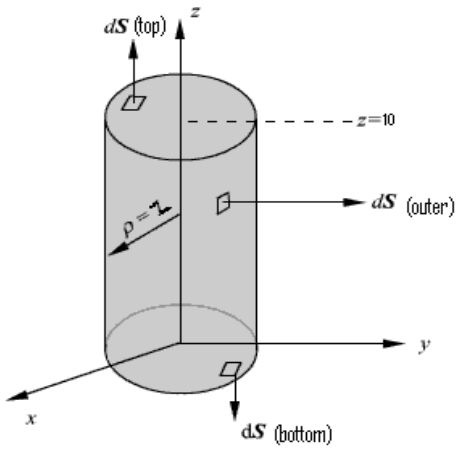
19. Given $\vec{D} = \frac{10\rho^3\vec{a_\rho}}{4}$ volume enclosed by ρ = 2, z = 0 and z = 10. Evaluate Divergence theorem on both sides.



Solution: From Divergence theorem $\oint \vec{D}.\vec{ds} = \iiint_v \nabla.\vec{D}dv$

RHS: $\int_{z=0}^{10}\int_{\emptyset=0}^{2\pi}\int_{\rho=0}^{2} \nabla.\vec{D}\rho d\rho\, d\emptyset\, dz$

$$\nabla.\vec{D} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\emptyset}{\partial\emptyset} + \frac{\partial D_z}{\partial z}$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\frac{\rho 10\rho^3}{4}\right) + 0$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\frac{10\rho^4}{4}\right) = \frac{10}{\rho}\frac{(4\rho^3)}{4}$$

$$\nabla.\vec{D} = \frac{10\rho^3}{\rho} = 10\,\rho^2$$

$$=> \int_{z=0}^{10}\int_{\emptyset=0}^{2\pi}\int_{\rho=0}^{2} 10\rho^2\rho d\rho\, d\emptyset\, dz = 10\int_{z=0}^{10}\int_{\emptyset=0}^{2\pi}\int_{\rho=0}^{2}\rho^3 d\rho\, d\emptyset\, dz$$

$$= 10\int_{z=0}^{10}\int_{\emptyset=0}^{2\pi}\left[\frac{\rho^4}{4}\right]_0^2 d\emptyset dz$$

$$= 10\int_{z=0}^{10}\int_{\emptyset=0}^{2\pi}\frac{16}{4}d\emptyset dz$$

$$= 40\int_{z=0}^{10}[0]_0^{2\pi}dz = 80\,\pi[z]_0^{10}$$

$$= 80\,\pi(10) = \mathbf{800\,\pi} \rightarrow \mathbf{RHS}$$

LHS:

$$\oint_S \vec{D}.\overrightarrow{ds} = \int_{top}\frac{\vec{D}.\overrightarrow{ds}}{(+\overrightarrow{a_z})} + \int_{bottom}\frac{\vec{D}.\overrightarrow{ds}}{(-\overrightarrow{a_z})} + \int_{outer}\frac{\vec{D}.\overrightarrow{ds}}{(\overrightarrow{a_\rho})}$$

$$\int_{top}\frac{\vec{D}.\overrightarrow{ds}}{(+\overrightarrow{a_z})} = \int\frac{10\rho^3}{4}\overrightarrow{a_\rho}.\,\rho d\rho\, d\emptyset\overrightarrow{a_z} = 0$$

$$\int_{bottom}\frac{\vec{D}.\overrightarrow{ds}}{(-\overrightarrow{a_z})} = \int\frac{10\rho^3}{4}\overrightarrow{a_\rho}.\,\rho d\rho\, d\emptyset\,(-\overrightarrow{a_z}) = 0$$

$$\int_{outer}\frac{\vec{D}.\overrightarrow{ds}}{(+\overrightarrow{a_l})} = \int\frac{10\rho^3}{4}\overrightarrow{a_\rho}.\,d\emptyset\, dz\overrightarrow{a_z} = \int_{z=0}^{10}\int_{\emptyset=0}^{2\pi}\frac{10\rho^4}{4}d\emptyset dz$$

$$= \int_{z=0}^{10}\int_{\emptyset=0}^{2\pi}\frac{10(16)}{4}d\emptyset dz$$

$$= 40\int_{z=0}^{10}[\emptyset]_0^{2\pi}dz = 80\pi\int_{z=0}^{10}dz$$

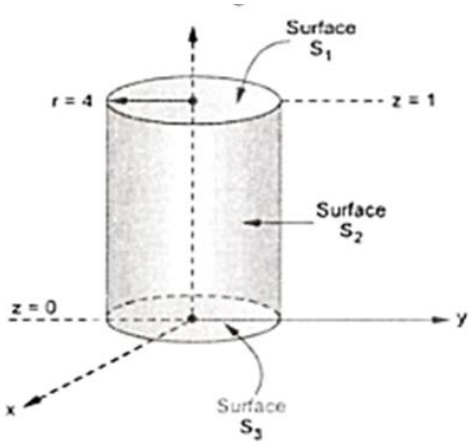
$$= \quad 80\pi[z]_0^{10} = \quad 80\pi(10)$$
$$= \mathbf{800\pi} \rightarrow \boldsymbol{LHS}$$

RHS=LHS

20. A particular vector field $\bar{F} = r^2 cos^2 \emptyset\, \bar{a}_r + zsin\, \emptyset\, \bar{a}_\emptyset$ is in cylindrical system. Find the flux emanating due to this field from the closed surface of the cylinder $0 \leq z \leq 1, r = 4$. Verefy the divergence theorem.

The outward flux is given by

$$\emptyset = \int_S \bar{F} \,.\, d\bar{S} \quad over\ a\ closed\ surface\ S$$



The total surface is made up of,

1. Top surface S_1 for which z = 1, r varies from 0 to 4 and ϕ varies from 0 to 2π.
2. Lateral surface for which z varies from 0 to 1, ϕ from 0 to 2π and r = 4.
3. Bottom surface S_3 for which z = 0, r varies from 0 to 4 and ϕ varies from 0 to 2π.

For S_1, $d\bar{S} = r\, dr\, d\phi\, \bar{a}_z$

For S_2, $d\bar{S} = r\, dz\, d\phi\, \bar{a}_r$

For S_3, $d\bar{S} = r\, dr\, d\phi\, (-\bar{a}_z)$

$$\therefore \quad \oint_{S_1} \bar{F} \cdot d\bar{S} = \oint_{S_1} (r^2 \cos^2\phi \bar{a}_r + z \sin\phi \bar{a}_\phi) \cdot (r\, dr\, d\phi \bar{a}_z)$$

$$= 0 \qquad \ldots \bar{a}_r \cdot \bar{a}_z = \bar{a}_\phi \cdot \bar{a}_z = 0$$

$$\oint_{S_3} \bar{F} \cdot d\bar{S} = \oint_{S_3} (r^2 \cos^2\phi \bar{a}_r + z \sin\phi \bar{a}_\phi) \cdot [r\, dr\, d\phi\, (-\bar{a}_z)]$$

$$= 0 \qquad \ldots \bar{a}_r \cdot \bar{a}_z = \bar{a}_\phi \cdot \bar{a}_z = 0$$

$$\oint_{S_2} \bar{F} \cdot d\bar{S} = \oint_{S_2} (r^2 \cos^2\phi \bar{a}_r + z \sin\phi \bar{a}_\phi) \cdot (r\, dz\, d\phi \bar{a}_r)$$

$$= \int_{z=0}^{1} \int_{\phi=0}^{2\pi} r^2 \cos^2\phi\, r\, dz\, d\phi \qquad \ldots \bar{a}_r \cdot \bar{a}_z = 1,\ \bar{a}_\phi \cdot \bar{a}_r = 0\ r = 4$$

$$= (4)^3 \int_{z=0}^{1}\int_{\phi=0}^{2\pi} dz \cos^2\phi \; d\phi = 64 \int_0^1 dz \int_{\phi=0}^{2\pi} \frac{1+\cos 2\phi}{2} d\phi$$

$$= 64 \times [z]_0^1 \times \frac{1}{2} \times \left\{ [\phi]_0^{2\pi} + \left[+ \frac{\sin 2\phi}{2} \right]_0^{2\pi} \right\}$$

$$= 64 \times 1 \times \frac{1}{2} \times 2\pi = 64\pi$$

$$\therefore \quad \oint_S \bar{F} \cdot d\bar{s} = 0 + 64\pi + 0 = 64\pi$$

Let us verify divergence theorem which states that,

$$\oint_S \bar{F} \cdot d\bar{s} = \oint_v (\nabla \cdot \bar{F}) \, dv$$

Now $dv = r \, dr \, d\phi \, dz$

$$\nabla \cdot \bar{F} = \frac{1}{r} \frac{\partial}{\partial r} (r F_r) + \frac{1}{r} \frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}$$

$$= \frac{1}{r} \frac{\partial}{\partial r} (r \times r^2 \cos^2\phi) + \frac{1}{r} \frac{\partial}{\partial \phi} (z \sin\phi) + 0$$

$$= \frac{\cos^2\phi}{r} \times 3r^2 + \frac{z}{r} (+\cos\phi) = 3 \, r \cos^2\phi + \frac{z\cos\phi}{r}$$

$$\therefore \quad \oint_v (\nabla \cdot \bar{F}) \, dv = \int_{z=0}^{1} \int_{\phi=0}^{2\pi} \int_{r=0}^{4} \left(3r\cos^2\phi + \frac{z\cos\phi}{r} \right) r \, dr \, d\phi \, dz$$

$$= \int_{z=0}^{1} \int_{\phi=0}^{2\pi} \left[\frac{3r^3}{3} \cos^2\phi + z\cos\phi \, r \right]_0^4 d\phi \, dz$$

$$= \int_{z=0}^{1} \int_{\phi=0}^{2\pi} \left\{ 4^3 \left[\frac{1+\cos 2\phi}{2} \right] + 4z\cos\phi \right\} d\phi \, dz$$

$$= \int_{z=0}^{1} \left\{ 32 \left[\phi + \frac{\sin 2\phi}{2} \right]_0^{2\pi} + 4z [\sin\phi]_0^{2\pi} \right\} dz$$

$$= \int_{z=0}^{1} \{ 32 \times [2\pi + 0] + 4z[0] \} \, dz = \int_{z=0}^{1} 64\pi \, dz$$

$$= 64\pi [z]_0^1 = 64\pi$$

Thus $\int_S \bar{F} \cdot d\bar{s} = \oint_v (\nabla \cdot \bar{F}) \, dv$ and divergence theorem is verified.

21. *V*erify Divergence theorem for the function $\vec{F} = 3x^2\vec{\imath} + (3y + z)\vec{\jmath} + (3z - x)\vec{k}$ $x^2 + y^2 = 9, Z = 0 and Z = 2$. Over the cylindrical region bounded by(Nov/Dec 2015)

$\overrightarrow{F} = 3x^2\overrightarrow{\imath} + (3y + z)\overrightarrow{\jmath} + (3z - x)\overrightarrow{k}$

$x^2 + y^2 = 9 \; and \; x = 0, \;\; y = 0, z = 0 \; and \; z = 2$

$\int\int\int and \; \overrightarrow{F} \; dv = \int\int \overrightarrow{F}.n^{\wedge} \; ds$

$\overrightarrow{F} = 6x + 3 + 3 = 6(x + 1)$

$L.H.S \;\; = \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{2} 6(x + 1)dz \; dy \; dx$

$= \; 6\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}[(x + 1)Z]_0^2 \; dy.dx$

$= \; 12\int_{-3}^{3}[(x + 1)y]dx$

$= \; 12\int_{-3}^{3}\left[(x + 1)\sqrt{9 - x^2} + (x + 1)\sqrt{9 - x^2}\right]dx$

$= \; 24\int_{-3}^{3}\left[x\sqrt{9 - x^2} + \sqrt{9 - x^2}\right]dx$

$= \; 24 \times 2\int_{0}^{3}\left[\sqrt{9 - x^2}\right]dx$

$= \; 48\left[\frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$

$= \; 48\frac{9}{2}\frac{\pi}{2}$

$= 108\,\pi$

$RHS = \int_{S_1}\int \overrightarrow{F}.\; n^{\wedge} \; ds + \int_{S_2}\int \overrightarrow{F}.\; n^{\wedge} \; ds + \int_{S_3}\int \overrightarrow{F}.\; n^{\wedge} \; ds$

To find $\int_{S_1}\int \overrightarrow{F}.\; n^{\wedge} \; ds$

$\int_{S_1}\int \overrightarrow{F}.\; n^{\wedge} \; ds = \int_{S_1}\int \overrightarrow{F}.\; (-\overrightarrow{h})dx \; dy = \int\int (x - 3z) \; dx \;\; dy$

$= \; \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} x \;\; dy \;\; dx$

$= \; \int_{-3}^{3}\left[x\sqrt{9 - x^2} + \sqrt{9 - x^2}\right]dx$

$= 0$

To find $\int_{S_2}\int \overrightarrow{F}.\ n^{\wedge}\ ds$:

$$\int_{S_2}\int \overrightarrow{F}.\ n^{\wedge}\ ds = \int\int \overrightarrow{F}\overrightarrow{h}\ dx\ \ dy = = \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}(3z-x)\ \ dx\ \ dy$$

$$= \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{2}(6-x)\ dx\ dy \qquad [\because\ \ Z=2\ \ in\ S_2]$$

$$= \int_{-3}^{3}(6-x)\sqrt{9-x^2}+(6-x)\sqrt{9-x^2}\,.dx$$

$$= 2\int_{-3}^{3}6\sqrt{9-x^2}+x\sqrt{9-x^2}\,.dx$$

$$= 2\int_{-3}^{3}6\sqrt{9-x^2}\,.dx$$

$$= 6\times 2\times 2\int_{0}^{3}\sqrt{9-x^2}\,.dx$$

$$= \ 24\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]\begin{matrix}3\\0\end{matrix}$$

$$= \ 24\frac{9}{2}\frac{\pi}{2}$$

$$= 54\,\pi$$

To find $\int_{S_3}\int \overrightarrow{F}.\ n^{\wedge}\ ds$:

$Let\ \phi = x^2+y^2=9$

$\nabla\phi = 2x\overrightarrow{\imath}+2y\overrightarrow{\jmath}$

$|\nabla\phi| = \sqrt{4x^2+4x^2} = 2\sqrt{x^2+y^2} = 2\sqrt{9} = 6$

$$n^{\wedge} = \frac{\nabla\phi}{|\nabla\phi|} = \frac{2x\overrightarrow{\imath}+2y\overrightarrow{\jmath}}{6} = \frac{x\overrightarrow{\imath}+y\overrightarrow{\jmath}}{3}$$

Here $S_3\ is\ the\ curved\ surface,$

By using cylindrical co – ordinates $x = 3\ \cos\theta\,, y = 3\ \sin\theta\ \ ds = 3\quad d\theta\quad dz.$

The curved surface $x^2+y^2=9\ \ v\ aries\ from\ 0\ to\ 3\ \ and\ \theta\ various\ from\ 0\ to\ 2\pi.$

$$\int_{S_3}\int \overrightarrow{F}.\ n^{\wedge}\ ds = \int_{S_3}\left(x^3+y^2+\frac{zy}{3}\right)ds$$

$$= \int_{0}^{2}\int_{0}^{2\pi}\left[(3\cos\theta)^3+(3\sin\theta)^2+\frac{z\ 3\sin\theta}{3}\right]3d\theta\ dz$$

$$= 3\int_{0}^{2}\int_{0}^{2\pi}[27\cos^3\theta+7\sin^2\theta+z\ \sin\theta]\,d\theta\ dz$$

$$= 3\int_{0}^{2}\int_{0}^{2\pi}\left[27\left(\frac{\cos 3\theta+3\cos\theta}{4}\right)+9\left(\frac{1-\cos 2\theta}{2}\right)+z\ \sin\theta\right]\,d\theta\ dz$$

$$= \int_0^{2\pi} \left[\frac{27}{4}(\cos 3\theta + 3\cos\theta)z + \frac{9}{2}(1-\cos 2\theta)z + \sin\theta\frac{z^2}{2}\right]_{z=0}^{2} d\theta$$

$$= \int_0^{2\pi} \left[\frac{27}{4}(\cos 3\theta + 3\cos\theta)(2) + \frac{9}{2}(1-\cos 2\theta)(2) + \sin\theta\frac{(2)^2}{2}\right] d\theta$$

$$= 3\left[\frac{27}{2}\left(\frac{\sin 3\theta}{3} + 3\sin\theta\right) + 9\left(\theta - \frac{\sin 2\theta}{2}\right) - 2\cos\theta\right]_0^{2\pi}$$

$$= 3[9(2\pi) - 2\cos(2\pi) + 2\cos 0]$$

$$= 3[182\pi - 2 + 2]$$

$$= 54\pi$$

$$RHS = \int_{S_1}\int \vec{F}.\,\hat{n}\,ds + \int_{S_2}\int \vec{F}.\,\hat{n}\,ds + \int_{S_3}\int \vec{F}.\,\hat{n}\,ds$$

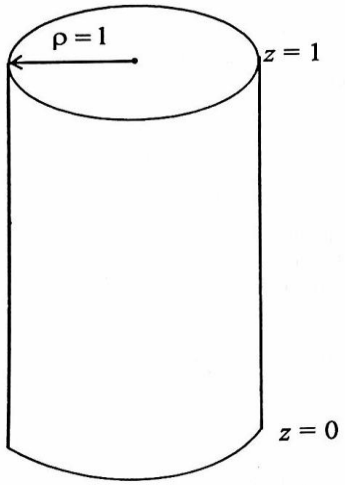
$$= 0 + 54\,\pi + 54\,\pi$$

$$= 108\,\pi$$

$$LHS = RHS$$

22. Verify the divergence theorem for a field vector $\bar{A} = 10e^{-2z}(\rho\bar{a}_\rho + \bar{a}_z)$ over the closed surface of the cylinder $\rho = 1$ *and* $0 \le z \le 1$ (Nov/Dec 2016) (PART C)

According to divergence theorem $\int_s \vec{A}.\vec{ds} = \int_v (\nabla.\vec{A})\,dv$



Here the surface of cylinder is divided into top, bottom and sides. At top surface $z = 1$, $0 \le \rho \le 1$, $0 \le \phi \le 2\pi$ and $\vec{ds} = \rho d\rho d\phi \vec{a}_z$. At bottom surface $z = 0$, $0 \le \rho \le 1, 0 \le \phi \le 2\pi$ and $\vec{ds} = \rho d\rho d\phi(-\vec{a}_z)$. At side surface $\rho = 1$, $0 \le z \le 1$, $0 \le \phi \le 2\pi$ and $\vec{ds} = \rho dz d\phi \vec{a}_\rho$. Therefore

$$\oint_s \vec{A}.\vec{ds} = \int_{s(\text{top})} \vec{A}.\vec{ds} + \int_{s(\text{bottom})} \vec{A}.\vec{ds} + \int_{s(\text{side})} \vec{A}.\vec{ds}$$

$$\int_{s(\text{top})} \vec{A}.\vec{ds} = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{1} 10e^{-2}\rho d\rho d\phi$$

$$= 10e^{-2}\left(\frac{\rho^2}{2}\right)_0^1 (\phi)_0^{2\pi}$$

$$= 10e^{-2}\left(\frac{1}{2}\right)(2\pi) = 10\pi e^{-2}$$

$$\int_{s(\text{bottom})} \vec{A}.\vec{ds} = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{1} 10e^{0}\left(\rho\vec{a_\rho}+\vec{a_z}\right).\rho d\rho d\phi(-\vec{a_z})$$

$$= \int_{\phi=0}^{2\pi}\int_{\rho=0}^{1} -10\rho d\rho d\phi = \rho d\rho d\,(-\vec{a_z})$$

$$= \int_{\phi=0}^{2\pi}\int_{\rho=0}^{1} -10\rho d\rho d\phi = -10\left(\frac{\rho^2}{2}\right)_0^1 (\phi)_0^{2\pi}$$

$$= -10\left(\frac{1}{2}\right)(2\pi) = -10\pi$$

$$\int_{s(\text{side})} \vec{A}.\vec{ds} = \int_{z=0}^{1}\int_{\phi=0}^{2\pi} 10e^{-2z}\,(\rho\vec{a_\rho}+\vec{a_z}).\rho dz d\phi\,\vec{a_\rho}$$

$$= \int_{z=0}^{1}\int_{\phi=0}^{2\pi} 10e^{-2z}\rho^2 dz d\phi = 10\left(\frac{e^{-2z}}{-2}\right)_0^1 (1)^2(\phi)_0^{2\pi}$$

$$= \frac{10}{2}\left(1-e^{-2}\right)(2\pi) = 10\pi - 10\pi e^{-2}$$

$$\oint_s \vec{A}.\vec{ds} = 10\pi e^{-2} - 10\pi + 10\pi - 10\pi e^{-2} = 0$$

Here the field have $A_\rho = 10e^{-2z}\rho$ and $A_z = 10e^{-2z}$ components. Therefore

$$\nabla.\vec{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho A_\rho + \frac{\partial}{\partial z}A_z$$

$$= \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho 10e^{-2z}\rho + \frac{\partial}{\partial z}10e^{-2z}$$

$$= \frac{1}{\rho}\ 2\rho 10e^{-2z} + 10e^{-2z}(-2)$$

$$= 20e^{-2z} - 20e^{-2z} = 0$$

$$\int_v (\nabla.\vec{A})\, dv = 0$$

Hence $\oint_s \vec{A}.\vec{ds} = \int_v (\nabla.\vec{A})\, dv$

23. Give that $\bar{D} = \frac{5r^2}{4}\bar{a}_r C/m^2$. Evaluate the both sides of divergence theorem for the volume enclosed by r=4m and $\theta = \frac{\pi}{4}$ (Apl/May 2017)

According to divergence theorem $\int_s D.\vec{ds} = \int_v (\nabla.\vec{D})\, dv$

Here $\vec{ds} = r^2 \sin\theta d\theta d\phi\ \vec{a}_r$, $dv = r^2 \sin\theta dr d\theta d\phi$, $0 \le \theta \le \frac{\pi}{4}$ and $0 \le \phi \le 2\pi$.

$$\oint_s \vec{D}.\vec{ds} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi/4}\left(\frac{5r^2}{4}\right) r^2 \sin\theta d\theta d\phi$$

$$= \frac{5r^4}{4}(-\cos\theta)_0^{\pi/4}(\phi)_0^{2\pi}, \text{ where } r = 4$$

$$= \frac{5}{4}(4)^4\ (0.29)\ (2\pi)\ \ 588.89$$

Here the flux have only $D_r = \frac{5r^2}{4}$component. Therefore

$$\nabla.\bar{D} = \frac{1}{r^2}\frac{\partial}{\partial r}r^2 D_r$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}r^2 \times \frac{5r^2}{4} = \frac{5}{4r^2}\frac{\partial}{\partial r} \times r^4$$

$$= \frac{5}{4r^2}(4r^3)$$

$$\int_v (\nabla.\bar{D})\, dv = \int_{\emptyset=0}^{2\pi}\int_{\theta=0}^{\pi/4}\int_{r=0}^{4} (5r)(r^2 \sin\theta\, dr\, d\theta\, d\emptyset)$$

$$= 5\left[\frac{r^4}{4}\right]_0^4 (-\cos\theta)_0^{\pi/4}(\emptyset)_0^{2\pi}$$

$$= \frac{5}{4}(4)^2(0.29)(2\pi) = 588.89$$

$$Hence \oint_s \bar{D}.\overline{ds} = \int_v (\nabla.\bar{D})\, dv$$

24. State and prove Stoke's theorem. (Apr 2011, Apr 2010)

The Stoke's theorem relates the line integral to a surface integral.

It states that,

The line integral of F around a closed path L is equal to the integral of curl of F over the open surface S enclosed by the closed path L.

Mathematically it is expressed as,

$$\oint_L \bar{F} \cdot d\bar{L} = \int_S (\nabla \times \bar{F}) \cdot d\bar{S} \quad \text{... (1.19.1)}$$

where dL = Perimeter of total surface S

Key Point *Stoke's theorem is applicable only when $\bar{F}$ and $\nabla \times \bar{F}$ are continuous on the surface S. The path L and open surface S enclosed by path L for which Stoke's theorem is applicable are shown in the Fig. 1.19.1.*

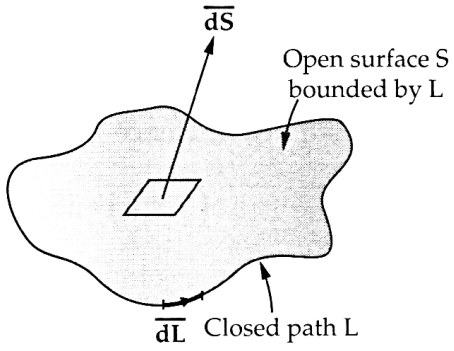


Fig. 1.19.1

Proof: Consider a surface S which is splitted into number of incremental surfaces. Each incremental surface is having area ΔS as shown in the Fig. 1.19.2 (a).

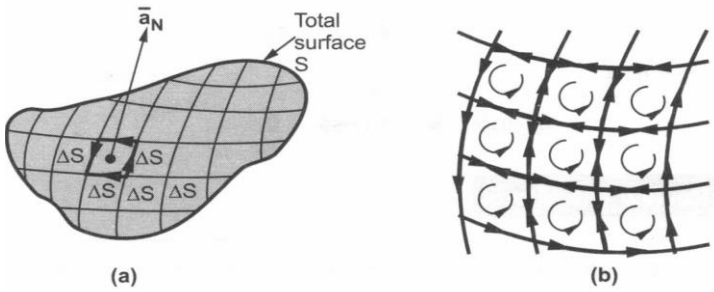


Fig. 1.19.2 Stoke's theorem

Applying definition of the curl to any of these incremental surfaces we can write,

$$\left(\nabla \times \bar{F}\right)_N = \frac{\oint \bar{F} \bullet d\bar{L}_{\Delta S}}{\Delta S} \qquad \ldots (1.19.1)$$

where N = Normal to ΔS according to right hand rule

$dL_{\Delta S}$ = Perimeter of the incremental surface ΔS

Now the curl of $\bar{F}$ in the normal direction is the dot product of curl of $\bar{F}$ with $\bar{a}_N$ where $\bar{a}_N$ is unit vector, normal to the surface ΔS, according to right hand rule.

$$\therefore \quad \left(\nabla \times \bar{F}\right)_N = \left(\nabla \times \bar{F}\right) \bullet \bar{a}_N \qquad \ldots \text{Using in (1.19.1) we get,}$$

$$\therefore \quad \oint \bar{F} \bullet d\bar{L}_{\Delta S} = \left(\nabla \times \bar{F}\right) \bullet \bar{a}_N \, \Delta S$$

$$\therefore \quad \oint \bar{F} \bullet d\bar{L}_{\Delta S} = \left(\nabla \times \bar{F}\right) \bullet \Delta\bar{S} \qquad \ldots (1.19.2)$$

- To obtain total curl for every incremental surface, add the closed line integrals for each ΔS. From the Fig. 1.19.2 (b), it can be seen that at a common boundary between the two incremental surfaces, the line integral is getting cancelled as the boundary is getting traced in two opposite directions.
- This happens for all the interior boundaries. Only at the outside boundary cancellation does not exist. Hence summation of all closed line integrals for each and every ΔS ends up in a single closed line integral to be obtained for the outer boundary of the total surface S. Hence the equation (1.19.2) becomes,

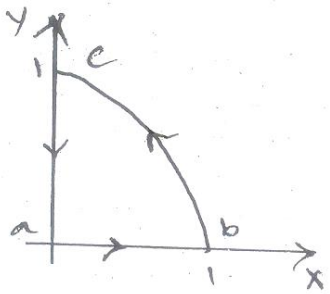
$$\therefore \quad \boxed{\oint_L \bar{F} \bullet d\bar{L} = \int_S \left(\nabla \times \bar{F}\right) \bullet d\bar{S}} \qquad \ldots (1.19.3)$$

Where dL = Perimeter of the total surface S

APPLICATION: The Stoke's theorem is applicable for the open surface enclosed by the given closed path. Any volume is a closed surface and hence application of Stoke's theorem to a closed surface which encloses certain volume, produces zero answer.

25. Given $\vec{A}$ =2r cosϕ $\vec{I}_r$ + r$\vec{I}_\phi$ in cylindrical co-ordinates for the contour shown in fig. Verify stokes' theorem.

Solution:



According to stoke's theorem, $\oint_c \vec{A}.\vec{dl} = \iint_s \nabla \times \vec{A}.\vec{ds}$

R.H.S:

ConvertingCylindrical to Cartesian co- ordinates,

$A_x = \frac{x}{\sqrt{x^2+y^2}} A_r + \frac{y}{\sqrt{x^2+y^2}} A_\phi$

$= \frac{x}{\sqrt{x^2+y^2}} 2r\cos\phi + \frac{y}{\sqrt{x^2+y^2}} r$

$A_x = 2x\cos\phi + y \qquad (\therefore r = \sqrt{x^2+y^2})$

Similarly,

$A_y = \frac{y}{\sqrt{x^2+y^2}} A_r + \frac{x}{\sqrt{x^2+y_2}} A_\phi = \frac{y}{\sqrt{x^2+y^2}} 2r\cos\phi + \frac{y}{\sqrt{x^2+y^2}} r$

$A_y = 2y\cos\phi + x$

$\vec{A} = A_x\vec{I}_x + A_y\vec{I}_y = (2x\cos\phi + y)\vec{I}_x + (2y\cos\phi + x)\vec{I}_y$

$$\nabla\times\vec{A} = \begin{vmatrix} \vec{I}_x & \vec{I}_y & \vec{I}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2x\cos\phi + y & 2y\cos\phi + x & 0 \end{vmatrix} = \vec{I}_x(0-0) - \vec{I}_y(0-0) + \vec{I}_z(1-1) = 0$$

$\Rightarrow \iint \nabla\times\vec{A} = 0$

L.H.S:

$\oint \vec{A}.\vec{dl} = \int_a^b A_{ab}.dl + \int_b^c A_{bc}.dl + \int_c^a A_{ca}.dl$

$\oint \vec{A}.\vec{dl} = \int_0^1 (2x\cos\phi + y)\,\vec{I}_x dx + \int_1^1 [(2x\cos\phi + y) + (2y\cos\phi + x)]\vec{I}_y]$

$$[\vec{I}_x dx+\vec{I}_y dy] + \int_1^0 (2y\cos\phi+x)\vec{I}_y dy$$

$= [x^2\cos\phi]_0^1 + [x^2\cos\phi + y^2\cos\phi]_1^1 + [y^2\cos\phi]_1^0$

$= \cos\phi + 0 - \cos\phi = 0$

$\Rightarrow \oint \vec{A}.\vec{dl} = 0$

From the above results, L.H.S = R.H.S.

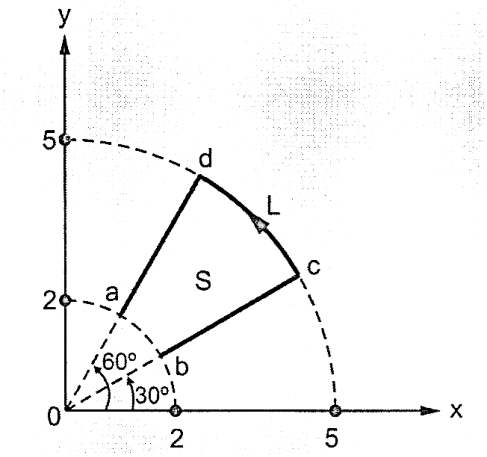
---

26. If $\vec{A}=\rho\cos\phi\vec{a}_\rho+\sin\phi\vec{a}_\phi$, evaluate $\oint_L \vec{A}.\vec{dl}$ around the path shown in fig. Confirm this using Stoke's theorem. (Apr 2010)

Using Stoke's theorem

$$\oint \vec{A}.\vec{dl} = \int_s (\nabla\times\vec{A}).\vec{ds}$$

LHS: $\oint_L \vec{A}.\vec{dl} = \left[\int_a^b + \int_b^c + \int_c^d + \int_d^a\right]\vec{A}.\vec{dl}$



Where, path L has been divided into segments ab,bc , cd and da.

Along ab, ρ =2 and dl = $\rho\, d\phi\, a_\phi$,

$$\int_a^b \vec{A}.\vec{dl} = \int_{\phi=60^\circ}^{30^\circ} \rho\sin\phi\, d\phi = 2(-\cos\phi)\Big]_{60^\circ}^{30^\circ} = -(\sqrt{3}-1)$$

Along bc,ϕ=30° and dl= $d\rho a_\rho$,

$$\int_b^c \vec{A}.\vec{dl} = \int_{\rho=2}^{5} \rho\cos\phi\, d\rho = \cos 30^\circ \frac{\rho 2}{2}\Big]_2^5 = \frac{21\sqrt{3}}{4}$$

Along cd, ρ =5,dl= $\rho\, d\phi a_\phi$

$$\int_c^d \vec{A}.\overrightarrow{dl} = \int_{\phi=30^\circ}^{60^\circ} \rho \sin\phi \, d\phi = 5\,(-\cos\phi)\Big|_{30^\circ}^{60^\circ} = \frac{5}{2}(\sqrt{3}-1)$$

Along da, $\phi=60^\circ$ and $dl = d\rho a_\rho$

$$\int_d^a \vec{A}.\overrightarrow{dl} = \int_{\rho=5}^{2} \rho \cos\phi \, d\rho = \cos 60^\circ \frac{\rho^2}{2}\Big|_5^2 = -\frac{21}{4}$$

Putting all these together results in

$$\oint \vec{A}.\overrightarrow{dl} = -\sqrt{3}+1+\frac{21\sqrt{3}}{4}+\frac{5\sqrt{3}}{2}-\frac{5}{2}-\frac{21}{4}=\frac{27}{4}(\sqrt{3}-1)=\mathbf{4.941 \rightarrow LHS}$$

RHS: $\int_s (\nabla\times\vec{A}).\overrightarrow{ds}$

But $\overrightarrow{ds} = \rho \, d\phi d\rho \, \vec{a}_z$

$$\nabla\times A=\vec{a}_\rho\left[\frac{1}{\rho}\frac{\partial A_z}{\partial\phi}-\frac{\partial A_\phi}{\partial z}\right]+\vec{a}_\phi\left[\frac{\partial A_\rho}{\partial z}-\frac{\partial A_z}{\partial\rho}\right]+\vec{a}_z\frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\phi)\frac{\partial A_\rho}{\partial\phi}\right]$$

$$=(0-0)\vec{a}_\rho+(0-0)\vec{a}_\phi+\frac{1}{\rho}(1+\rho)\sin\phi\vec{a}_z$$

Hence

$$\int_s (\nabla\times\vec{A}).\overrightarrow{ds} = \int_{\phi=30^\circ}^{60^\circ}\int_{\rho=2}^{5}\frac{1}{\rho}(1+\rho)\sin\phi \, d\rho \, d\phi$$

$$=\int_{30^\circ}^{60^\circ}\sin\phi d\phi\int_2^5(1+\rho)\,d\rho$$

$$=-\cos\phi\Big|_{30^\circ}^{60^\circ}\left(\rho+\frac{\rho^2}{2}\right)\Big|_5^2=\frac{27}{4}(\sqrt{3}-1)=\mathbf{4.941 \rightarrow RHS}$$

RHS = LHS. Hence Proved.

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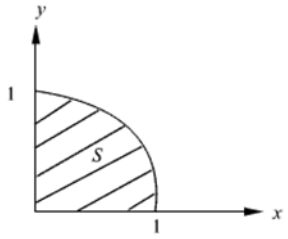
27. For $\vec{A} = 2r\cos\emptyset\overrightarrow{a_r} + r\overrightarrow{a_\emptyset}$ in the cylindrical coordinate for the contour shown in figure verify both side of the Stokes theorem.



Solution:

Given: $\vec{A} = 2r\cos\emptyset\overrightarrow{a_r} + r\overrightarrow{a_\emptyset}$

Assume $r = \rho \Rightarrow \vec{A} = 2\rho\, cos\emptyset \overrightarrow{a_\rho} + \rho \overrightarrow{a_\emptyset}$

Using Stokes Theorem $\oint_L \vec{A}.\overrightarrow{dl} = \iint_s \nabla \times \vec{A}.\overrightarrow{dS_n}$

RHS: $\iint \nabla \times \vec{A} \overrightarrow{ds_n}$

$$\nabla \times \vec{A} = \frac{1}{\rho}\begin{vmatrix} \overrightarrow{a_l} & \overrightarrow{\rho a_\emptyset} & \overrightarrow{a_z} \\ \partial/\partial\rho & \partial/\partial\emptyset & \partial/\partial z \\ A_x & \rho A_\emptyset & A_z \end{vmatrix}$$

$$= \frac{1}{\rho}\begin{vmatrix} \overrightarrow{a_l} & \overrightarrow{\rho a_\emptyset} & \overrightarrow{a_z} \\ \partial/\partial\rho & \partial/\partial\emptyset & \partial/\partial z \\ 2\rho cos\emptyset & \rho^2 & 0 \end{vmatrix}$$

$$= \frac{1}{\rho}\left[\left(\partial/\partial\emptyset\,(0) - \partial/\partial z\,(\rho)\right)\overrightarrow{a_\rho} - \left(\partial/\partial\rho\,(0) - \partial/\partial z\,(2\rho cos\emptyset)\right)\overrightarrow{a_\emptyset} + \left(\left(\partial/\partial\rho\,(\rho) - \partial/\partial\emptyset\,(2\,\rho cos\emptyset)\right)\overrightarrow{a_z}\right)\right]$$

$$= \frac{1}{\rho}\left[0\overrightarrow{a_\rho} - 0\vec{a}_\emptyset + \overrightarrow{a_z}(2\rho + 2sin\emptyset)\right]$$

$$= \frac{1}{\rho}[2\rho(1 + sin\emptyset)\overrightarrow{a_z}] = \quad (2 + 2sin\emptyset)\overrightarrow{a_z}$$

$$\Rightarrow \iint_s \nabla \times \vec{A}.\overrightarrow{ds_z} = \int_{\emptyset=0}^{\pi/2}\int_{\rho=0}^{1}(2\rho + 2sin\emptyset)d\rho d\emptyset$$

$$= \int_{\emptyset=0}^{\pi/2}\left[\frac{2\rho^2}{2} + \frac{2\rho^2}{2}sin\emptyset\right]_0^1 d\emptyset = \int_{\emptyset=0}^{\pi/2}[1 + (1)sin\emptyset]\,d\emptyset$$

$$= \quad [\emptyset - 2cos\emptyset]_0^{\pi/2} \quad = \pi/2 - 2cos\,\pi/2 - 0 + (cos0)$$

$$= \pi/2 + 2 = \quad \boldsymbol{\pi/2 + 1} \rightarrow \textbf{RHS}$$

LHS: $\oint_l \vec{A}.\overrightarrow{dl}$

$$\oint_l \vec{A}.\overrightarrow{dl} = \int_{0a} \vec{A}.\overrightarrow{\underset{(\overrightarrow{a_l})}{dl}} + \oint_{ab} \vec{A}.\overrightarrow{\underset{(\overrightarrow{a_\emptyset})}{dl}} + \oint_{b0} \vec{A}.\overrightarrow{\underset{(-\overrightarrow{a_l})}{dl}}$$

$$= \int \left(2\rho\, cos\emptyset\overrightarrow{a_\rho} + \rho\overrightarrow{a_\emptyset}\right).d\rho\overrightarrow{a_\rho} + \int \left(2\rho cos\emptyset\overrightarrow{a_\rho} + \rho\overrightarrow{a_\emptyset}\right).\rho d\emptyset\overrightarrow{a_\emptyset} + \int \left(2\rho\, cos\emptyset\overrightarrow{a_\rho} + \rho\overrightarrow{a_\emptyset}\right).d\rho\left(-\vec{a}_\rho\right)$$

$$= 2\rho\, cos\emptyset d\rho + \int_{\emptyset=0}^{\pi/2}\rho^2 d\emptyset - \int_{\rho=0}^{1} 2\rho\, cos\emptyset d\rho$$

$$= \left[\frac{2\rho^2}{2}\right]_0^1 + [\emptyset]_0^{\pi/2} - 0 \qquad = (1 - 0) + \pi/2 - 0 \quad = \boldsymbol{1 + \frac{\pi}{2}} \rightarrow \textbf{LHS}$$

28. Verify Stokes theorem for a vector field $\overline{F} = r^2 cos \ \emptyset \, \overline{a}_r + zsin \, \emptyset \overline{a}_z$ around the path L defined by $0 \leq r \leq 3, 0 \ \leq \emptyset \leq 45°$ *and* $z = 0$ *as shown in the below figure*.

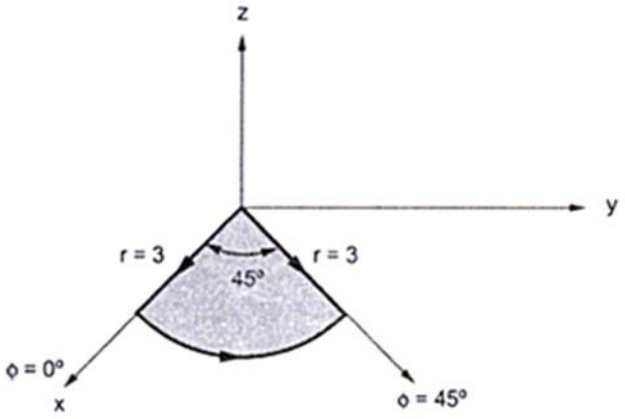
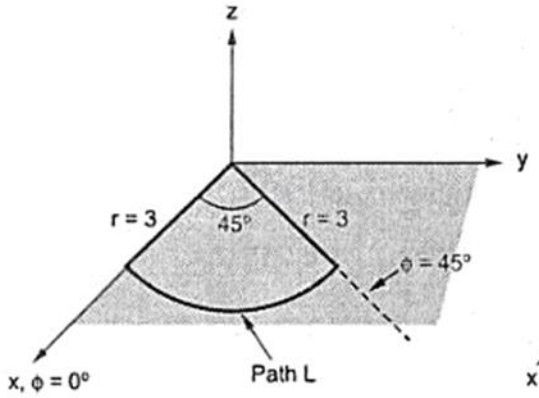


Fig. 1.52

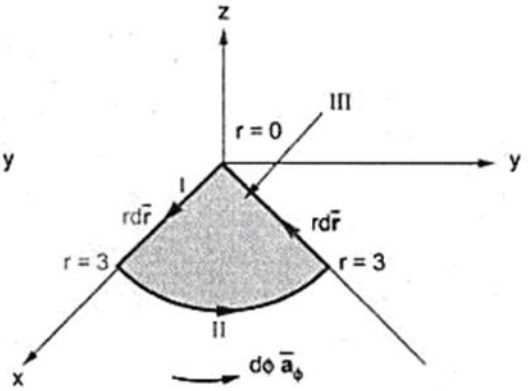
Solution : From Stoke's theorem,

$$\oint_L \overline{F} \cdot d\overline{L} = \int_S (\nabla \times \overline{F}) \cdot d\overline{S}$$

L.H.S:



(a)



(b)

Divide the given path L into three sections.

Section I : r varies from 0 to 3, $\phi = 0°$ and $z = 0$

$\therefore \quad d\overline{l} = dr \, \overline{a}_r$ ... Along radial direction

$$\therefore \quad \int_1 \overline{F} \cdot d\overline{l} = \int_{r=0}^{3} (r^2 \cos \phi \ \overline{a}_r + z \sin \phi \, \overline{a}_z) \cdot dr \, \overline{a}_r$$

$$= \int_{r=0}^{3} r^2 \cos \phi \ dr \quad \ldots \ \overline{a}_r \cdot \overline{a}_r = 1, \ \overline{a}_z \cdot \overline{a}_r = 0$$

$$= \left[\frac{r^3}{3}\right]_0^3 \cos 0° = \left[\frac{27}{3}\right][1] = 9$$

$$\therefore \quad \int_{II} \bar{F} \cdot d\bar{l} = \int_{\phi=0}^{45^\circ} (r^2 \cos\phi \ \bar{a}_r + z \sin\phi \ \bar{a}_z) \cdot d\phi \ \bar{a}_\phi$$

$$= 0 \qquad \ldots \bar{a}_r \cdot \bar{a}_\phi = \bar{a}_z \cdot \bar{a}_\phi = 0$$

Section III : r varies from 3 to 0, $\phi = 45^\circ$ and $z = 0$

$$d\bar{l} = dr \ \bar{a}_r$$

Note that $d\bar{l}$ is always positive, limits of integration from r = 3 to 0 taking care of direction.

$$= \int_{r=3}^{0} r^2 \cos\phi \ dr \qquad \ldots \bar{a}_r \cdot \bar{a}_r = 1, \bar{a}_z \cdot \bar{a}_r = 0$$

$$= \cos 45^\circ \left[\frac{r^3}{3}\right]_3^0 = 0.7071 \left[\frac{-27}{3}\right] = -6.3639$$

$$\therefore \quad \oint_L \bar{F} \cdot d\bar{l} = 9 + 0 - 6.3639 = \mathbf{2.636}$$

To evaluate R.H.S., find $\nabla \times \mathbf{F}$

$$\nabla \times \bar{F} = \left[\frac{1}{r}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right]\bar{a}_r + \left[\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right]\bar{a}_\phi + \left[\frac{1}{r}\frac{\partial (r F_\phi)}{\partial r} - \frac{1}{r}\frac{\partial F_r}{\partial \phi}\right]\bar{a}_z$$

$$F_r = r^2 \cos\phi, \quad F_\phi = 0, \quad F_z = z \sin\phi$$

$$\therefore \quad \nabla \times \bar{F} = \left[\frac{1}{r} \times 0 - 0\right]\bar{a}_r + [0 - 0]\ \bar{a}_\phi + \left[\frac{1}{r}(0) - \frac{1}{r}(r)^2(-\sin\phi)\right]\bar{a}_z$$

$$= r \sin\phi \ \bar{a}_z$$

$d\bar{S} = r \ dr \ d\phi \ \bar{a}_z$ as surface is in x-y plane i.e. z = 0 plane for which normal direciton is $\bar{a}_z$.

$$\therefore \int_S (\nabla \times \bar{F}) \cdot d\bar{S} = \int_S (r \sin\phi \bar{a}_z) \cdot (r \ dr \ d\phi)\bar{a}_z = \int_{\phi=0}^{45^\circ} \int_{r=0}^{3} r^2 \sin\phi \ dr \ d\phi = \left[\frac{r^3}{3}\right]_0^3 [-\cos\phi]_0^{45^\circ}$$

$$= [9]\ [-0.707 - (-1)] = 9 \times 0.2928 = \mathbf{2.636}$$

Thus **Stoke's theorem is verified.**

29. State and prove coulombs Law.(Dec 2015)

Coulombs law :

Charles coulomb performed an elaborate series of experiments to determine quantitatively the force exerted between two objects each having a static change

Coulomb stated that

The force between two very small object separated by a distance which is large compared to their size is proportional to the change to the change on each and inversely proportional to the square of the distance between them

A small charged particle which negligible dimension is called a point change.the coulomb's law refers to the force of interaction between two point charges. The force is repulsive when both the charges are like and attractive when the charges are unlike the coulomb's law can be stated that.

The force of attraction or repulsion between any two point charges is directly proportional to the product of two change and inversely proportional to the square of the distance between them.

Consider the point charges $Q_1$and Q_2 separated by a distance 'r'. the force of interaction between two point charges is given as follows:

$$F \propto \frac{Q_1 Q_2}{r^2}$$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2} \text{Newtons}$$

Where ε is permittivity of the medium or dielectric constant which is written as

$\varepsilon = \varepsilon_0 \varepsilon_r$ Farads/meter

Relative permittivity of the medium $\varepsilon_r \frac{\varepsilon}{\varepsilon_0}$

$$\varepsilon_0 = \frac{permittivity of the medium}{permittivity of the free space (or) vacuum}$$

Permittivity of the free space or vacuum $\varepsilon_0 = \frac{1}{36\pi \times 10^9} 8.854 \times 10^{-12} F/m$

Vector Form of Coulomb's Law

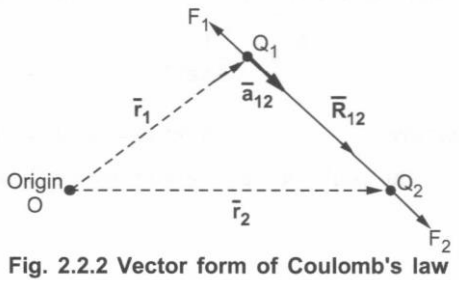


Fig. 2.2.2 Vector form of Coulomb's law

$$\bar{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\bar{a}_{12} \qquad \ldots (2.2.9)$$

where $\bar{a}_{12}$ = Unit vector along $\bar{R}_{12} = \dfrac{\text{Vector}}{\text{Magnitude of vector}}$

$$\therefore \quad \bar{a}_{12} = \frac{\bar{R}_{12}}{|\bar{R}_{12}|} = \frac{\bar{r}_2 - \bar{r}_1}{|\bar{R}_{12}|} = \frac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|} \qquad \ldots (2.2.10)$$

where $|\bar{R}_{12}| = R$ = Distance between the two charges

Example 2.2.1 *Two point charges of 0.7 mC and 4.9 μC are situated in free space at (2, 3, 6) and (0, 0, 0). Calculate the force acting on the 0.7 mC charge.* **Dec.-08, Marks 8**

Solution : The charges are shown in the Fig. 2.2.5.

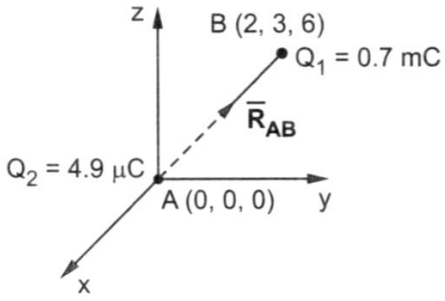


Fig. 2.2.5

$$\bar{F}_{21} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{AB}^2}\bar{a}_{AB}$$

$$\bar{R}_{AB} = (2-0)\bar{a}_x + (3-0)\bar{a}_y + (6-0)\bar{a}_z$$

$$= +2\bar{a}_x + 3\bar{a}_y + 6\bar{a}_z$$

$$|\bar{R}_{AB}| = \sqrt{4+9+36} = 7$$

$$\bar{a}_{AB} = \frac{\bar{R}_{AB}}{|\bar{R}_{AB}|} = \frac{+2\bar{a}_x + 3\bar{a}_y + 6\bar{a}_z}{7}$$

$$\therefore \quad \bar{F}_{12} = \frac{0.7 \times 10^{-3} \times 4.9 \times 10^{-6}}{4\pi \times 8.854 \times 10^{-12} \times 7^2}\left[\frac{2\bar{a}_x + 3\bar{a}_y + 6\bar{a}_z}{7}\right]$$

$$= \mathbf{0.1797\,\bar{a}_x + 0.2696\,\bar{a}_y + 0.5392\,\bar{a}_z\ N}$$

$$|\bar{F}_{12}| = \sqrt{(0.1797)^2 + (0.2696)^2 + (0.5392)^2} = \mathbf{0.6291\ N} \quad \ldots \text{Magnitude}$$

30. Three point charges in free space are located as follows : 50nC at(0,0)m; 40nC at(3,0)m; -60nC at(0,4)m; Find the electric field intensity at(3,4)m; (May 2016)(Nov/Dec 2017)

Solution:

Q_1 = 50nC at(0,0)m;

Q_2 = 40nC at(3,0)m;

Q_3 = -60nC at (0,4)m;

The field E_1 at `P' due to the point at A

$$E1 = \frac{Q_1}{4\pi\mathcal{E}r^2}\overrightarrow{a_{12}}$$

$$= \frac{50 * 10 - 9}{4\pi * 8.854 \times 10 - 12 * 5}\left[\frac{3\overrightarrow{a_x} + 4\overrightarrow{a_y}}{5}\right]\left[\therefore \overrightarrow{a_{12}} = \frac{r_2 - r_1}{\sqrt{{r_2}^2 - {r_1}^2}}\right]$$

$$E1 = 17.975[3\overrightarrow{a_x} + 4\overrightarrow{a_y}]$$

The field E_1 at `P' due to the point at B

$$E1 = \frac{Q_1}{4\pi\mathcal{E}r^2}\overrightarrow{a_{12}}$$

$$= \frac{40 * 10 - 9}{4\pi * 8.854 \times 10 - 12 * 4}\left[\frac{4\overrightarrow{a_y}}{4}\right]$$

$$\left[\therefore \overrightarrow{a_{12}} = \frac{r_2 - r_1}{\sqrt{{r_2}^2 - {r_1}^2}}\right]$$

$$E2 = 22.46[\overrightarrow{4a_y}]$$

The field E_1 at `P' due to the point at C

$$E1 = \frac{Q_1}{4\pi\mathcal{E}r^2}\overrightarrow{a_{12}}$$

$$= \frac{-60 * 10 - 9}{4\pi * 8.854 \times 10 - 12 * 5}\left[\frac{3\overrightarrow{a_x} - 4\overrightarrow{a_y}}{5}\right]$$

$$\left[\therefore \overrightarrow{a_{12}} = \frac{r_2 - r_1}{\sqrt{{r_2}^2 - {r_1}^2}}\right]$$

$$E1 = -21.57[3\overrightarrow{a_x} - 4\overrightarrow{a_y}]$$

Electric field intensity at P (3,4)

$E = E_1+E_2+E_3$

$= 17.975[3\overrightarrow{a_x} + 4\overrightarrow{a_y}] + 22.46[\overrightarrow{4a_y}] + 21.57[3\overrightarrow{a_x} - 4\overrightarrow{a_y}]$

$\overrightarrow{E}$ = -10.785$\overrightarrow{a_x}$ + 75.45 $\overrightarrow{a_y}$

31. A Charge Q1 =100nC is located in vacuum at P1 (-0.03,0.01,0.04)m. Find the force on Q1 due to (i) Q2 = 120μC at P2(0.03,0.08,-0.02)m (ii) Q3 = 120μC at P3(-0.09,-0.06,0.10) (iii) Q2 and Q3.(May 2016)

Solution:

The force on Q1 due to Q2

$$F_{21} = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2}$$

$$\therefore r = \sqrt{(0.03-(-0.03))^2 + (0.08-0.01)^2 + (-0.02-0.04)^2}$$

$$= 0.11$$

$$= \frac{(120\times10-6)\times(100\times10-9)}{4\pi*8.854*10^{-12}*0.11^2}$$

$$F_{21} = 8.913N$$

The force on Q1 due to Q3

$$F_{21} = \frac{Q_1 Q_3}{4\pi\varepsilon_o r^2}$$

$$\therefore r = \sqrt{(-0.09-(-0.03))^2 + (-0.06-0.01)^2 + (-0.10-0.04)^2}$$

$$= 0.11$$

$$= \frac{(120\times10-6)\times(100\times10-9)}{4\pi*8.854*10^{-12}*0.11^2}$$

$$F_{21} = 8.913$$

The force on Q1 due to Q2 and Q3

$$F = F1 + F2$$

$$= 17.828 \text{ N.}$$

---

32. Find the force in Newton on charge Q_1=20 μc situated at (0,1,2) m due to charge Q_2=-300μC situated at (2,0,0) m. (Nov/Dec 2016)

Solution:

$$\vec{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}} \vec{a}_{12}$$

$$\vec{F}_{21} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{21}} \vec{a}_{21}$$

$$\vec{R}_{21} = \vec{R}_1 - \vec{R}_2 = (0,1,2)-(2,0,0) = -2\vec{a}_x + \vec{a}_y + 2\vec{a}_z$$

$$\left|\vec{R}_{21}\right| = \sqrt{(2^2 + 1^2 + 2^2}\ = 3$$

$$\vec{a}_{21} = \frac{-2\vec{a}_x + \vec{a}_y + 2\vec{a}_z}{3}$$

$$\vec{F}_1 = \frac{20\times10^{-6}\times(-300\times10^{-6})}{4\pi\times8.854\times10^{-2}\times3^2}\cdot\frac{(-2\vec{a}_x + \vec{a}_y + 2\vec{a}_z)}{3} = 4\vec{a}_x - 2\vec{a}_y - 4\vec{a}_z \quad \text{N}$$

$$\left|\vec{F}1\right| = \sqrt{(4^2 + 2^2 + 4^2}\ = \mathbf{6N}$$

33. A charge 20μC is located at A(-6,4,7) and a second charge 50μC is at B(5,8,-2) in free space. If distance are given in metres, determine the vector force excerted by the fisrst charge on the second one.(Nov/Dec 2017)

$$Q_1 = 20\mu C = 20\times10^{-6}C$$
$$Q_2 = 50\mu C = 50\times10^{-6}C$$

A(-6,4,7) and a second charge 50μC is at B(5,8,-2)

$$r_1 = -6\vec{a}x + 4\vec{a}y + 7\vec{a}z$$
$$r_2 = 5\vec{a}x + 8\vec{a}y - 2\vec{a}z$$

$$\boldsymbol{r} = r_2 - r_1 = 11\vec{a}x + 4\vec{a}y - 9\vec{a}z$$
$$|r| = \sqrt{\mathbf{121 + 16 + 81}} = \mathbf{14.76}$$

Unit vector$= \frac{\boldsymbol{r}}{|r|} = \frac{11\vec{a}x + 4\vec{a}y - 9\vec{a}z}{\mathbf{14.76}}$

$$= \frac{20\times10^{-6}\times50\times10^{-6}}{\mathbf{4\pi\times8.85\times10^{-12}\times217.8}}\cdot\frac{11\vec{a}x + 4\vec{a}y - 9\vec{a}z}{\mathbf{14.76}}$$
$$= \frac{\mathbf{1000}}{\mathbf{24222}}\frac{11\vec{a}x + 4\vec{a}y - 9\vec{a}z}{\mathbf{14.76}}$$
$$= \mathbf{0.0028}11\vec{a}x + 4\vec{a}y - 9\vec{a}z$$

Electric Field Intensity due to Various Charge Distributions

$$\bar{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\bar{a}_R$$

$\bar{E}$ due to Line Charge

$$\therefore \quad \bar{E} = \int_L \frac{\rho_L \, dl}{4\pi\varepsilon_0 R^2}\bar{a}_R$$

E due to Surface Charge

$$\therefore \quad \bar{E} = \int_S \frac{\rho_S \, dS}{4\pi\varepsilon_0 R^2}\bar{a}_R$$

E due to Volume Charge

$$\therefore \quad \bar{E} = \int_{Vol} \frac{\rho_v \, dv}{4\pi\varepsilon_0 R^2}\bar{a}_R$$

34. Find the Electric Field due to Infinite Line Charge.

Consider an infinitely long straight line carrying uniform line charge having density $\rho_L C/m$. Let this line lies along z-axis from $-\infty$ *to* $+\infty$to 00 and hence called **infinite line** charge. Let point P is on y-axis at which electric field intensity is to be determined. The distance of point P from the origin is 'r' as shown in the Fig. 2.6.1.

Consider a small differential length *dl* carrying a charge dQ, along the line as shown
It is along z-axis hence *dl* = dz.

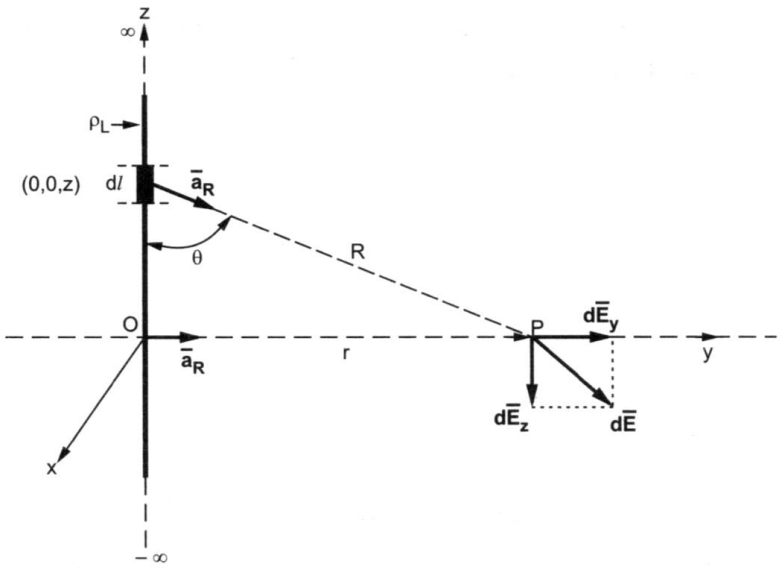


Fig. 2.6.1 Field due to infinite line charge

$$\therefore \quad dQ = \rho_L \, dl = \rho_L \, dz \quad \ldots (2.6.1)$$

The co-ordinates of dQ are (0, 0, z) while the co-ordinates of point P are (0, r, 0). Hence the distance vector $\bar{R}$ can be written as,

$$\bar{R} = \bar{r}_P - \bar{r}_{dl} = \left[r\bar{a}_y - z\bar{a}_z \right]$$

$$\therefore \quad |\bar{R}| = \sqrt{r^2 + z^2}$$

$$\therefore \quad \bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{r\bar{a}_y - z\bar{a}_z}{\sqrt{r^2 + z^2}} \quad \ldots (2.6.2)$$

$$\therefore \quad d\bar{E} = \frac{dQ}{4\pi\varepsilon_0 R^2} \bar{a}_R$$

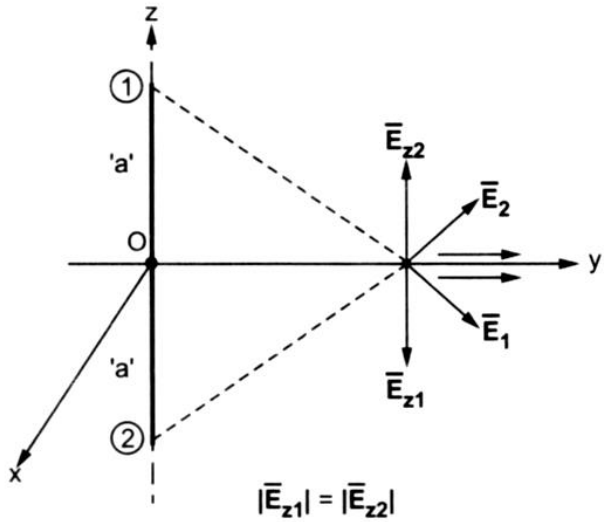


Fig. 2.6.2

$$= \frac{\rho_L \, dz}{4\pi\varepsilon_0 \left(\sqrt{r^2 + z^2} \right)^2} \left[\frac{r\bar{a}_y - z\bar{a}_z}{\sqrt{r^2 + z^2}} \right] \quad \ldots (2.6.3)$$

Note For every charge on positive z-axis there is equal charge present on negative z-axis. Hence the z component of electric field intensities produced by such charges at point P will cancel each other. Hence effectively there will not be any z component of $\bar{E}$ at P. This is shown in the Fig. 2.6.2.

Hence the equation of $d\bar{E}$ can be written by eliminating $\bar{a}_z$ component,

$$\therefore \quad d\bar{E} = \frac{\rho_L \, dz}{4\pi\varepsilon_0 \left(\sqrt{r^2 + z^2} \right)^2} \frac{r\bar{a}_y}{\sqrt{r^2 + z^2}} \quad \ldots (2.6.4)$$

Now by integrating $d\bar{E}$ over the z axis from $-\infty$ to ∞ we can obtain total $\bar{E}$ at point P.

$$\therefore \quad \bar{E} = \int_{-\infty}^{\infty} \frac{\rho_L}{4\pi\varepsilon_0 \left(r^2 + z^2 \right)^{3/2}} r \, dz \, \bar{a}_y$$

Note For such an integration, use the substitution

$$z = r \tan\theta \quad \text{i.e.} \quad r = \frac{z}{\tan\theta}$$

$$\therefore \quad dz = r \sec^2\theta \, d\theta$$

Here r is not the variable of integration.

For $z = -\infty$, $\theta = \tan^{-1}(-\infty) = -\pi/2 = -90°$

For $z = +\infty$, $\theta = \tan^{-1}(\infty) = \pi/2 = +90°$

$$\therefore \quad \bar{E} = \int_{\theta=-\pi/2}^{\pi/2} \frac{\rho_L}{4\pi\varepsilon_0\left[r^2 + r^2\tan^2\theta\right]^{3/2}} r \times r\sec^2\theta \, d\theta \, \bar{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{r^2\sec^2\theta \, d\theta}{r^3\left[1+\tan^2\theta\right]^{3/2}} \bar{a}_y$$

But $1 + \tan^2\theta = \sec^2\theta$

$$\therefore \quad \bar{E} = \frac{\rho_L}{4\pi\varepsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta \, d\theta}{r\sec^3\theta} \bar{a}_y$$ } changing the limits

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta \, \bar{a}_y \qquad \ldots \sec\theta = \frac{1}{\cos\theta}$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r} \left[\sin\theta\right]_{-\pi/2}^{\pi/2} \bar{a}_y = \frac{\rho_L}{4\pi\varepsilon_0 r}\left[\sin\frac{\pi}{2} - \sin\left(\frac{-\pi}{2}\right)\right] \bar{a}_y$$

$$= \frac{\rho_L}{4\pi\varepsilon_0 r}\left[1-(-1)\right] \bar{a}_y = \frac{\rho_L}{4\pi\varepsilon_0 r} \times 2 \, \bar{a}_y$$

$$\therefore \quad \boxed{\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r} \bar{a}_y \ \text{V/m}} \qquad \ldots (2.6.5)$$

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r} \bar{a}_r \ \text{V/m}} \qquad \ldots (2.6.6)$$

35. Find the Electric Field due to Charged Circular Ring.

Consider a charged circular ring of radius r placed in xy plane with centre at origin, carrying a charge uniformly along its circumference. The charge density is ρ_L C/m .

The point P is at a perpendicular distance 'z' from the ring as shown in the Fig. 2.7.1.

Consider a small differential length dl on this ring. The charge on it is dQ.

$$\therefore \quad dQ = \rho_L \, dl$$

$$\therefore \quad d\bar{E} = \frac{\rho_L \, dl}{4\pi\varepsilon_0 R^2} \bar{a}_R \qquad \ldots (2.7.1)$$

where R = Distance of point P from dl

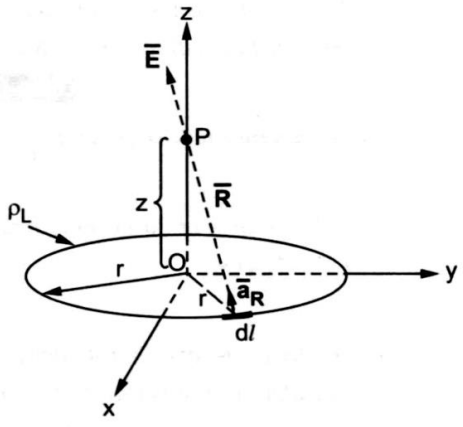


Fig. 2.7.1

Consider the cylindrical coordinate system. For dl we are moving in ϕ direction where $dl = r\,d\phi$.

$$\therefore \quad dl = r\,d\phi \qquad \ldots (2.7.2)$$

Now $R^2 = r^2 + z^2$ ... from Fig. 2.7.1

While $\bar{R}$ can be obtained from its two components, in cylindrical system as shown in the Fig. 2.7.1 (a). The two components are,

1. Distance r in the direction of $-\bar{a}_r$, radially inwards i.e. $-r\bar{a}_r$.
2. Distance z in the direction of $\bar{a}_z$ i.e. $z\,\bar{a}_z$

$$\therefore \quad \bar{R} = -r\bar{a}_r + z\bar{a}_z \qquad \ldots (2.7.3)$$

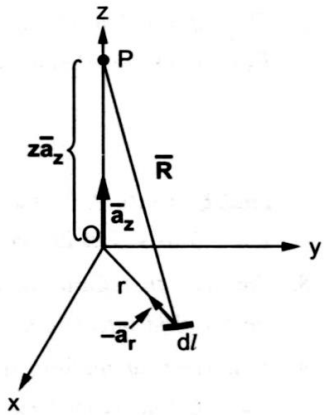


Fig. 2.7.1(a)

Key Point *This method can be used conveniently to obtain $\bar{R}$ by identifying its components in the direction of unit vectors in the co-ordinate system considered.*

$$\therefore \quad |\bar{R}| = \sqrt{(-r)^2 + (z)^2} = \sqrt{r^2 + z^2} \qquad \ldots (2.7.4)$$

$$\therefore \quad \bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{-r\bar{a}_r + z\bar{a}_z}{\sqrt{r^2 + z^2}} \qquad \ldots (2.7.5)$$

$$\therefore \quad d\bar{E} = \frac{\rho_L \, dl}{4\pi\varepsilon_0 \left(\sqrt{r^2 + z^2}\right)^2} \times \frac{-r\bar{a}_r + z\bar{a}_z}{\sqrt{r^2 + z^2}}$$

$$\therefore \quad d\bar{E} = \frac{\rho_L (r\,d\phi)}{4\pi\varepsilon_0 \left(r^2 + z^2\right)^{3/2}} [-r\bar{a}_r + z\bar{a}_z] \qquad \ldots (2.7.6)$$

Hence neglecting $\bar{a}_r$ component from $d\bar{E}$ we get,

$$d\bar{E} = \frac{\rho_L (r\, d\phi)}{4\pi\varepsilon_0 \left(r^2 + z^2\right)^{3/2}} z\bar{a}_z \quad \ldots (2.7.7)$$

$$\therefore \bar{E} = \int_{\phi=0}^{2\pi} \frac{\rho_L\, r\, d\phi}{4\pi\varepsilon_0 \left(r^2 + z^2\right)^{3/2}} z\bar{a}_z$$

$$= \frac{\rho_L\, r}{4\pi\varepsilon_0 \left(r^2 + z^2\right)^{3/2}} z\, \bar{a}_z \, [\phi]_0^{2\pi}$$

... Integration w.r.t. ϕ

$$\therefore \quad \boxed{\bar{E} = \frac{\rho_L\, r\, z}{2\varepsilon_0 \left(r^2 + z^2\right)^{3/2}} \bar{a}_z}$$

36. Find the Electric Field due to Infinite Sheet of Charge.

Consider an infinite sheet of charge having uniform charge density ρ_S C/m^2, placed in xy plane as shown in the Fig. 2.8.1. Let us use cylindrical coordinates.

The point P at which $\bar{E}$ to be calculated is on z-axis.

Consider the differential surface area dS carrying a charge dQ. The normal direction to dS is z direction hence dS normal to z direction is r dr dϕ .

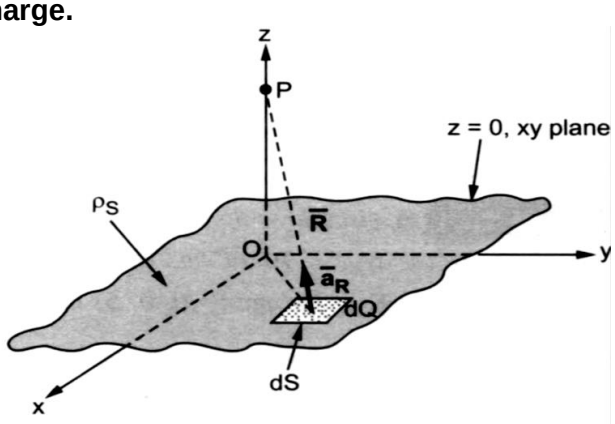


Fig. 2.8.1

Now $dQ = \rho_S\, dS = \rho_S\ r\, dr\, d\phi$ ... (2.8.1)

Hence, $d\bar{E} = \frac{dQ}{4\pi\varepsilon_0 R^2} \bar{a}_R = \frac{\rho_S\, r\, dr\, d\phi}{4\pi\varepsilon_0 R^2} \bar{a}_R$

... (2.8.2)

The distance vector $\bar{R}$ has two components as shown in the Fig. 2.8.2.

1. The radial component r along $-\bar{a}_r$ i.e. $-\,r\,\bar{a}_r$.
2. The component z along $\bar{a}_z$ i.e. $z\,\bar{a}_z$.

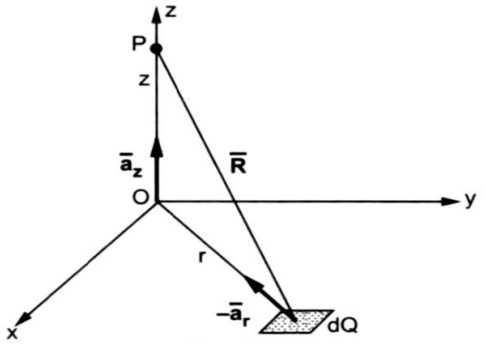


Fig. 2.8.2

With these two components $\bar{R}$ can be obtained from the differential area towards point P as,

$$\therefore \quad \bar{R} = -r\bar{a}_r + z\bar{a}_z \qquad \ldots (2.8.3)$$

$$\therefore \quad |\bar{R}| = \sqrt{(-r)^2 + (z)^2} = \sqrt{r^2 + z^2} \qquad \ldots (2.8.4)$$

$$\therefore \quad \bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{-r\bar{a}_r + z\bar{a}_z}{\sqrt{r^2 + z^2}} \qquad \ldots (2.8.5)$$

$$\therefore \quad d\bar{E} = \frac{\rho_S \, r \, dr \, d\phi}{4\pi\varepsilon_0 \left(\sqrt{r^2 + z^2}\right)^2} \left[\frac{-r\bar{a}_r + z\bar{a}_z}{\sqrt{r^2 + z^2}}\right]$$

$$\therefore \quad \bar{E} = \int_{\phi=0}^{2\pi} \int_{r=0}^{\infty} d\bar{E} = \int_0^{2\pi} \int_0^{\infty} \frac{\rho_S \, r \, dr \, d\phi}{4\pi\varepsilon_0 \left(r^2 + z^2\right)^{3/2}} (z\bar{a}_z)$$

Put $r^2 + z^2 = u^2$ hence $2r\,dr = 2u\,du$

For $r = 0$, $u = z$ and $r = \infty$, $u = \infty$ ... Changing limits

$$\therefore \quad \bar{E} = \int_0^{2\pi} \int_{u=z}^{\infty} \frac{\rho_S}{4\pi\varepsilon_0} \frac{u\,du}{\left(u^2\right)^{3/2}} d\phi \; z\,\bar{a}_z$$

$$= \int_0^{2\pi} \int_{u=z}^{\infty} \frac{\rho_S}{4\pi\varepsilon_0} \frac{du}{u^2} d\phi \; (z\,\bar{a}_z)$$

$$= \int_0^{2\pi} \frac{\rho_S}{4\pi\varepsilon_0} d\phi \; z\,\bar{a}_z \left[-\frac{1}{u}\right]_z^{\infty} \qquad \ldots \text{as} \int \frac{1}{u^2} = \int u^{-2} = \frac{u^{-1}}{-1} = -\frac{1}{u}$$

$$= \frac{\rho_S}{4\pi\varepsilon_0} [\phi]_0^{2\pi} \; (z\bar{a}_z) \left[-\frac{1}{\infty} - \left(-\frac{1}{z}\right)\right] = \frac{\rho_S}{4\pi\varepsilon_0} (2\pi)\,\bar{a}_z$$

$$\therefore \quad \bar{E} = \frac{\rho_S}{2\varepsilon_0} \bar{a}_z \ \text{V/m} \qquad \ldots \text{For points above xy plane}$$

37. Find the force on a point charge q located at (0, 0, h) m due to charge of Surface charge density ρ_S C/m^2 uniformly distributed over the circular disc r <= a, z=0m. Also find electric field intensity at the same point.

Solution : The charges are shown in the Fig. 2.8.4.

Consider the differential area dS carrying the charge dQ. The normal direction to dS is $\bar{a}_z$ hence

$$dS_z = r\, dr\, d\phi$$

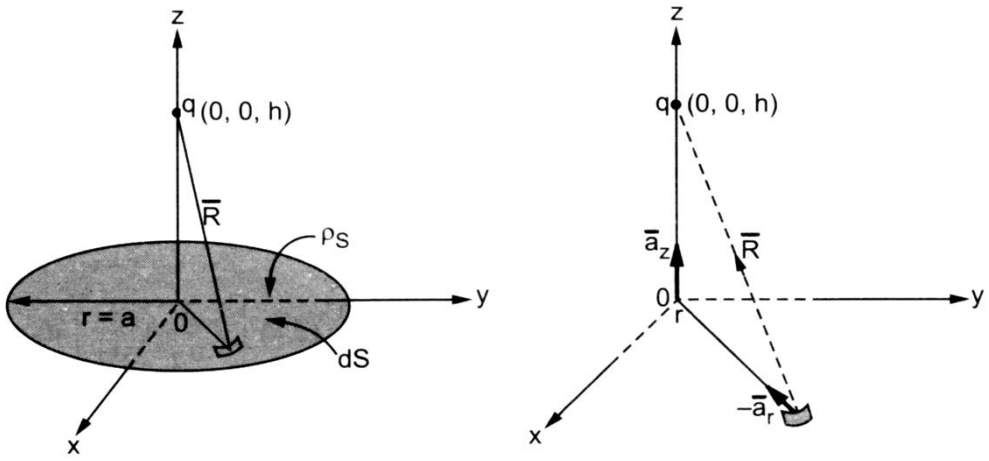


Fig. 2.8.4

$$\therefore \quad dQ = \rho_S\, dS = \rho_S\, r\, dr\, d\phi$$

Thus the force on a point charge q due to dQ is,

$$d\bar{F} = \frac{q\, dQ}{4\pi\varepsilon_0\, R^2}\, \bar{a}_R$$

The $\bar{R}$ can be splitted as shown in the Fig. 2.8.4 (b),

1. The component along $-\bar{a}_r$ having length r i.e. $-r\bar{a}_r$.
2. The component z = h along $\bar{a}_z$ i.e. $h\,\bar{a}_z$.

$$\therefore \quad \bar{R} = -r\bar{a}_r + h\bar{a}_z$$

$$\therefore \quad |\bar{R}| = \sqrt{(-r)^2 + h^2} = \sqrt{r^2 + h^2}$$

$$\therefore \quad \bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{-r\bar{a}_r + h\bar{a}_z}{\sqrt{r^2 + h^2}}$$

$$\therefore \quad d\bar{F} = \frac{dQ\, q}{4\pi\varepsilon_0 \left(\sqrt{r^2 + h^2}\right)^2} \left[\frac{-r\bar{a}_r + h\bar{a}_z}{\sqrt{r^2 + h^2}}\right]$$

$$\therefore \quad \bar{F} = \int d\bar{F} = \int_{\phi=0}^{2\pi} \int_{r=0}^{a} \frac{\rho_S \, r \, dr \, d\phi \, q}{4\pi\varepsilon_0 \left(r^2 + h^2\right)^{3/2}} \left(h\bar{a}_z\right)$$

Put $r^2 + h^2 = u^2$ hence $2r\,dr = 2u\,du$ and for $r = 0$, $u_1 = h$ while $r = a$, $u_2 = \sqrt{a^2 + h^2}$

$$\therefore \quad \bar{F} = \int_{\phi=0}^{2\pi} \int_{u=u_1}^{u_2} \frac{\rho_S \, q}{4\pi\varepsilon_0} \frac{u \, du}{\left(u^2\right)^{3/2}} \, d\phi \; h \, \bar{a}_z$$

$$= \frac{\rho_S \, h \, q}{4\pi\varepsilon_0} \int_{\phi=0}^{2\pi} \int_{u_1}^{u_2} \frac{1}{u_2} \, du \, d\phi \; \bar{a}_z \qquad \ldots \int \frac{1}{u^2} = \int u^{-2} = -\frac{1}{u}$$

$$= \frac{\rho_S \, h \, q}{4\pi\varepsilon_0} [\phi]_0^{2\pi} \left[-\frac{1}{u}\right]_{u_1}^{u_2} \bar{a}_z = \frac{q\rho_S \, h}{4\pi\varepsilon_0} (2\pi) \left[-\frac{1}{u_2} + \frac{1}{u_1}\right] \bar{a}_z$$

$$= \frac{\rho_S \, q \, h}{2\varepsilon_0} \left[-\frac{1}{\sqrt{a^2 + h^2}} + \frac{1}{h}\right] \bar{a}_z \; \text{N}$$

The electric field intensity is given by,

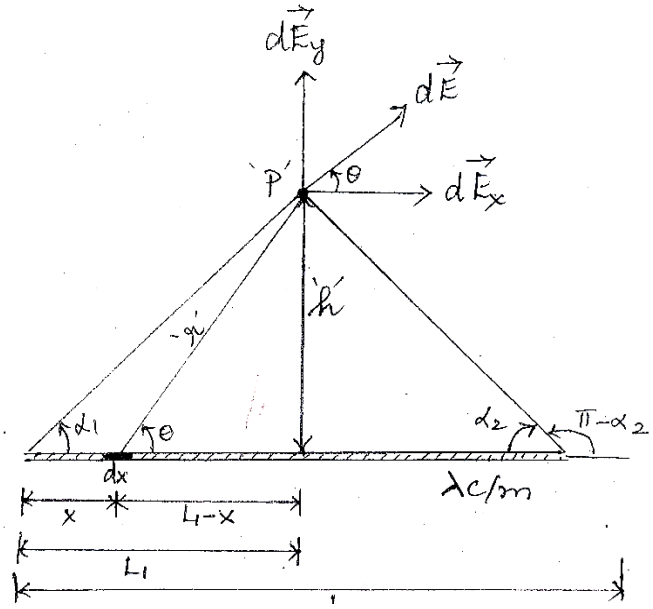
$$\bar{E} = \frac{\bar{F}}{\text{Charge}} = \frac{\bar{F}}{q}$$

$$\therefore \quad \boxed{\bar{E} = \frac{\rho_S h}{2\varepsilon_0} \left[\frac{1}{h} - \frac{1}{\sqrt{a^2 + h^2}}\right] \bar{a}_z \; \text{V/m}}$$

38. Find the elective field intensity of a straight uniformly charged wire of length 'L' m and having linear charge density of +λ c/m, at any point 'p' at a distance of 'h' m. The angles made by the two ends of the charged wire with the point 'p' are given as α_1 and α_2 with respect to x- axis.

Obtain an expression for electric field intensity due to a uniformly charged line of length 'l'. (Apr/May 2015)(Nov/Dec 2015)

A Charge is distributed along a finite straight line with constant density C/m along "X axis. Develop an Expression for E at an arbitrary point P.(Apl/May 2018)



Assumptions:

Let the point 'p' be at a distance of 'L_1' meter from one end of the charged wire along 'x' axis, and at a height 'h' meter along 'y' axis.

Consider a small element 'dx' at a distance 'x' from one end of wire.

Charge element dq=λdx

Total charge 'q' due to full length will be the integration of dq.

Let 'dE' be the field due to charge element λdx

$$\vec{dE} = \vec{dE}\,x + \vec{dE}\,y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

From figure;

$\vec{dE}_x = dE\cos\theta\,\vec{a}_x$

$\vec{dE}_y = dE\sin\theta\,\vec{a}_y$

By definition;

$dE = \frac{dq}{4\pi\varepsilon_0 r^2}\,\vec{a}_r$ v/m

$$\Rightarrow \vec{dE}_x = \frac{\lambda dx}{4\pi\varepsilon_0 r^2}\cos\theta$$

$$\vec{E}_x = \int_{x=0}^{L} \frac{\lambda dx}{4\pi\varepsilon_0 r^2}\cos\theta\,\vec{a}_x \quad \text{.....(2)}$$

From the fig, $\tan\theta = \frac{h}{L_1 - x} \Rightarrow \cot\theta \frac{L_1 - x}{h}$

$\Rightarrow L_1 - x = h\cot\theta$

$\Rightarrow -dx = -h\,\text{cosec}^2\theta\, d\theta$ .....(3)

Also, $\sin\theta = \frac{h}{r} \Rightarrow r = h\,\text{cosec}\theta$ .....(4)

Sub 3 and 4 in equation 2

$$\vec{Ex} = \int_0^L \frac{\lambda(h\,\text{cosec}^2\theta.d\theta)}{4\pi\varepsilon_0(h\,\text{cosec}\theta)^2}.\cos\theta\,\vec{a}_x$$

NOTE: [dx→become dθ,so change limits also ie, x=0 to L, becomes $\theta=\alpha_1$, to $\pi - \alpha_2$]

$$\vec{E}_x = \int_{\alpha_1}^{\Pi-\alpha_2} \frac{\lambda}{4\pi\varepsilon_0 h}.\cos\theta\, d\theta$$

$$\Rightarrow \vec{Ex} = \frac{\lambda}{4\pi\varepsilon_0 h}[\sin\theta\ ]_{\alpha_1}^{\Pi-\alpha_2} = \frac{\lambda}{4\pi\varepsilon_0 h}\ [\sin(\pi - \alpha_2) - \sin\alpha_1]$$

$$\vec{E}_x = \frac{\lambda}{4\pi\varepsilon_0 h}\ \mathbf{[\sin\ \alpha_2 - \sin\alpha_1]\ V/m}$$

Similarly

$$d\vec{Ey} = \frac{\lambda dx}{4\pi\varepsilon_0 r^2}\ \sin\theta\,\vec{ay} \qquad \Rightarrow \vec{Ey} = \int_{x=0}^{L} \frac{\lambda dx}{4\pi\varepsilon_0 r^2}\ \sin\theta$$

Substituting for 'θ' and 'dx'

$$\vec{Ey} = \int_{\alpha_1}^{\Pi=\alpha_2} \frac{\lambda h\,\text{cosec}^2\theta d\theta}{4\pi\varepsilon_0(h^2\,\text{cosec}^2\theta)}\sin\theta\,\vec{ay} = \int_{\alpha_1}^{\Pi=\alpha_2} \frac{\lambda}{4\pi\varepsilon_0 h}\ \sin\theta\ d\theta\,\vec{ay} = \frac{\lambda}{4\pi\varepsilon_0 h}\ [1-\cos\theta]_{\alpha_1}^{\Pi-\alpha_2}\ \vec{a}_y$$

$$\Rightarrow \vec{Ey} = \frac{\lambda}{4\pi\varepsilon_0 h}[\cos\alpha_1 + \cos\alpha_2]\ \vec{a}_y\ \text{V/m}$$

Case (i)

If point 'p' is along $\perp$ bisector of wire, than $\alpha_1 = \alpha_2 = \alpha \therefore \vec{a}_x$ component get cancelled and $\vec{a}_y$ component adds.

$$\vec{E}_y = \vec{E} = \frac{\lambda}{4\pi\varepsilon_0 h}[\cos\alpha + \cos\alpha]$$

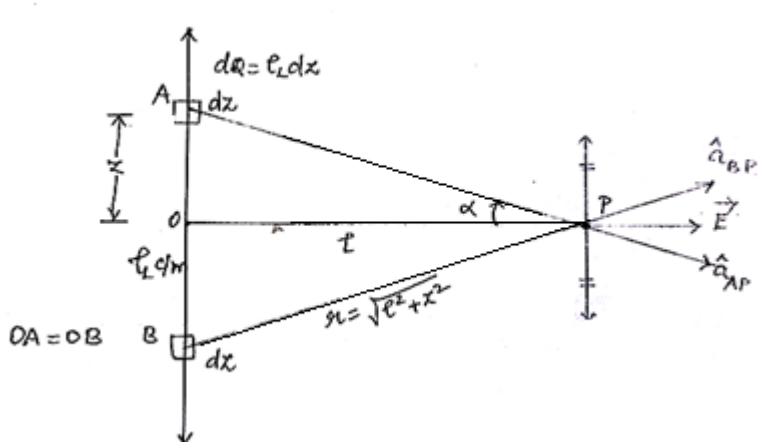
$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 h}.\cos\alpha\ \vec{a}_y \text{ V/m}$$

Case (ii)

In case of infinitely long conductor, α =0

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 h}\ \vec{a}_y \quad \text{v/m}$$

39. An infinite long line charge of uniform density ρ_Lc/m is situated along the Z axis as shown obtain the Electric field intensity along point due to this line charge. (June 2012)(April/May 2017)



First divide the line into a no of infinitesimal segment each of length dz, such that $dq=\rho_L dz$

- Consider a point 'p' at a distance 'ρ' from z-axis with the projection of point 'p'on to 'z' axis being 'o'.
- $\vec{E}$ at a point p, due to charge elements at A and B is given as

$$\vec{E}p = \vec{E}_{AP} + \vec{E}_{BP} = \vec{E}_{AP\rho} + \vec{E}_{AP\rho} + \vec{E}_{BP\rho} + \vec{E}_{BPz}$$

- Since $\vec{E}_{APz}$ and $\vec{E}_{BPz}$ are equal and opposite they cancel each other.
- Since $\vec{E}_{AP\rho}$ and $\vec{E}_{BP\rho}$ are in same direction, they add up.

$\vec{E}_P = \vec{E}_{AP\rho} + \vec{E}_{BP\rho}$

By definition $d\vec{E}_{AP} = \frac{dq}{4\pi\varepsilon_0 r^2}\ \vec{a}_{AP}$

But we know dq =$\rho_L$dz and r=$\sqrt{\rho^2 + z^2}$

$\Rightarrow d\vec{E}_{AP} = \frac{\rho_L d_z}{4\pi\varepsilon_0(\rho^2 + z^2)}\ .\ \vec{a}_{AP}$

The component of $\vec{E}$ along OP is given as

$d\vec{E}_{oP} = \frac{\rho_L d_z}{4\pi\varepsilon_0(\rho^2 + z^2)}\cos\alpha\ \vec{a}_{oP}$

$\cos\alpha = \frac{\rho}{\sqrt{\rho^2 + z^2}}, \vec{a}_{oP} = \vec{a}_\rho$

$\Rightarrow d\vec{E}_{oP} = \frac{\rho_L d_z}{4\pi\varepsilon_0(\rho^2 + z^2)} . \frac{\rho}{\sqrt{(\rho^2 + z^2)^{1/2}}} . \vec{a}\rho$

$\Rightarrow d\vec{E}_{oP} = \frac{\rho.\rho_L d_z}{4\pi\varepsilon_0(\rho^2 + z^2)^{3/2}} . \vec{a}_\rho$

Resultant $d\vec{E}$ at 'p' due to 2 charge segments at A+B is given as,

$\vec{dE} = \frac{2\rho_L.\rho d_z}{4\pi\varepsilon_0(\rho^2 + z^2)^{3/2}} . \vec{a}_\rho$

$\vec{E} = \int_{z=0}^{z=\infty} \text{dE}\,\vec{a}_\rho = \frac{2\rho_L}{4\pi\varepsilon_0}\int_0^\infty \frac{\rho d_z}{(\rho^2 + z^2)^{3/2}}\ \vec{a}_\rho$

From the fig.

Z= ρtan α , dz= ρ^2sec$^2\alpha$ dα

ρ^2+z^2 = ρ^2sec$^2\alpha$

Changing the limits, when z=0

tanα =0 $\therefore \alpha$ =0

When z= ∞

tanα = ∞ $\therefore \alpha$ =90°

$\Rightarrow \vec{E} = \frac{2\rho_L}{4\pi\varepsilon_0}\int_{0°}^{90°} \frac{\rho.\rho.sce^2\alpha d\alpha}{(\rho^2\sec^2\alpha)^{3/2}} . \vec{a}_\rho$

$$\Rightarrow \vec{E} = \frac{\rho_L}{2\pi\varepsilon_0}\int_{0^\circ}^{90^\circ} \cos\alpha \, d\alpha \; \vec{a}_\rho = \frac{\rho_L}{2\pi\varepsilon_0\rho}[\sin\alpha \;]_{0^\circ}^{90^\circ} \vec{a}_\rho = \frac{\rho_L}{2\pi\varepsilon_0\rho}\vec{a}_\rho$$

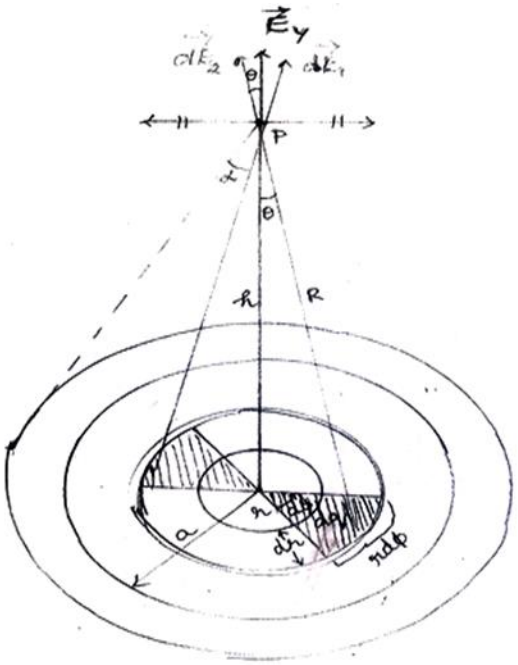
Ans, $\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\vec{a}_\rho$

---

40. A circular disc of radius 'a'm charged uniformly with a charge density of ρ c/m2. Find the Electric field at a point 'h' m from the disc along its axis. (June 2013)

Derive the expression for electrical field intensity due to uniformly charged circular disc of ρ c/m2.(Nov/Dec 2016)

- The point 'p' is at 'h' m along the axis of the disc.
- Let 'α' be the angle made by any point on the circumference of disc with the point 'p'.
- Consider the disc surface area consisting of a number of annular area elements of varying radius 'r' meter and radial thickness 'dr' m.
- From the figure, it can be seen that for every small elemental charge in the annular area of disc, there will be on equal and opposite charge in the diametrically opposite end. So, the horizontal component of the field gets neutralized.
- Therefore, the field intensity at 'p' is given by summation of vertical components alone.
- Consider an elemental area ds=2πr dr and its contribution in producing the field dE at P



$$d\vec{E} = \frac{\sigma ds}{4\pi\varepsilon_0 R^2}\ \vec{a}_R$$

Its vertical component is dEcosθ

$$\vec{dE}_y = \frac{\sigma ds}{4\pi\varepsilon_0 R^2}\cos\theta$$

Total field at P,is given as

$$\vec{E}y = \int_{r=0}^{r=a} \frac{\sigma}{4\pi\varepsilon_0 R^2} 2\pi r dr\cos\theta = \frac{\sigma}{4\pi\varepsilon_0}.2\pi\int_{r=0}^{r=a} \frac{r}{R^2} dr\cos\theta = \frac{\sigma}{2\varepsilon_0}\int_{r=0}^{a} \frac{\cos\theta}{R^2}.rdr$$

For the figure

r=htanθand R = h secθ

$\Rightarrow$dr = hsec^{2}θdθ

$$\Rightarrow \vec{E}y = \frac{\sigma}{2\varepsilon_0}\int_{\theta=0}^{\theta=\alpha} \frac{(h\tan\theta)(h\sec^2\theta)}{(h\ \sec^2\theta)}\cos\theta.d\theta\ \vec{a}_y$$

$$= \frac{\sigma}{2\varepsilon_0}\int_{\theta=0}^{\alpha} \frac{h^2\sec^2\theta\tan\theta\cos\theta)}{(h^2\sec^2\theta)} d\theta\ \vec{a}_y$$

$$= \frac{\sigma}{2\varepsilon_0}\int_{\theta=0}^{\alpha} \sin\theta\ d\theta\ \vec{a}_y = \frac{\sigma}{2\varepsilon_0}[-\cos\ ]_0^{\alpha}\ \vec{a}_y = \frac{\sigma}{2\varepsilon_0}[-\cos\alpha - (-\cos\theta)]\ \vec{a}_y$$

$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\ \mathbf{[1-\cos\alpha\ ]}\ \vec{a}\ \mathbf{y}$$

The result may be extended to an infinite plane sheet of charge

- For an infinite plane sheet α =90°

$$\Rightarrow \vec{E} = \frac{\sigma}{2\varepsilon_0}\ \vec{a}_y$$

---

41. A 50cm length coaxial cable having an inner radius 1mm and an outer radius4mm, has its inner space between the conductors filled with air. The total charge on the inner conductor is 30nC. Find the charge density on each conductor.(Nov/Dec 2017)

$$L = 50 \times 10^{-2} m$$
$$a = 1 \times 10^{-3} m$$
$$b = 4 \times 10^{-3} m$$
$$Q = 30 \times 10^{-9} C$$
$$Q = 2\pi a L \rho_s$$
$$\rho_s = \frac{Q}{2\pi a L}$$

$$= \frac{30 \times 10^{-9}}{2 \times \pi \times 1 \times 10^{-3} \times 50 \times 10^{-2}}$$
$$= \frac{30 \times 10^{-4}}{314}$$
$$= 9\mu C/m^2$$

42. Find the total charge enclosed in an increamental volume of $10^{-9}m^3$ located at the origin, if $A = e^{-x}sin\ ya_x + 2za_zC/m^2$ (Nov/Dec 2017)

$$Q = \int (\nabla \cdot A)dv \text{ and } dQ = (\nabla \cdot A)dv$$

Here, dv = $10^{-9}m^3$ at origin x=y=z=0

$$(\nabla \cdot A) = \frac{\partial Ax}{\partial x} + \frac{\partial Ay}{\partial y} + \frac{\partial Az}{\partial z}$$
$$= -e^{-x}siny + 2$$
$$dQ = (\nabla \cdot A)\text{at origin} \times dv = (0 + 2) \times 10^{-9}$$
$$= 2 \times 10^{-9}C = 2nC$$

43. State and prove Gauss Law. Write the applications of Gauss Law. (Apr/May 2015)(Nov/Dec 2017)

Although coulomb's law is applicable in finding the electric field due to any charge distribution, it is easier to use Gauss's law when charge distribution is symmetrical.

Gauss Law:

It states that, the electric flux passing through closed surface is equal to the total charge enclosed by that surface.

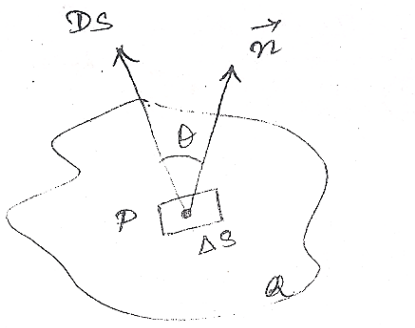
Is $\psi = \oint \vec{D} . d\vec{s} = Q$

Proof:

Assume any closed surface in an isotropic medium which consists of charge. If the total charge inside the surface is 'Q' coulomb's then Q coulombs of electric flux will pass through the enclosing surface.

At every point on the surface, the electric flux density $\vec{D}$ will have come value of Ds where the subscript 's' indicates the value of 'D' at that surface and Ds will vary in magnitude and in direction from one point to the other.

Consider an incremental element of the surface with an area of ΔS. ΔS is the scalar of the surface area. It can be represented as a vector as $\vec{n}\Delta S$, Where $\vec{n}$ is a unit vector given by outward normal to the surface.



At any point p, consider an incremental element of surface ΔS and let Ds make an angleθ with ΔS as show in fig.

The flux crossing ΔS is then, the product of normal component of Ds and ΔS.

Δψ=flux crossing ΔS

Δψ=Ds. $\vec{n}\Delta S$ …….(1)

Δψ=Ds. $\vec{n}\Delta_S\cos\theta$ …….(2)

*but we know, $\vec{E}=\frac{Q}{4\pi\varepsilon_0 R^2}\vec{a}_R$

$\Rightarrow \varepsilon_\circ \vec{E}=\frac{Q}{4\pi R^2}\vec{a}_R \qquad [\because \varepsilon_0 \vec{E}=\vec{D}]$

$\Rightarrow \vec{D}=\frac{Q}{4\pi R^2}\vec{a}_R$**……. (3)**

From eqn (2) &(3),

$\Delta\psi=\frac{Q}{4\pi R^2}\Delta s\cos\theta$ ……..(4)

In solid geometry, the solid angle is defined as the ratio of surface area subtended to the square of the radius.

$d\omega=\frac{\Delta s\cos\theta}{R^2}$ ……(5)

From eqn (1)and (4)

Δψ= Ds. $\vec{n}\Delta S = \frac{Q}{4\pi R^2}\Delta s\cos\theta$ ……. (6)

Ds. $\vec{n}\Delta S = \frac{Q}{4\pi R^2}d\omega$……..(7)

In solid geometry, the total angle subtended by any closed surface is 4π radians.

$$\int \Delta\psi = \oint_s Ds.\vec{n}\,\Delta S = \frac{Q}{4\pi}\oint_s d\omega$$

$$\psi = \oint_s Ds.\vec{n}\,\Delta S = \frac{Q}{4\pi}\times 4\pi = Q$$

A small circle on the integral sign indicates that the integration is to be performed over a closed surface. Such surface is often called Gaussian surface.

Mathematical formulation of gauss's law is

$\psi = \oint_s \vec{D}\,s.\,\vec{\Delta}_S$ =charge enclosed =Q

$\Delta\vec{s}$ Can be replaced by $d\vec{s}$ for convenience $\psi = \oint_s \vec{D}\,s.\,\vec{\Delta}_S = Q$

Gauss's law may be written in terms of the charge distribution as

$$\oint_s \vec{D}\,s.\,\vec{\Delta}_S = \int_{vol} \rho_v\, d_V$$

Applications of Gauss's law:-

Gauss's law is used to find on to the Electric field intensity (or) Electric flux density only for symmetrical charge distributions.

Example: Infinite line charge, infinite surface charge and spherical charges.

44. (i) A volume charge is distributed throughout a sphere of radius 'a' with uniform density $\rho_v c/m^2$.
(ii) Obtain the Electric field intensity due to this volume charge using Gauss's Law.
Show that over the closed surface of a sphere of radius b, ds = 0.(Apr/May 2015)(Apr/May 2018)

Spherical Shell of Charge

The charge is uniformly distributed over its surface with a density ρ_S C/m^2. Let us find $\bar{E}$ at a point P located at a distance r from the centre such that r >a and r≤ a, using Gauss's law.
The shell is shown in the Fig. 3.7.5.

Case 1: Point P outside the shell (r > a)

Consider a point P at a distance r from the origin such that r > a. The Gaussian surface passing through point P is a concentric sphere of radius r. Due to spherical Gaussian surface, the flux lines are directed radially outwards and are normal to the surface. Hence electric flux density D is also directed radially outwards at point P and has component only in $\bar{a}_r$ direction. Consider a

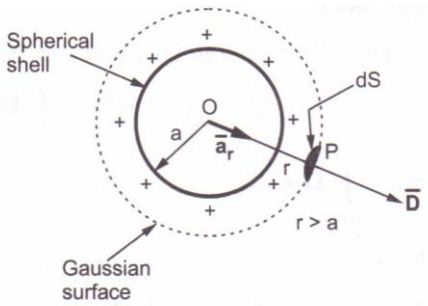


Fig. 3.7.5 Spherical shell of charge

differential surface area at P normal to $\bar{a}_r$ direction hence $dS = r^2 \sin\theta \, d\theta \, d\phi$ in spherical system.

$$\therefore \quad d\psi = \bar{D} \bullet d\bar{S} = [D_r \bar{a}_r] \bullet [r^2 \sin\theta \, d\theta \, d\phi \, \bar{a}_r] = D_r \, r^2 \sin\theta \, d\theta \, d\phi$$

$$\therefore \quad \psi = \oint_S D_r \, r^2 \sin\theta \, d\theta \, d\phi = D_r \, r^2 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta \, d\theta \, d\phi$$

$$\therefore \quad \psi = D_r \, r^2 [-\cos\theta]_0^{\pi} [\phi]_0^{2\pi} = 4\pi r^2 D_r \qquad \text{... (3.7.18)}$$

But $\psi = Q$ ... Gauss's law

$$\therefore \quad Q = 4\pi r^2 D_r$$

$$\therefore \quad D_r = \frac{Q}{4\pi r^2}$$

$$\therefore \quad \bar{D} = D_r \bar{a}_r = \frac{Q}{4\pi r^2} \bar{a}_r \ C/m^2 \qquad \text{... (3.7.19)}$$

And $$\bar{E} = \frac{\bar{D}}{\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2} \bar{a}_r \ V/m \qquad \text{... (3.7.20)}$$

Thus for r > a, the field $\bar{E}$ is inversely proportional to the square of the distance from the origin.

If the surface charge density is $\rho_S \ C/m^2$ then

$$Q = \rho_S \times \text{Surface area of shell}$$

$$\therefore \quad Q = \rho_S \times 4\pi a^2$$

$$\therefore \quad \bar{E} = \frac{\rho_S 4\pi a^2}{4\pi\varepsilon_0 r^2} \bar{a}_r = \frac{\rho_S \, a^2}{\varepsilon_0 r^2} \bar{a}_r \ V/m \qquad \text{... (3.7.21)}$$

and $$\boxed{\bar{D} = \varepsilon_0 \bar{E} = \frac{\rho_S \, a^2}{r^2} \bar{a}_r \ C/m^2} \qquad \text{... (3.7.22)}$$

Case 2 : Point P is on the shell (r = a)

On the shell, r = a

The Gaussian surface is same as the shell itself and $\bar{E}$ can be obtained using r = a in the equation (3.7.20).

$$\therefore \quad \bar{E} = \frac{Q}{4\pi\varepsilon_0 a^2} \bar{a}_r \ V/m \qquad \text{... (3.7.23)}$$

Case3 : Point P inside the shell (r < a)

The Gaussian surface, passing through the point P is again a spherical surface with radius r < a.

But it can be seen that the entire charge is on the surface Gaussian and no charge is enclosed by the spherical shell. And when surface the Gaussian surface is such that no charge is enclosed, irrespective of any charges present outside, the total charge enclosed is zero.

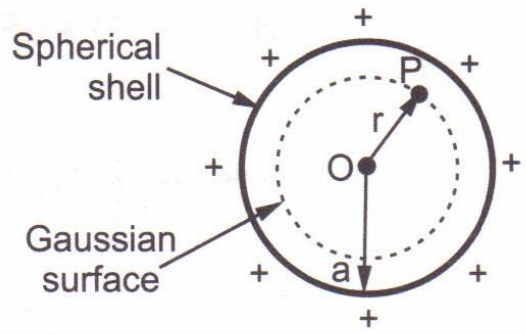


Fig. 3.7.6

$$\therefore \quad \psi = Q = \oint_S \overline{D} \bullet d\overline{S} = 0 \qquad \text{... As per Gauss's law}$$

$$\text{Now} \quad \oint_S d\overline{S} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin\theta \, d\theta \, d\phi = 4\pi r^2$$

$$\text{Thus} \quad \oint_S d\overline{S} \neq 0$$

Hence to satisfy that total charge enclosed is zero, inside the spherical shell.

$$\boxed{\overline{D} = 0 \quad \text{and} \quad \overline{E} = \frac{\overline{D}}{\varepsilon_0} = 0} \qquad \text{... (3.7.24)}$$

Thus electric flux density and electric field at any point **inside a spherical shell is zero.**

Uniformly Charged Sphere

Consider a sphere of radius 'a' with a uniform charge density of ρ_v C/m^3. Let us find $\bar{E}$ at a point P located at a radial distance r from centre of the sphere such that $r \leq a$ and $r > a$, using Gauss's law.

The sphere is shown in the Fig. 3.7.8.

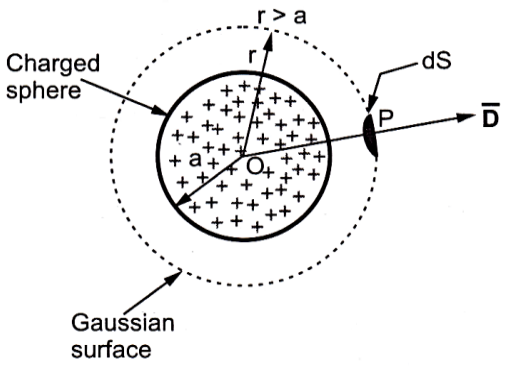


Fig. 3.7.8 Uniformly charged sphere

Case 1 : The point P is outside the sphere ($r > a$).

The Gaussian surface passing through point P is a spherical surface of radius r.

The flux lines and $\bar{D}$ are directed radially outwords along $\bar{a}_r$ direction.

The differential area dS is considered at point P which is normal to $\bar{a}_r$ direction.

$$\therefore \quad dS = r^2 \sin\theta \, d\theta \, d\phi$$

$$\therefore \quad d\psi = \bar{D} \bullet d\bar{S} = D_r \bar{a}_r \bullet r^2 \sin\theta \, d\theta \, d\phi \, \bar{a}_r$$

$$= D_r \, r^2 \sin\theta \, d\theta \, d\phi \qquad \ldots (\bar{a}_r \bullet \bar{a}_r = 1)$$

$$\therefore \quad \psi = Q = \oint_S \bar{D} \bullet d\bar{S} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} D_r \, r^2 \sin\theta \, d\theta \, d\phi$$

$$= D_r \, r^2 \, [-\cos\theta]_0^{\pi} \, [\phi]_0^{2\pi} = D_r \, r^2 \, 4\pi$$

$$\therefore \quad D_r = \frac{Q}{4\pi r^2}$$

$$\therefore \quad \bar{D} = \frac{Q}{4\pi r^2} \bar{a}_r \ \text{C/m}^2 \qquad \ldots (3.7.25)$$

$$\therefore \quad \bar{E} = \frac{\bar{D}}{\varepsilon_0} = \frac{Q}{4\pi\varepsilon_0 r^2} \bar{a}_r \ \text{V/m} \qquad \ldots (3.7.26)$$

The total charge enclosed can be obtained as,

$$Q = \int_v \rho_v \, dv = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{a} \rho_v \, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \rho_v \left[\frac{r^3}{3}\right]_0^a [-\cos\theta]_0^{\pi} \, [\phi]_0^{2\pi} = \frac{4}{3}\pi a^3 \rho_v \ \text{C} \qquad \ldots (3.7.27)$$

$$\therefore \quad \bar{E} = \frac{\frac{4}{3}\pi a^3 \rho_v}{4\pi\varepsilon_0 r^2}\bar{a}_r = \frac{a^3 \rho_v}{3\varepsilon_0 r^2}\bar{a}_r \qquad \ldots (3.7.28)$$

while $$\boxed{\bar{D} = \frac{a^3 \rho_v}{3r^2}\bar{a}_r} \qquad \ldots (3.7.29)$$

These are the expressions for $\bar{D}$ and $\bar{E}$ outside the uniformly charged sphere.

Case 2 : The point P on the sphere (r = a).

The Gaussian surface is same as the surface of the charged sphere. Hence results can be obtained directly substituting r = a in the equation (3.7.28) and (3.7.26).

$$\therefore \quad \bar{E} = \frac{a^3 \rho_v}{3\varepsilon_0 a^2}\bar{a}_r = \frac{\rho_v a}{3\varepsilon_0}\bar{a}_r \qquad \ldots (3.7.30)$$

and $$\boxed{\bar{D} = \varepsilon_0 \bar{E} = \frac{\rho_v a}{3}\bar{a}_r} \qquad \ldots (3.7.31)$$

Case 3 : The piont P is inside the sphere (r < a) the Gaussian surface is a spherical surface of radius r where r < a.

Consider differential surface area dS as shown in the Fig. 3.7.9.

Again $d\bar{S}$ and $\bar{D}$ are directed radially outwards.

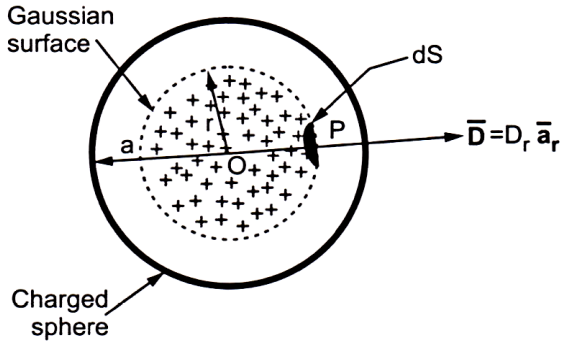


Fig. 3.7.9

$$\therefore \quad \bar{D} = D_r \bar{a}_r \quad \text{while} \quad d\bar{S} = r^2 \sin\theta\, d\theta\, d\phi\, \bar{a}_r$$

$$\therefore \quad d\psi = \bar{D} \bullet d\bar{S} = D_r r^2 \sin\theta\, d\theta\, d\phi \qquad \ldots (\bar{a}_r \bullet \bar{a}_r = 1)$$

$$\therefore \quad \psi = Q = \oint_S \bar{D} \bullet d\bar{S} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} D_r r^2 \sin\theta\, d\theta\, d\phi$$

$$= D_r r^2 \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi} = 4\pi r^2 D_r$$

$$\therefore \quad D_r = \frac{Q}{4\pi r^2}$$

$$\therefore \quad \bar{D} = \frac{Q}{4\pi r^2}\bar{a}_r \ C/m^2 \qquad \ldots (3.7.32)$$

Now the charge enclosed is by the sphere of radius r only and not by the entire sphere. **The charge outside the Gaussian surface will not affect $\overline{D}$.**

$$\therefore \quad Q = \int_v \rho_v dv = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{r} r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \frac{4}{3}\pi r^3 \rho_v \qquad \text{where } r < a \qquad \ldots (3.7.33)$$

Using in equation (3.7.32) we get,

$$\overline{D} = \frac{\frac{4}{3}\pi r^3 \rho_v}{4\pi r^2} \bar{a}_r$$

$$\therefore \quad \boxed{\overline{D} = \frac{r}{3}\rho_v \bar{a}_r \qquad \ldots 0 < r \leq a} \qquad \ldots (3.7.34)$$

$$\therefore \quad \overline{E} = \frac{\overline{D}}{\varepsilon_0} = \frac{r}{3\varepsilon_0}\rho_v \bar{a}_r \qquad 0 < r \leq a \qquad \ldots (3.7.35)$$

PART -A

1. Define electric potential and potential difference. (April 2010)

The potential at a distance 'r' from the point charge is the work done per unit charge by the external agent in bringing a test charge from infinity to that point that is

$$V_{(r)} = -\int_{\infty}^{r} \vec{E} \cdot \vec{d\ell}$$ Against a field $\vec{E}$,

The potential difference between two points A and B, is defined as work done per unit charge by an external agent in bringing a test charge Q from point A and B, against the field $\vec{E}$.

$$V_{AB} = -\int_{A}^{B} \vec{E} \cdot \vec{d\ell}$$

2. Write down Laplace's and Poisson's equations. (April 2010 & April 2011, April 2016)(Nov/Dec 2016) .(Nov/Dec 2017)

Laplace equation in Cartesian, cylindrical and spherical Co-ordinates respectively is given by.

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \frac{\partial^2 v}{\partial z^2} = 0$$

Cylindrical Co-ordinates

$$\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial v}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 v}{\partial \phi^2} + \frac{\partial}{\partial z^2} 2v$$

Spherical Co-ordinates

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial v}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial v}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 v}{\partial \theta^2} = 0$$

In general $\nabla^2 V = 0$ ------ Laplace's equation

Poisson's equation $\nabla^2 V = -\frac{\rho v}{\varepsilon}$

3. Define electric dipole and electric dipole moment. (Nov 2010)

Two equal and opposite point charges separated by a very small distance is defined as an electric dipole.

If the separation of the charges is 'd' m and the magnitude of charges are 'Q' coulombs, then the product of magnitude of charge and the separation 'd' is defined as electric dipole moment, denoted as m.

m = Qd Coulomb meter

4. Define polarization. (April 2011)

The process of forming dipoles is known as polarization. It is defined as the total dipole moment per unit volume.

$$\vec{P} = \frac{m}{v} = \frac{Qd}{v} \quad c/m^2$$

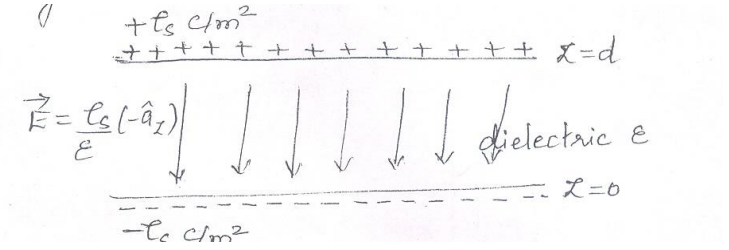
5. Define dielectric strength of materials and give its unit.(April 2010) (Nov/Dec 2017)

The maximum electric field intensity that a dielectric material can withstand without breakdown is the dielectric strength of the material.

For example: Air can withstand up to 30 kV/cm unit of dielectric strength is V/m or kV/cm.

6. Draw the equipotential lines and electric field lines for a parallel plate capacitor. (April 2008)

Any surface on which the potential is same throughout is known as an equipotential surface.



8. What is the practical significance of dielectric strength? (April 2008)

When the applied electric field in a dielectric is sufficiently large, it begins to pull electrons completely out of molecules and the dielectric becomes conducting. When a dielectric becomes conducting it is said that dielectric breakdown has occurred. The dielectric strength of a dielectric material is defined as the maximum electric field which a dielectric can tolerate or withstand breakdown.

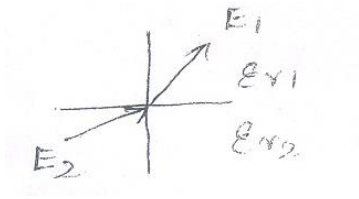
9. Give examples for uniform and non-uniform electric fields? (May 2007)

Uniform electric fields: Field in a parallel plate capacitor, field due to infinite sheet charge, field due to infinite line charge.

Non-uniform electric fields: Field due to point charge, field due to finite line and sheet (disc) charge, field due to spherical volume charge.

10. Comment on the values of the relative permittivity.

E_1 and E_2 for the field at the charge free interface of two dielectric media.At charge free interface of two dielectric medium.



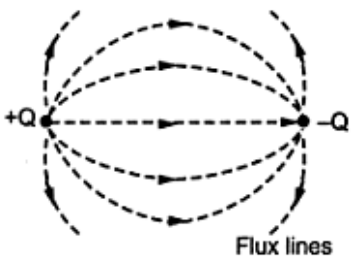
$$\frac{E_{1n}}{E_{2n}} = \frac{\varepsilon r_2}{\varepsilon r_1}$$

ie The normal component of $\vec{E}$is discontinuous at the boundary.

11. State the properties of electric flux lines. (Nov/Dec 2014)(Apl/May 2018)

Electric flux lines are lines of force, around a charge. Such electric flux lines have following properties:

1. They start from positive charge and terminate on negative charge.
2. If negative charge is absent, then the lux lines terminate at infinity. While in absence of positive charge, the electric flux terminates on negative charge from infinity.



3. There is crowding of flux lines if electric field is stronger.
4. These lines are parallel and never cross each other.
5. They are independent on the medium in which the charges are present.
6. The lines always enter or leave the charged surface, normally.

12. Give the significant physical differences between Poisson's and Laplace's equations. (Nov/Dec 2014)

$\nabla . \vec{D} = \rho_v$ --------- point from of gauss law

$\Rightarrow \nabla . \vec{D} = \nabla . \varepsilon \vec{E} = \rho_v$ ----------(1)

But $\vec{E}$ = - ∇V $\Rightarrow \nabla . \varepsilon \vec{E} = \nabla .(-\varepsilon \nabla v)$ ---------- (2)

From eqn(1)&(2),we can write $\nabla .(-\varepsilon \nabla v) = \rho_v$ for an inhomogeneous medium

For homogeneous medium, the equation becomes

$\boxed{\nabla^2 V = -\frac{\rho_v}{\varepsilon}}$ --------- POISSON'S EQUATION

If in a certain region, volume charge density is zero ($\rho_v = 0$), which is true for dielectric medium then the Poisson's equation takes the form,

$\boxed{\nabla^2 V = 0}$ ---------- LAPLACE'S EQUATION (For charge free region)

14. What do you understand by linear, surface and volume charge densities?

Linear charge density (ρ_l): It is the charge p-er unit length, at a point on the line of charge

$$\rho_L = \lim_{\Delta l \to 0} \left(\frac{\Delta Q}{\Delta l} \right) \text{ c/m}$$

Surface charge density (ρ_s): It is the charge per surface area, at a point on the surface of the charge.

$$\rho_s = \lim_{\Delta s \to 0} \left(\frac{\Delta Q}{\Delta s} \right) \text{ c/m}^2$$

Volume charge density (ρ_v): It is the charge per unit volume, at a point on the volume charge.

$$\rho_v = \lim_{\Delta v \to 0} \left(\frac{\Delta Q}{\Delta v} \right) \text{ c/m}^3$$

15. What is the relation between intensity of electric field $\vec{E}$ and electric flux density $\vec{D}$ in free space?

$$\vec{\mathbf{D}} = \boldsymbol{\varepsilon}\vec{\mathbf{E}}, \quad \textbf{where } \boldsymbol{\varepsilon} - \textbf{permittivity of the medium}$$

16. State the principle of superposition of fields.

If there are more than two point charges, then each charge will exert force on the other. Then the net force on any charge can be obtained by the principle of superposition.

i.e. The vector sum of all the forces calculated earlier is the resultant force.

$\vec{F}_Q = \vec{F}_{Q1Q} + \vec{F}_{Q2Q} \ldots\ldots\ldots\ldots \vec{F}_{QnQ}$

17. What is principle of conservation of charges?

The principle of conservation of charges states that charges can neither be created nor be destroyed, but equal amount of +ve and –ve charges may be simultaneously created, by separation method.

18. Write the relation between potential and electric field. (Dec 2007)

$$\vec{E} = -\nabla V$$

The electric field strength at any point is the negative of the potential gradient at that point.

19. Define electric scalar potential.

Absolute electric scalar potential at a point is defined as the work done in moving a unit test charge from infinity (or reference point at which potential in zero) to the point against the direction of the field.

20. Define conduction and displacement current.

Current flowing in a conductor is called conduction current. It is caused by an electric field E.

Conduction current density: $\mathbf{J_c = \frac{dQ}{dt} = \sigma E}$

Displacement or convection current is that which flows through an insulating medium such as liquid, vacuum, etc.

Example: Beam of electrons in a vacuum tube.

Displacement current density: $\mathbf{J_D = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t}}$

21. State boundary conditions for two different dielectric medium. (Apl/May 2018)

i. $E_{t1} = E_{t2}$

ii. $\frac{D_{t1}}{D_{t2}} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$

iii. $D_{N1} = D_{N2}$

iv. $\frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_2}{\varepsilon_1} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}$

22. Write boundary conditions on a perfect conductor interface.

$E_t = D_t = 0$

$D_N = \rho_s$

$E_N = \frac{\rho_s}{\varepsilon} = \frac{\rho_s}{\varepsilon_0 \varepsilon_r}$

23. Why is the electrostatic potential continuous at a boundary?

At the boundary $E_{t1} = E_{t2}$. Thus the tangential components of the field intensity at the boundary in both the dielectrics remain same. Thus electric field intensity is continuous at the boundary. Since $E = -\nabla V$, the electrostatic potential is also continuous at the boundary.

24. Write the equation of continuity.

$$\nabla . \vec{J} = -\frac{\partial \rho}{\partial t}$$

Is called continuity equation or law of conservation of charges in differential form.

25. What is the practical significance of Lorentz's Force? (Apr/May 2015)(Nov/Dec 2015)

$$\vec{F} = \vec{F}_e + \vec{F}_m = Q(\vec{E} + \vec{v} \times \vec{B})\ Newton$$

Above equation is called Lorentz Force equation which relates mechanical force to the electrical force.

- First term $Q\vec{E}$ does not depend on v and Fe is in same direction as $\vec{E}$.
- Second term does $Q(\vec{v} x \vec{B})$ depends on v and is perpendicular to both $\vec{v}$ and $\vec{B}$.

The solution of this equation is useful in the determination of electron orbits in magnetron, proton paths in cyclotron and plasma characteristics in Magneto hydrodynamicGenerator (MHD generator).

26. Find the electric field intensity in free space if D = 30 a_x C/m^3. (Apr/May 2015)

In general, $\vec{D} = \varepsilon\vec{E}$

For free space $\varepsilon = \varepsilon_0$=>$\vec{\mathbf{D}} = \boldsymbol{\varepsilon_0}\vec{\mathbf{E}}$

$$\Rightarrow \mathbf{E} = \frac{D}{\varepsilon_0} = \frac{30}{8.854\times10^{-12}} = 3.388 \times 10^{12}\ N/m$$

27. What is capacitor and capacitance?(May 2016)

A Capacitor is a passive electronic component that stores energy in the form of an electrostatic field. Capacitance is a measure of the amount of the electrical charge a capacitor can store. The S.I unit of capacitance is Farad (F).

$$\mathbf{C} = \frac{\boldsymbol{\varepsilon_0\varepsilon_0 A}}{\mathbf{d}}\ \mathbf{farad}$$

---

28. Calculate the capacitance per km between a pair of parallel wires each of diameter 1cm at a spacing of 50cms (Nov/Dec 2015)

Given:

D=1cm

R=d/2=0.5cm

=0.005m

Solution:

The capacitance between two parallel wires

$C=\frac{\pi\varepsilon_\circ}{ln\frac{D}{r}}$

$= \pi\times8.854\times10^{-12}/\ln\frac{1}{0.005}$

$=5.25\times10^{-12}$

=5.25PF

---

29. What is a conservative field? (April/May 2017)

- The work done in moving a test charge around any closed path in a static field E is zero.
- This is because starting and terminating point is same for a closed path.
- Hence the work done becomes zero.

$$\oint \bar{E}.\ d\bar{L} = 0$$

30. Find the capacitance of an isolated spherical shell of radius a.(Nov/Dec 2016)

Consider a single isolated sphere of radius 'a', given a charge + Q. It forms a capacitance with an outer plate which is infinitely large hence b = ∞**.**

$$\therefore C = \frac{4\pi\varepsilon}{\left[\frac{1}{a} - \frac{1}{\infty}\right]}$$

$$\therefore C = 4\pi\varepsilon a\ F$$

31.Find the magnitude of D for a dielectric material in which E=0.15MV/m and $\varepsilon_r = 5.25$ (Nov/Dec 2016)

$$\bar{D} == \epsilon_0\epsilon_R\bar{E}$$

$$\therefore\ D = 8.854 \times 10^{-12} \times 5.25 \times 0.15 \times 10^6 = 6.9725 \times 10^{-6}\ C/m^2$$

32. The electrical potential near the origin of a system of co-ordinates is $V=5x^2+8y^2+10z^2$. Find the field at (1, 2, 3) (April/May 2017)

Here the potential is in Cartesian co-ordinates.
The relation between potential and electric field is given by $\bar{E} = -\nabla V$

$$\bar{E} = -\left[\frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z\right]$$
$$= -\left[\frac{\partial}{\partial x}(5x^2 + 8y^2 + 10z^2)\,\bar{a}_x + \frac{\partial}{\partial y}(5x^2 + 8y^2 + 10z^2)\,\bar{a}_y + \frac{\partial}{\partial z}(5x^2 + 8y^2 + 10z^2)\,\bar{a}_z\right]$$
$$= -\left[10x\,\bar{a}_x + 16y\,\bar{a}_y + 20z\,\bar{a}_z\right]$$

$\bar{E}$ at point (1,2,3)

Where x=1, y=2, z =3

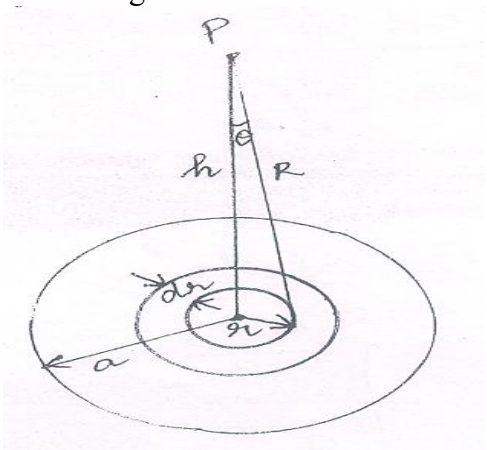
$$\bar{E} = -\left[10\,\bar{a}_x + 32\,\bar{a}_y + 60z\,\bar{a}_z\right],\ {}^{V}/_{m}$$

PART- B

1. Find the potential at any point along the axis of uniformly charged disc of$\rho_s C/m^2$. The disc has radius of 'a' m.(Dec 2010)

Let the point 'p' be along the axis of circular disc at a distance of 'h' m.

Potential at p due to the circular disc can be evaluated by considering the circular disc as composed of a large no of elemental ring of thickness 'dr'.



Consider one annular ring of radius 'r'and thickness 'dr'.

The surface area is equal to 2πr and this surface element carries a charge dq= ρ 2πrdr

Potential dV at any point 'P'due todq is given by

$$dV = \frac{dq}{4\pi\varepsilon_0 R} = \frac{\rho_s 2\pi r dr}{4\pi\varepsilon_0\sqrt{h^2+r^2}}$$

$$\Rightarrow V = \int_{r=0}^{r=a} \frac{\rho_s 2\pi r dr}{4\pi\varepsilon_0\sqrt{h^2+r^2}}$$

From the fig,

$r = h\tan\theta, dr = h\sec^2\theta\, d\theta$

$$V = \int \frac{2\pi\rho_s h\tan\theta}{4\pi\varepsilon_0 h\sec\theta}.h\sec^2\theta d\theta$$

$$= \frac{\rho_s h}{2\varepsilon_0}\int_{r=0}^{r=a} \tan\theta.\sec\theta d\theta$$

$$= \frac{\rho_s h}{2\varepsilon_0}[\sec\theta\ ]_{r=0}^{r=a}$$

$$= \frac{\rho_s h}{2\varepsilon_0}[\sqrt{1+\tan^2\theta}\ ]_{r=0}^{r=a}$$

$$= \frac{\rho_s h}{2\varepsilon_0}[\sqrt{1+\frac{r^2}{h^2}}\ ]_{r=0}^{r=a}$$

$$\Rightarrow \mathbf{V} = \frac{\rho_s h}{2\varepsilon_0}\left[\frac{\sqrt{a^2+h^2}}{h} \mathbf{-1}\right]$$

2. Determine the variation of field from point to point due to

(a) Single spherical shell of charged with radius R_1,

(b) Two concentric spherical shells of charge of radii R. (inner) and R2 (outer).

Evaluate D and E in all regions for a concentric spherical shell containing charge Q on it. Assume the charge distributions are infinite in extent. (April/May 2017)

(a). Single spherical shell of charge

Let us assume that a total charge Q coulombs to be uniformly distributed over an imaginary shell of radius R_1, in a medium of free space (E= E0).

Case (i) r <R1 (inside)

At any radius r<R1 inside the shell of charge no charge is enclosed by this surface so the flux density and Electric flux intensity $\vec{E}$ are zero.

According to Gauss's Law $\oint_s \vec{E} \cdot \vec{ds} = \frac{Q}{\varepsilon_0} = 0 [\because \ Q=0]$

$\vec{E} = 0$ for(r<R1)

Case (ii) r>R1 (outside)

At any radius r >R1, construct a Gaussian sphere. The charge enclosed by the Gaussian sphere is equal to the charge over the spherical shell Q coulombs.

Applying Gauss's Law:- $\oint_s \vec{E} \cdot \vec{ds} = \frac{Q}{\varepsilon_0}$

$$\vec{E} \times 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\ \vec{a}_r \quad (r \geq R1)$$

The solution is identical with the point charge.

So, the field produced just outside a spherical shell of charge Q uniformly distributed over the surface is same as it the charge were concentrated at the center of the sphere as a point charge.

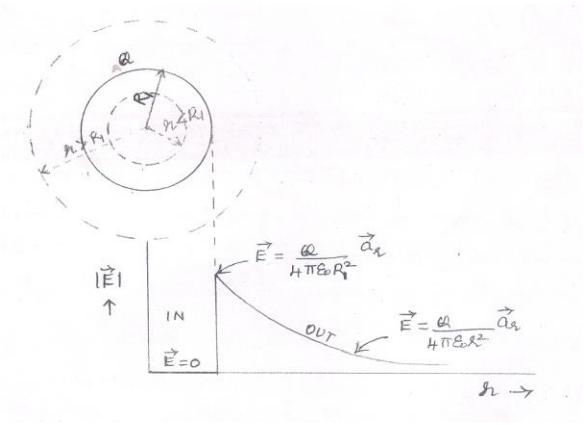


Figure (i) Single spherical shell of charge

(b) Two concentric spherical shell of charge

Consider the two spherical shells of charges Q1 and Q2, coulombs at radii R1 and R2 respectively. The charge is uniformly distributed.

Case (i) r<R

As no charge is enclosed within Gaussian sphere of radius r<R1.

$\vec{E}$ =0 (for r≤R1)

Case (ii) R1<r<R2

In the region between the spherical shells, the charge enclosed is Q_1, coulombs.

So, according to Gauss's Law,

$$\oint_s \vec{E} \cdot \vec{ds} = Q1 \Rightarrow \vec{E} \ \frac{Q}{4\pi\varepsilon_0 r^2} \ ((R1 \le r \le R2)$$

Case (iii) r>R2

In the region just outside the shells of charges Q_1 and Q_2, the charge enclosed is (Q_1+Q_2) coulombs.

By Gauss's Law

$$\oint_s \vec{E} \cdot \vec{ds} = Q_1+Q_2 \qquad \Rightarrow \vec{E} = \frac{Q_1+Q_2}{4\pi\varepsilon_0 r^2} \ ((r \ge R2)$$

Two concentric spherical shells of charges

3. Derive Poisson's and Laplace's equation.(May 2010) (Apl/May 2018)

From Gauss's Law

$$\oint_s \vec{D} \cdot \vec{ds} = Q$$

Assume the Gaussian surface contains the charge which is distributed uniformly over a volume.

Then $Q = \oint_v \rho_v dv$

$$\oint_s \vec{D} \cdot \vec{ds} = \oint_v \rho_v dv$$

Using divergence theorem L.H.S can be written as

$$\oint_s \vec{D} \cdot \vec{ds} = \oint_v \nabla \cdot \vec{D} \ dv \Rightarrow \oint_v \nabla \cdot \vec{D} \ dv = \oint_v \rho_v dv$$

$$\Rightarrow \nabla \cdot \vec{D} = \rho_v \text{ ---------- point from of gauss law}$$

$$\Rightarrow \nabla \cdot \vec{D} = \nabla \cdot \varepsilon \ \vec{E} = \rho_v \text{ ----------(1)}$$

But $\vec{E}$ =- ∇v

$$\Rightarrow \nabla \cdot \varepsilon \ \vec{E} = \nabla \cdot (-\varepsilon \nabla v) \ \text{……(2)}$$

From eqn (1)+(2) ,we can write

$$\nabla.(-\varepsilon \nabla v)= \rho_v \text{ for an inhomogeneous medium}$$

For homogeneous medium, the equation becomes

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} \text{ --------- POISSON'S EQUATION}$$

A special case of the equation occurs when $\rho_v = 0$ i.e. For charge free region.

$$\nabla^2 V = 0 \text{--------LAPLACE'S EQUATION}$$

4. Derive the expression for boundary conditions at the charge interface of two dielectric media. (May 2010) (OR)

At an interface separating dielectric1 (ε_{r1}) and dielectric2 (ε_{r2}), show that the tangential component of E is continuous across the boundary, whereas the normal component of E is discontinuous at the boundary. (Nov/Dec 2014)

State and prove the electro-static boundary conditions.(Nov/Dec 2015))(April/May 2017) .(Nov/Dec 2017) (Apl/May 2018)

If an electric field exists in a region consisting of two different media, the conditions that the field must satisfy at the interface separating the media are called boundary conditions.

These conditions are helpful determining the field on one side of the boundary if the field on the other side is known.

Depending upon the types of material the media are made of.

- Dielectric ε_{r1} and dielectric ε_{r2}
- Conductor and dielectric
- Conductor and free space

To determine the boundary conditions the following two equations of Electric and field are used.

(i) Conservative property

$$\oint_{\rho} \vec{E} . \vec{dl} = 0 \text{----------(1)}$$

(ii) Gauss's law

$$\oint_{s} \vec{D} . \vec{ds} = Q_{enc} \text{----------(2)}$$

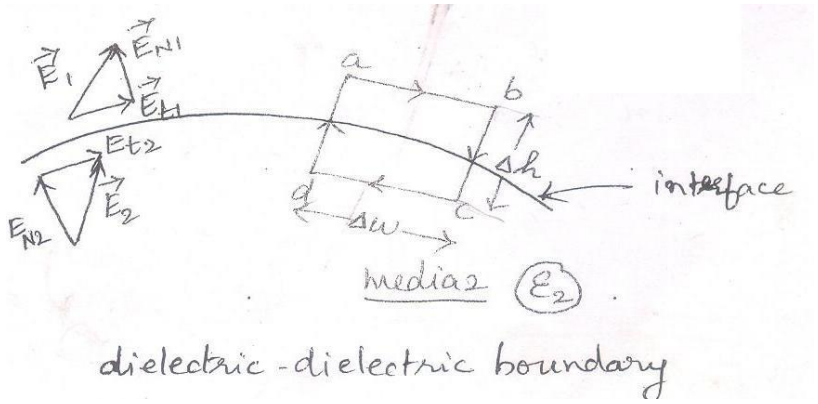
Also, the Electric field and Electric flux density are decomposed into two orthogonal components as

$$\vec{E} = \vec{E}_t + \vec{E}_n \text{ and } \vec{D} = \vec{D}_t + \vec{D}_n$$

Where $\vec{E}_t$ and $\vec{D}_t$ are tangential components and $\vec{E}_n$ and $\vec{D}_n$ are normal components respectively.

(a) Dielectric – Dielectric boundary:-

Consider the Electric field $\vec{E}$ existing in a region consisting of two different dielectrics



ε_{r1} and ε_{r2}

Electric field in media 1 and 2 can be written as

$$\vec{E}_1 = \vec{E}_{1t} + \vec{E}_{1n}$$

Applying equation 1 to the closed path abcda,

$$\vec{E}_{t1}\,\Delta\omega - \vec{E}_{N1}\,\frac{\Delta h}{2} - \vec{E}_{N2}\,\frac{\Delta h}{2} - \vec{E}_{t2}\,\Delta\omega + \vec{E}_{N2}\,\frac{\Delta h}{2} + \vec{E}_{N1}\,\frac{\Delta h}{2} = 0$$

As $\Delta h \to 0$ it at the interface, the above equation becomes

$$(\vec{E}_{t1} - \vec{E}_{t2})\Delta\omega = 0$$

$$\Rightarrow \vec{E}_{t1} = \vec{E}_{t2}$$

Thus the tangential components of $\vec{E}$ are the same on the two sides of the boundary. In other words, $\vec{E}_t$ undergoes no change on the boundary and it is said to be continuous across the boundary.

Since $D = \varepsilon E$

$$\frac{Dt1}{\varepsilon 1} = \vec{E}_{t1} \text{ and } \frac{Dt2}{\varepsilon 2} = \vec{E}_{t2}$$

As $\vec{E}_{t1} = \vec{E}_{t2} \Rightarrow \frac{Dt1}{\varepsilon 1} = \frac{Dt2}{\varepsilon 2}$

$$\Rightarrow \frac{Dt1}{Dt2} = \frac{\varepsilon 1}{\varepsilon 2}$$

The tangential components of electric flux density are discontinuous across the surface.

Similarly equation (2) is applied to the pill box and allowing $\Delta h \to 0$

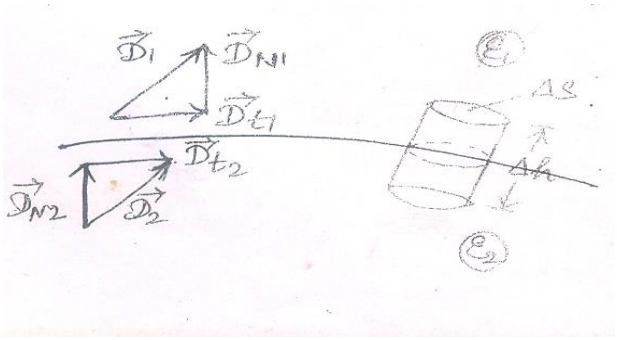
$$D_{N1}\,\vec{a}_z \cdot \Delta s\,\vec{a}_z + D_{N2}\,\vec{a}_z \cdot \Delta s\,(-\vec{a}_z) = \Delta Q$$

(Top) (Bottom)

As $\Delta Q = \rho_s \, \Delta s$, where ρ_s is the charge density placed at the boundary.

$D_{N1}\Delta s - D_{N2}\Delta s = \Delta s \ \rho_s$

$\Rightarrow D_{N1} - D_{N2} = \rho_s$



Note that $\vec{D}$ is directed from region 2 to 1

If no free charges exist at the interface, $\rho_s = 0$

$$D_{N1} = D_{N2}$$

Thus normal components of $\vec{D}$ is continuous across the surface (i.e.) in undergoes no charge at the boundary.

Since $\vec{D} = \varepsilon \vec{E}$

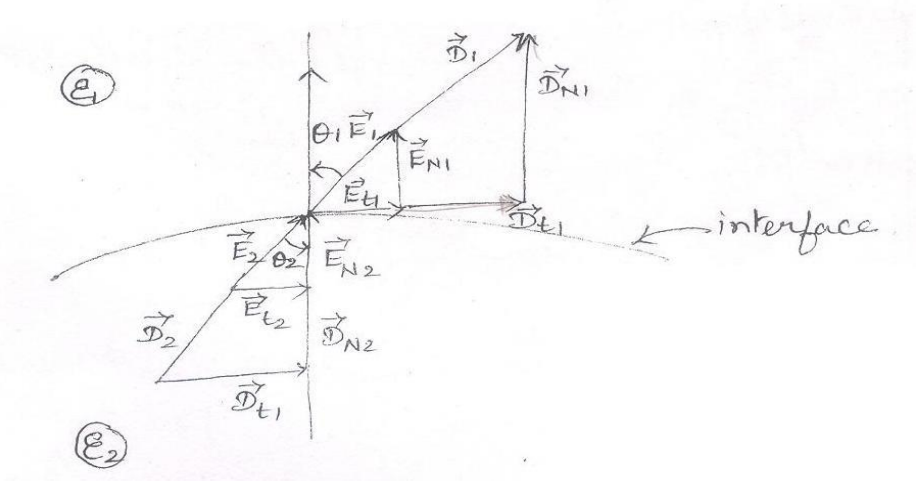
$$\vec{\varepsilon}_1 \ \vec{E}_{N1} = \vec{\varepsilon}_2 \ \vec{E}_{N2}$$

$$\Rightarrow \frac{E_{N1}}{E_{N2}} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}}$$

The normal components of $\vec{E}$ is discontinuous at the boundary.

This boundary conditions are used to determine the 'refraction' of the electric field across the surface.

Consider D_1 or $E_1$andD_2andE_2 making angle $Q_1$and $Q_2$with the normal to the interface



$E_1 \sin\theta_1 = E_{t1} = E_{t2} = E_2 \sin\theta_2$

$E_1 \sin\theta_2 = E_2 \sin\theta_2$ ---------(a)

We know $D_{N1} = D_{N2}$

$\varepsilon_1 E_1 \cos\theta_1 = \varepsilon_2 E_2 \cos\theta_2$---------- (b)

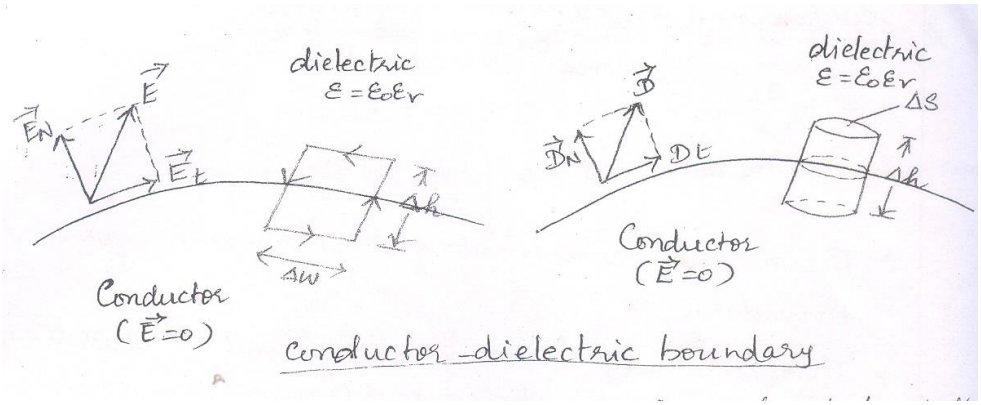
Dividing eqn (a) by (b)

$$\frac{E_1 \sin\theta_1}{\varepsilon_1 E_1 \cos\theta_1} = \frac{E_2 \sin\theta_2}{\varepsilon_2 E_2 \cos\theta_2}$$

$$\Rightarrow \frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

$$\Rightarrow \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_1}{\varepsilon_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

(b) Conductor – dielectric boundary



The same procedure is followed here also to find the boundary conditions for a conductor a dielectric interface. It should be remembered that Electric field intensity is zero inside the conductor.

Applying conservative law,

$0.\Delta\omega + 0\frac{\Delta h}{2} + E_n\frac{\Delta h}{2} - Et\,\Delta\omega - E_n\frac{\Delta h}{2}.0\frac{\Delta h}{2} = 0$

As $\Delta h \rightarrow 0$, $E_t = 0$

Where E_t is the tangential component of field in the dielectric medium?

Similarly by applying Gauss's law to the pill box and letting $\Delta h \rightarrow 0$ we get

$\Delta Q = Dn.\Delta s - 0\Delta s$

Since the normal component of $\vec{D}$ is zero in the conductor medium.

$$Dn = \frac{\Delta Q}{\Delta s} = \rho s \Rightarrow Dn = \rho s$$

Conclusions:-

No electric field may exist within a conductor

ie. $\rho_{v=0}$, E=0

Since $\vec{E} = -\nabla V = 0$ there is no potential inside a conductor (i.e) a conductor is an equippotential body

The electric field $\vec{E}$ can only be external to the conductor and normal to its surface.

$\vec{D}_t = \varepsilon_0 \varepsilon_r \vec{E}_t = 0$

$\vec{D}_n = \varepsilon_0 \varepsilon_r \vec{E}_n = \ell s$

(c) Conductor – free space boundary

The boundary conditions can be obtained from the previous question by replacing $\varepsilon_r = 1$ for free space.

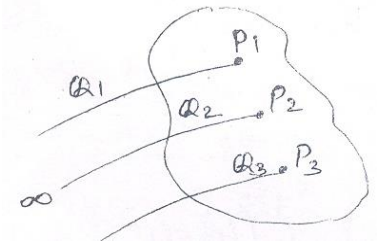
$D_t = \varepsilon_0 E_t = 0$

$D_n = \varepsilon_0 E_n = \rho s$

5. Derive the expression for energy density in electrostatic fields. (Nov/Dec 2012,2015,2017)

To determine the energy present in an assembly of charges, initially the amount of work necessary to assemble them must be determined.

Let us assume that three point charges are to be positioned in an initially empty space.



No work is required to transfer Q_1 from infinity to P_1 because the space is initially charge free and there is no electric field so $W_1=0$

The work is done in transferring Q_2 from infinity to P_2 is equal to the product of Q_2 and potential v_{21} at p_2 due to Q_1.

Similarly, the work done in positioning Q_3 at p_3 is equal to Q_3 ($V_{32}+V_{31}$) where v_{32} and v_{31} are the potentials at p_3 due to Q_2 and Q_1 respectively.

Hence the that work done in positioning the three charges is

$$W_E=W_1+W_2+W_3 =0+Q_2v_{21}+Q_3(v_{31}+v_{32}) \text{ ---------- (1)}$$

If the charges were positioned in reverse order

$$W_E=W_3+W_2+W_1 =0+Q_2V_{23}+Q_1(V_{12}+V_{13}) \text{---------- (2)}$$

Where V_{23} is the potentials at p_2 due to Q_3, V_{12} and V_{13} are potentials at p_1 due to Q_2 and Q_3

*Adding the eqn (1) and (2)

$$2W_E=Q_1(V_{12}+V_{13})+Q_2(V_{21}+V_{23})+Q_3(V_{31}+V_{32})=Q_1V_1+Q_2V_2+Q_3V_3$$

$$\Rightarrow W_E=\frac{1}{2}[Q_1V_1+Q_2V_2+Q_3V_3] \text{……..(3)}$$

Where V_1, V2and V_3 are total potentials at P_1, P2 and P_3 respectively

*In general,if there are ‘n’ point charges, then

$$W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k \text{ joule} \text{ ---------- (4)}$$

Instead of point charges, it the region has a continuous charge distribution of $\rho_v$then the summation becomes integration.

$$W_E = \frac{1}{2}\int_v (\rho_v \, dv)\, VQ = \int_v \rho_v \, dv$$

From Gauss law (point from)

$\rho_v = \nabla . \vec{D}$

$$W_E = \frac{1}{2}\int (\nabla . \vec{D}) V \, dv \text{ ---------- (5)}$$

Using vector identity

$\nabla .(V\vec{A}) = \vec{A} . \nabla V + V(\nabla .A)$

$$\therefore (\nabla . \vec{A})V = (\nabla . V\vec{A}) - (\vec{A} . \nabla V) \text{ ---------- (6)}$$

Let $\vec{A} = \vec{D}$ and substitute in eqn, (5)

$$W_E = \frac{1}{2}\int (\nabla . V\vec{D}) \, dv - \frac{1}{2}\int (\vec{D} . \nabla V)\, d\theta \text{ ---------- (7)}$$

Applying divergence to the first term on RHS

$$W_E = \frac{1}{2}\oint_s (V\vec{D}) . ds - \frac{1}{2}\int (\vec{D} . \nabla V) \, dv \text{ ---------- (8)}$$

We know that 'v' varies inversely to radius r ($V \times \frac{1}{r}$) and $\vec{D}$ varies inversely to square of radius r ($\vec{D} \times \frac{1}{r^2}$), While ds varies directly are square of radius ($ds \times r^2$)

Consequently the first integral must tend to zero as the surface Sbecomes very large of radius infinity.

$$\oint_{r\to\infty} (V\vec{D}).d\vec{s} \times \frac{1}{r} = 0$$

Hence

$$W_E = \frac{-1}{2}\oint_v (\vec{D} . \nabla V)\, dv = \frac{1}{2}\oint_v (\vec{D} . \vec{E}) \, dv$$

Since $\vec{D} = \varepsilon_0 \vec{E}$, $W_E = \frac{1}{2}\int \varepsilon_0 \vec{E}^2 \, dv$ joules

Electrostatic energy density W in (J/m^3) is

$$W_E = \frac{dW_E}{dv} = \frac{1}{2}\varepsilon_0 E^2 = \frac{D^2}{2\varepsilon_0}$$

$$\boxed{W_E = \frac{1}{2}\varepsilon_0 E^2}$$

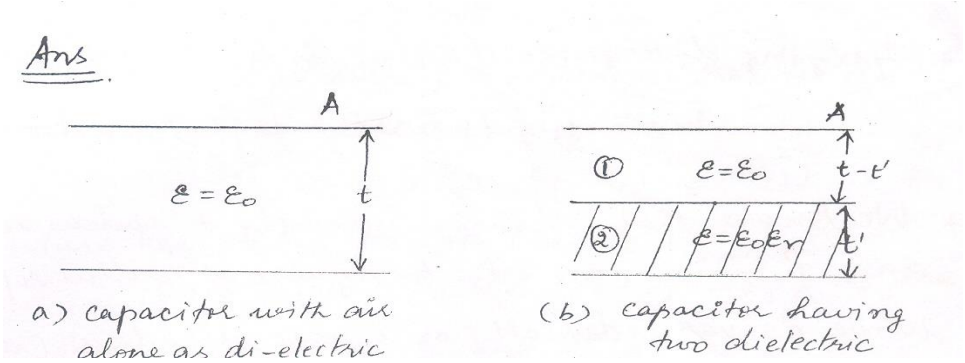
6. Deduce an expression for the capacitance of a parallel plate capacitor having two dielectric media. (Dec 2010)(May 2016)(April/May 2017) .(Nov/Dec 2017)

(OR)

A parallel plate capacitor has a plate separation t. The capacitance with air only between the plates is 'c' farads. When a slab of thickness't' and relative permittivity ε_r is placed on one of the plates, the capacitance is 'C'. Show that $\frac{C'}{C} = \frac{\varepsilon_0 t}{t' + \varepsilon_r (t - t')}$ (Apl/May 2018)

(OR)

Obtain an expression for the capacitance of a parallel plate capacitor with two dielectrics of relative permittivity ε_1 and ε_2 respectively interposed between the plates. (Apr/May 2015)



With air alone as dielectric as in figure (a)

Capacitance $c = \frac{\varepsilon_0 A}{t}$ (as $\varepsilon_{r=1}$)----------(1)

With the introduction of a slab of thickness'', the thickness of air film is reduced to (t-t')

Let E_1 and E_2 be the field intensities in the air film and slab respectively.Let V_1 and V_2 be the potential difference across them, If V is the total potential difference across the capacitor.

$$V = V_1 + V_2 \quad \ldots\ldots\ldots\ldots \quad (2)$$

$$V_1 = E_1 (t - t') \ldots\ldots\ldots\ldots (3)$$

$$V_2 = E_2 (t') \ldots\ldots\ldots\ldots (4)$$

If Q is the charge on the plates, then from the boundary conditions, the normal component of electric field densities $\vec{D}$ being equal in two dielectrics.

$$\vec{D_1} = \vec{D_2} = \vec{D}$$

Where $\vec{D} = \frac{Q}{A}$,but $\vec{D} = \varepsilon \vec{E}$

$$E_1 = \frac{D}{\varepsilon_0}, \; E_2 = \frac{D}{\varepsilon_0 \varepsilon_r} = E_1 = \frac{D}{\varepsilon} \ldots\ldots\ldots\ldots\ldots (5)$$

- Sub eqn (5) in (3)and(4)

$$V_1 = \frac{D}{\varepsilon_0} (t - t') \text{ and } V_2 = \frac{D}{\varepsilon_0 \varepsilon_r} (t') \ldots\ldots\ldots (6)$$

- Sub (6) in (2) , $V=\frac{D}{\varepsilon_0}(t-t')+\frac{D}{\varepsilon_0\varepsilon_r}(t')$

$$\Rightarrow V=\frac{D}{\varepsilon_0}[(t-t')+\frac{t'}{\varepsilon_r}]$$

$$V=\frac{Q}{A}[\frac{(t-t1)}{\varepsilon_0}+\frac{t'}{\varepsilon_0\varepsilon_r}] \ldots\ldots\ldots (7)$$

$$V=\frac{Q}{A\varepsilon_0}[\frac{t'}{\varepsilon_r}+(t-t')]$$

$$C'=\frac{Q}{V}=\frac{A\varepsilon_0}{\frac{t'}{\varepsilon_r}+(t-t')} \ldots\ldots\ldots (8)$$

$$C'=\varepsilon_0\frac{A\varepsilon_r}{t'+\varepsilon_r(t-t')} \ldots\ldots\ldots (9)$$

Dividing eqn (9) by (1)

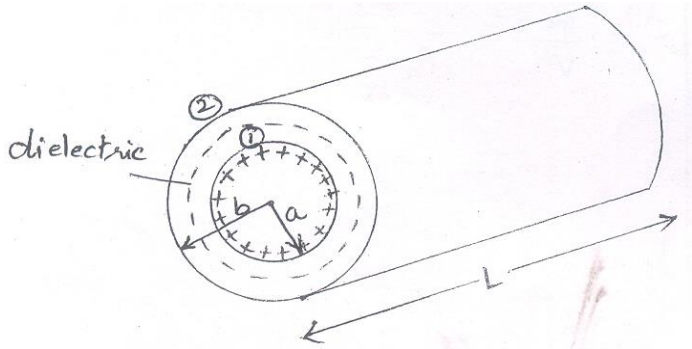
$$\boxed{\frac{C'}{C}=\frac{\varepsilon_0 t}{t'+\varepsilon_r(t-t')}} \quad \ldots\ldots\ldots (10)$$

Hence the proof

---

7. Derive the expression for capacitance between two co-axial cylinders of radii 'a'(inner) and 'b'(outer) respectively.(May 2010)

Consider two co-axial conductors of length 'L' m, having inner radius ' a' and outer radius ' b ' as in figure



Let the space between the conductors be filled with a homogeneous di-electric with permittivity ε_r. Assume the conductors 1 and 2 carry +Q and –Q coulombs of charges uniformly distributed on them.For convenience, assume cylindrical co-ordinate system.

The linear charge density $\rho_L$is given as$\rho_L=\frac{Q}{L}$ C/m ---------- (1)

Electric field intensity between two co-axial conductor is obtained by applying Gauss's law to an arbitrary Gaussian cylindrical surface of radius ' r'

- We know, $\vec{E}=\frac{\rho_L}{2\pi\varepsilon_0\rho}\vec{a}_\rho$ (a<r<b)-------(2)

- Where 'r' is the distance of point from the linear change
- Sub (1) in (2) ,

$\vec{E} = \frac{Q}{2\pi\varepsilon_0\rho_l}\ \vec{a}_\rho$ [here the medium of dielectric is not free space so ε_0 is replaced by ε]

Neglecting flux fringing of the cylindricalends.

$$V = -\int_2^1 \vec{E}\cdot\vec{dl} = -\int_{r=b}^{r=a} E\,\vec{a}_\rho \cdot d\rho\,\vec{a}_\rho = -\int_b^a \frac{Q}{2\pi\varepsilon_0\rho_l}\,d\rho = \frac{Q}{2\pi\varepsilon\rho_l}\ln[b/a]$$

Thus the capacitance of co-axial cylinder is

$$\boxed{C = \frac{2\pi\varepsilon\rho_l}{ln\frac{b}{a}}\ \text{Henry}}$$

8. Find the force in Newton on charge Q_1=20 µc situated at (0,1,2) m due to charge Q_2=-300µC situated at (2,0,0) m. (Dec 2012)

Solution:

$$\vec{F}_{12} = \frac{Q_1Q_2}{4\pi\varepsilon_0 R_{12}}\ \vec{a}_{12}$$

$$\vec{F}_{21} = \frac{Q_1Q_2}{4\pi\varepsilon_0 R_{21}}\ \vec{a}_{21}$$

$$\vec{R}_{21} = \vec{R}_1 - \vec{R}_2 = (0,1,2)-(2,0,0) = -2\vec{a}_x + \vec{a}_y + 2\vec{a}_z$$

$$\left|\vec{R}_{21}\right| = \sqrt{(2^2+1^2+2^2)} = 3$$

$$\vec{a}_{21} = \frac{-2\vec{a}_x + \vec{a}_y + 2\vec{a}_z}{3}$$

$$\vec{F}_1 = \frac{20\times10^{-6}\times(-300\times10^{-6})}{4\pi\times8.854\times10^{-2}\times3^2}\cdot\frac{(-2\vec{a}_x + \vec{a}_y + 2\vec{a}_z)}{3} = 4\vec{a}_x - 2\vec{a}_y - 4\vec{a}_z \quad \text{N}$$

$$\left|\vec{F}1\right| = \sqrt{(4^2+2^2+4^2)} = \mathbf{6N}$$

9. Determine the electric field intensity at P (-0.2, 0, -2.3)m due to a point charge of +5nc at Q(0.2,0.1,-2.5)m in air.(May 2010)

Solution:

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\ \vec{a}_R,\ \vec{a}_R = \frac{\vec{R}_{QP}}{\left|\vec{R}_{QP}\right|}$$

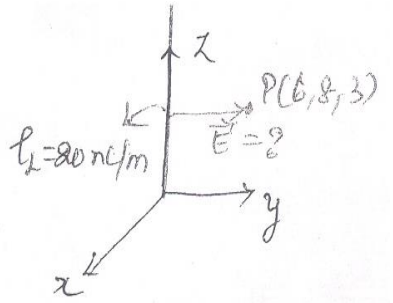
$$\vec{R}_{QP} = \vec{P} - \vec{Q} = -0.4\,\vec{a}_x - 0.1\,\vec{a}_y + 0.2\,\vec{a}_z$$

$$\vec{a}_R = \frac{\vec{R}_{QP}}{|\vec{R}_{QP}|} = \frac{-0.4\,\vec{a}_x - 0.1\,\vec{a}_y + 0.2\,\vec{a}_z}{0.45825}$$

$$\vec{E} = \frac{5\times10^{-9}}{4\pi\times8.845\times10^{-12}\times(0.45825)} \cdot \left[\frac{-0.4\,\vec{a}_x - 0.1\,\vec{a}_y + 0.2\,\vec{a}_z}{(0.45825)}\right]$$

$$\vec{E} = \mathbf{-186.779\,\vec{a}_x - 46.699\,\vec{a}_y + 93.389\,\vec{a}_z\ V/m}$$

10. A uniform line charge, infinite in extent with ρ_L = 20 nC/m lies along the axis. Find the $\vec{E}$ at (6,8,3)m



Solution:

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\,\vec{a}_r$$

$$\vec{a}_r = \frac{\vec{r}}{|\vec{r}|} = \frac{(6-0)\,\vec{a}_x + (8-0)\,\vec{a}_y}{\sqrt{6^2+8^2}}$$

$$\vec{a}_r = 0.6\,\vec{a}_x + 0.8\,\vec{a}_y$$

$$\Rightarrow \vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\,\vec{a}_r = \frac{20\times10^{-9}}{2\pi\times8.854\times10^{-12}\times10}\,[0.6\,\vec{a}_x + 0.8\,\vec{a}_y]$$

$$\Rightarrow \vec{E} = \mathbf{10.785\,\vec{a}_x + 14.38\,\vec{a}_y\ V/m}$$

11. Determine the potential difference between the points A and B which are at a distance of 0.5 m and 0.1 m respectively from a negative charged of 20×10^{-10} C.
Assume ε_0 =8.854 p F/m.

Solution:

Potential due to the point charge is given as

$$V = \frac{Q}{4\pi\varepsilon_0 r} \Rightarrow V_A = \frac{Q}{4\pi\varepsilon_0 r_A} \text{ and } V_B = \frac{Q}{4\pi\varepsilon_0 r_B}$$

$V_{BA}=V_A-V_B=\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A}-\frac{1}{r_B}\right]=\frac{-20\times10^{-10}}{4\pi\times8.854\times10^{-12}}\left[\frac{1}{0.5}-\frac{1}{0.1}\right]$

V_{BA}=143.80838 V

12. Calculate the work done in moving a 4c charge from B (1,0,0) to A (0,2,0) along a straight path in the field $\vec{E}=5\vec{a}_x$ V/m

Solution:

$\vec{E}=5\vec{a}_x$, Q=4C

$W=-Q\int_B^A E.dl$

$\vec{dl}=dx\vec{a}_x+dy\vec{a}_y+dz\vec{a}_z$ V/m $\vec{a}_x.\vec{a}_{x=1}$

$\vec{E}.dl=5\vec{dx}$

$W=-Q\int_B^A 5dx=-4\int_{x=1}^{x=0}5dx=-20[x]_1^0=20J$

13. The electric potential near the origin of a system of co-ordinates is V=$5x^2+8y^2+10z^2$ Find the electric field at (1, 2, 3).(April/May 2017)

Solution:

$V=ax^2+by^2+cz^2$

$\vec{E}=-\nabla V=-\frac{\partial v}{\partial x}\vec{a}_x+\frac{\partial v}{\partial y}\vec{a}_y+\frac{\partial v}{\partial z}\vec{a}_z$

$=-\left[10x\vec{a}_x+16y\vec{a}_y+20z\vec{a}_z\right]$

At given point, x=1,y=2, and z=3

$\Rightarrow\vec{E}=-\left[10(1)\vec{a}_x+16(2)\vec{a}_y+20(3)\vec{a}_z\right]$

$\vec{E}=-10\vec{a}_x-32b\vec{a}_y-60c\vec{a}_z$ V/m

14. A uniform surface charge of $\sigma=2\mu C/m^2$ situated at z = -2plane. What is the value of flux density at (1, 1, 1

Solution:

$D=\frac{\rho_s}{d}\vec{a}_x=\frac{2\times10^{-6}}{2}\vec{a}_x$

$D=10^{-6}\vec{a}_x$ c/m^2

15. Given field $\vec{E}=\left(-\frac{6y}{x^2}\right)\vec{I}_x+\left(\frac{6}{x}\right)\vec{I}_y+5\vec{I}_z$ V/m find the potential difference V_{AB} given A(-7,2,1) and B (4, 1, 2).

Solution:

$$\vec{E}=-\frac{\partial v}{\partial x}\vec{a}_x-\frac{\partial v}{\partial y}\vec{a}_y-\frac{\partial v}{\partial z}\vec{a}_z$$

$$\frac{dv}{dx}=\frac{6y}{x^2}\Rightarrow V_x=\int\frac{6y}{x^2}d_x=-\frac{6y}{x}+C_1$$

$$\frac{dv}{dy}=\frac{6}{x}\Rightarrow V_y=\int-\frac{6}{x}dy=-\frac{6y}{x}+C_2$$

$$\frac{dv}{dz}=-5\Rightarrow V_z=-\int 5dz=-5z+C_3$$

$$\Rightarrow V=\frac{-6y}{x}-5z+C$$

Let C=0 $\Rightarrow V=\frac{-6y}{x}-5z$

V_A at (-7, 2, 1)$=\frac{-6\times 2}{-7}-5=-3.29$

V_B at (4, 1, 2) $=-\frac{6}{4}-10=-11.5V$

$V_{AB}=V_B-V_A=-11.5+3.29=-8.21V$

16. Conducting spherical shells with radii a= 8cm and b= 20cm are maintained at a potential difference of 100V such that V(r= b) =0 and V(r=a) =70V. Determine V and E in the region between the shells. If $\varepsilon_r=2$ in the region, determine the total charge induced on the shells and the capacitance of the capacitor. (May 2011)

Solution:

Laplace equations in spherical co-ordinates is

$$\nabla^2V=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial v}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v}{\partial\phi^2}=0$$

As V depends only on 'r' Laplace's equation becomes

$$\nabla^2V=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v}{\partial r}\right)=0$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial v}{\partial r}\right)=0$$

Integrating once, $\left(r^2\frac{\partial v}{\partial r}\right)=A\Rightarrow\frac{\partial v}{\partial r}=\frac{A}{r^2}$

Again integrating, $V=\frac{-A}{r}+B$

The constants A and B determined from the boundary conditions, as below

When r=b, v=0

$$\Rightarrow 0=-\frac{A}{b}+B\Rightarrow B=\frac{A}{b}$$

$V = -\frac{A}{r} + \frac{A}{b}$

$V = A\left[\frac{1}{b} - \frac{1}{r}\right]$

When r=a, V=V_0

$\Rightarrow V_0 = A\left[\frac{1}{b} - \frac{1}{r}\right]$

$\Rightarrow A = \frac{V_0}{\left[\frac{1}{b} - \frac{1}{a}\right]}$

*thus $V = V_0 \frac{\left[\frac{1}{r} - \frac{1}{b}\right]}{\left[\frac{1}{b} - \frac{1}{a}\right]}$ ----------(1)

$\vec{E} = -\nabla V = -\frac{\partial v}{\partial r}\vec{a}_r$

$\vec{E} = -\frac{A}{r^2}\vec{a}_r$

$\Rightarrow \vec{E} = \frac{V_0}{r^2\left[\frac{1}{a} - \frac{1}{b}\right]}\vec{a}_r$----------(2)

*Q can also be found out using

$Q = \int_s \vec{D}.\vec{ds} = \int \varepsilon.\vec{E}.\vec{ds} = \int_{\theta=0}^{\Pi} \int_{\phi=0}^{2\Pi} \frac{\varepsilon_0 \varepsilon_r V_0 r^2}{r_2\left[\frac{1}{a} - \frac{1}{b}\right]} \sin\theta d\Phi d\theta$

$\Rightarrow Q = \frac{4\pi\varepsilon_0\varepsilon_r V_0}{\left[\frac{1}{a} - \frac{1}{b}\right]}$ …….(3)

Sub (3) in (2), $\vec{E} = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}\vec{a}r$

$C = \frac{Q}{V_0} = \frac{4\pi\varepsilon_0\varepsilon_r}{\left[\frac{1}{a} - \frac{1}{b}\right]}$ ---------- (4)

Now sub, a=0.08m,b=0.2 m,V_0=100 V

$V = 100\frac{\left[\frac{1}{r} - \frac{1}{0.2}\right]}{\left[\frac{1}{0.08} - \frac{1}{0.2}\right]} = 13.33\left[\frac{1}{r} - 5\right] = \frac{13.33}{r} - 66.65 \quad \vec{V} = \frac{13.33}{r^2}a_r$ V/m

Q= $\frac{4\pi \times 8.854x10^{-12} \times 2*(100)}{[\frac{1}{0.08}-\frac{1}{0.02}]}$ =2.96x10^{-9} C=2.96n C

C= $\frac{Q}{V_0}$ **=** $\frac{2.96*10^{-9}}{100}$ **=2.96x10^{-11} F=2.96 pF**

17. A spherical volume charge density distribution is given by

$$\rho = \rho_0\left(1-\frac{r^2}{a^2}\right); (r \le a)$$
$$= 0; (r > a)$$

(i) Calculate the total charge Q
(ii) Find the electric field intensity outside the charge distribution
(iii) Find the electric field intensity $\vec{E}$ inside
(iv) Show that the maximum value of $\vec{E}$ is at r= 0.745a

(i) Calculate of total charge Q :

$$Q = \iiint_{vol} \rho.dv$$

In spherical co-ordinate $dv = r^2 sin\theta dr d\theta d\phi$

$$Q = \int_{\emptyset=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{a} \rho.r^2\ sin\theta dr d\theta d\phi$$

$$= \iiint \rho_0\left(1-\frac{r^2}{a^2}\right).r^2 sin\theta dr d\theta d\phi$$

$$= \iiint \rho_0 .r^2 sin\theta dr d\theta d\phi - \iiint \rho_0 \frac{r^4}{a^2} sin\theta dr d\theta d\phi$$

$$= \iint \left(\frac{\rho_0 r^3}{3}\right)_0^a sin\theta d\theta d\phi \quad - \iint \left(\frac{\rho_0}{a^2}\frac{r^5}{5}\right) sin\theta d\theta d\phi$$

$$= \int_{\emptyset=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{\rho_0 a^3}{3}\ sin\theta d\theta d\phi - \int_{\emptyset=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{\rho_0 a^3}{5}\ sin\theta d\theta d\phi$$

$$= \int_{\emptyset=0}^{2\pi} \left(\frac{-\rho_0 a^3}{3}\right)(cos\theta)_0^{\pi} d\phi - \int_{\emptyset=0}^{2\pi} \left(\frac{-\rho_0 a^3}{5}\right)(cos\theta)_0^{\pi}.d\phi$$

$$= \int_0^{2\pi} 2.\frac{\rho_0 a^3}{3} d\phi + \int_0^{2\pi} \frac{\rho_0 a^3}{5}(-2).d\phi$$

$$= \frac{2\rho_0 a^3}{3}(\emptyset)_0^{2\pi} - \frac{2\rho_0 a^3}{5}(\emptyset)_0^{2\pi}$$

$$= \frac{2\rho_0 a^3}{3} \times 2\pi - \frac{2\rho_0 a^3}{5}(2\pi)$$

$$= 4\,\rho_0 a^3\,\pi\left(\frac{1}{3} - \frac{1}{5}\right)$$

$$\boldsymbol{Q = \frac{8\pi\rho_0 a^3}{15}}\ \textbf{Coulombs}$$

(ii) To find electric field intensity outside consider a Gaussian sphere of radius $r > a$ Using gauss's law:

$$\oint \vec{E}.\overrightarrow{ds} = \frac{Q}{\varepsilon_0}$$

$$\oint \vec{E}.\overrightarrow{a_r}\,.ds\,\vec{a}_r = \frac{8\pi\rho_0 a^3}{15\varepsilon_0}$$

$$E\int ds = \frac{8\pi\rho_0 a^3}{15\varepsilon_0}$$

$$for\ a\ spherical\ surface \int ds = 4\pi r^2$$

$$E(4\pi r^2) = \frac{8\pi\rho_0 a^3}{15\varepsilon_0}$$

$$\vec{E} = \frac{2\rho_0 a^3}{15\varepsilon_0 r^2}\vec{a}_r$$

(iii) To find electric field intensity inside the sphere
Consider a Gaussian sphere of radius $r < a$ and applying Gauss law:

$$\oint \vec{E}.\overrightarrow{ds} = \frac{Q}{\varepsilon_0}$$

$$\Longrightarrow E_{in} \times 4\pi r^2 = \iiint \frac{\rho dv}{\varepsilon_0}$$

$$\Longrightarrow E_{in} \times 4\varepsilon o\pi r^2 = \iiint \frac{\rho_0}{1}\left(1 - \frac{r^2}{a^2}\right) r^2\,.sin\theta\,.d_\theta\,d_\emptyset\,d_r$$

$$= \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \rho_0.r^2 d_r\,sin\theta d\theta d\phi \int_0^{2\pi}\int_0^{\pi}\int_0^{r} \frac{\rho_0.r^4}{a^2} d_r sin\theta d\theta d\phi$$

$$= \int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{\rho_0.r^3}{3} sin\theta d\theta d\phi - \int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi} \frac{\rho_0.r^5}{5a^2} sin\theta d\theta d\phi$$

Cancelling r^2 on both sides

$$E_{in} \times 4\pi\varepsilon_0 = \int_0^{2\pi} \frac{-\rho_0 r}{3}(cos)_0^{\pi} d_\emptyset + \int_0^{2\pi} \frac{\rho_0 r^3}{5a^2}(cos)_0^{\pi} d_\emptyset$$

$$= \int_0^{2\pi} \frac{2\rho_0 r}{3} d_\emptyset - \int_0^{2\pi} \frac{2\rho_0 r^3}{5a^2} d_\emptyset$$

$$= \frac{2\rho_0 r}{3}(\emptyset)_0^{2\pi} - \frac{2\rho_0 r^3}{5a^2}(\emptyset)_0^{2\pi}$$

$$= \frac{4\pi\rho_0 r}{3} - \frac{4\pi r^3}{5a^2}\rho_0$$

$$E_{in} \times 4\pi\varepsilon_0 = 4\pi\left[\frac{\rho_0 r}{3} - \frac{\rho_0 r^3}{5a^2}\right]$$

$$E_{in} = \frac{1}{\varepsilon_0}\rho_0 r\left[\frac{1}{3} - \frac{r^2}{5a^2}\right]$$

$\overrightarrow{E_{in}} = \frac{\rho_0 r}{\varepsilon_0}\left[\frac{1}{3} - \frac{r^2}{5a^2}\right]\vec{a}_r V/\text{m}$

(iv) To find the maximum value of $\vec{E}$

$$\frac{dE_{in}}{dr} = 0$$

$$\frac{\rho_0}{3\varepsilon_0} - \frac{\rho_0 3r^3}{5a^2} = 0$$

$$\frac{\rho_0}{\varepsilon_0}\left(\frac{1}{3} - \frac{3}{5}\frac{r^2}{a^2}\right) = 0$$

$$\frac{1}{3} - \frac{3}{5}\frac{r^2}{a^2} = 0$$

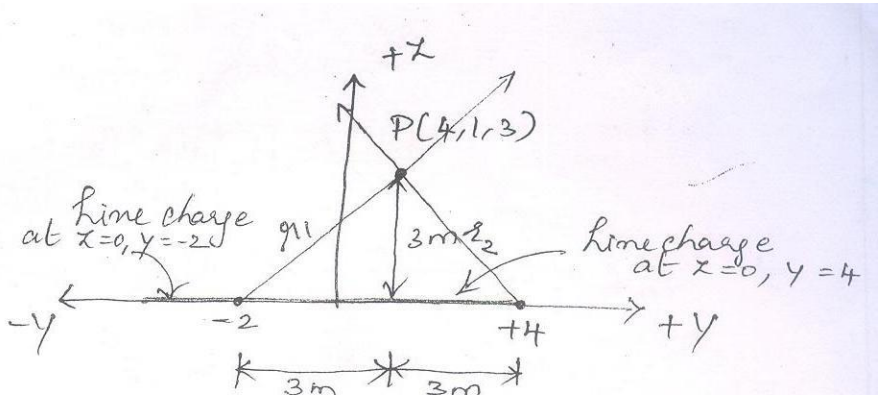
$$\frac{3}{5}\frac{r^2}{a^2} = \frac{1}{5}$$

$$\frac{r^2}{a^2} = \frac{5}{9}$$

$$r = 0.745a$$

$\vec{E}$ is maximum at $r = 0.745a$

18. Two uniformly distributed line charges of $\lambda = 5n\ c/m$ each are parallel to the x axis, one at z=0 and y=-2m and other at z=0 and y=4m determine the field at (4, 1, 3) m.



Let r_1 and r_2 be the directed line segments from the line 1 and 2 respectively to the point P (in YZ plane)

$$r_1 = 3\vec{a}_y + 3\,\vec{a}_z \Longrightarrow |r_1| = \sqrt{3^2 + 3^2} = 4.24$$
$$r_2 = -3\vec{a}_y + 3\,\vec{a}_z \Longrightarrow |r_2| = \sqrt{3^2 + 3^2} = 4.24$$

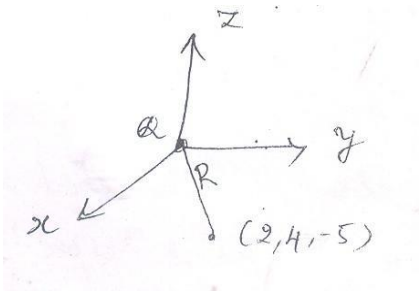
Electric fields $\overrightarrow{E_1} = \frac{\lambda}{2\pi\epsilon_0 r_1}\left(\frac{3\vec{a}_y + 3\,\vec{a}_z}{r_1}\right)$-------(1) and $\overrightarrow{E_2} = \frac{\lambda}{2\pi\epsilon_0 r_2}\left(\frac{-3\vec{a}_y + 3\,\vec{a}_z}{r_2}\right)$------- (2)

Adding (1) and (2)

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0}\left[\frac{1}{r_1^2} + \frac{1}{r_2^2}\right](3\vec{a}_z)$$

$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0}\left[\frac{1}{18} + \frac{1}{18}\right]3\,\vec{a}_z = \frac{\lambda}{2\pi\epsilon_0}\left(\frac{2}{18}\right)\vec{a}_z = \frac{5\times10^{-9}}{6\pi\varepsilon_0}\vec{a}_z$$
$$\vec{E} = \frac{59.92}{2}\vec{a}_z \;\; V/m$$
$$\Longrightarrow \;\; \boldsymbol{\vec{E} = 30\vec{a}_z \;\; V/m}$$

19. A point charge Q = 50nc is located at the origin. Find the electric density D at (2, 4,-5m) in Cartesian system of co-ordinates.



The electric flux density at a distance R from the change in given by

$$\vec{D} = \frac{Q}{4\pi R^2}\overrightarrow{a_r}$$

When $\vec{R} = 2\overrightarrow{a_x} + 4\overrightarrow{a_y} - 5\overrightarrow{a_z}$

$$\overrightarrow{a_r} = \frac{\vec{R}}{|\vec{R}|} = \frac{2\overrightarrow{a_x} + 4\overrightarrow{a_y} - 5\overrightarrow{a_z}}{\sqrt{2^2 + 4^2 + 5^2}}$$

$R^2 = 2^2 + 4^2 + 5^2$=45

$$\vec{D} = \frac{50\times10^{-9}}{4\pi(45)} \cdot \frac{2\overrightarrow{a_x} + 4\overrightarrow{a_y} - 5\overrightarrow{a_z}}{\sqrt{45}}$$

$$\boldsymbol{\vec{D} = 2.64\overrightarrow{a_x} + 5.28\overrightarrow{a_y} - 6.60\overrightarrow{a_z} c/m^2}$$

20. Determine the Capacitance of a capacitor consisting of two parallel metal plates 30cm x 30cm surface area, separated by 5mm in air. What is the total energy stored by the capacitor, if it is charged to a potential difference of 500V? What is the energy density? (June 2013, Nov/Dec 2014))

Given data:

$\varepsilon_0 = 8.854 \times 10^{-2}$

$\varepsilon_r = 1, A = 0.3 \times 0.3 = 0.09\ m^2$

$t = 5 \times 10^{-2}\ m$

(i) Capacitance $C = \varepsilon_0 \varepsilon_r \frac{A}{t} = 8.854 \times 10^{-12} \times \frac{0.09}{5\times10} - 2$ = 159.2 pF

(ii) Energy stored $= \frac{1}{2}CV^2 = \frac{1}{2} \times 159.2 \times 10^{-12} \times (500)^2 = 19.90 \times 10^{-6}\ J$

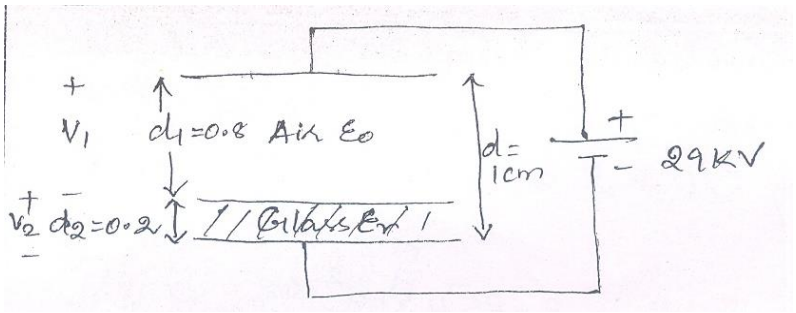
(iii) Energy density $= 1/2\varepsilon_0 E^2 = 1/2 \times 8.854 \times 10^{-12} \times 10^{10} = 4.427 \times 10^{-2}\ J/m^2$

For parallel plate capacitor,

$\left|\vec{E}\right| = \frac{Q}{A\varepsilon_0\varepsilon_r} = \frac{C\,V}{A\varepsilon_0\varepsilon_r} = \frac{159.2\times10^{-12}\times500}{(0.09\times8.54\times10^{-2}\times1} = 99.89 \times 10^3\ V/m$

$Note$: $\Longrightarrow E^2 = 1 \times 10^{10}$ This is using to find energy density

21. A parallel plate capacitor with a separation of 1cm has 29kV applied, when air was used as dielectric. Assume that the dielectric strength of air as 30 kV/cm. A piece of glass with $\varepsilon_r = 6.5$ with a dielectric strength of 290 kV/cm with thickness 0.2cm is inserted. Find whether glass will breakdown or air? (Dec2010)



Potential across the capacitor is $V = 29\ KV$

When a glass dielectric is inserted $V = V_1 + V_2$

V_1 = Potential across air

V_2 = Potential across glass

$$V_1 = E_1 d_1 \Longrightarrow V_1 = E_1 \times 0.8$$
$$V_2 = E_2(d - d_1) \Longrightarrow V_2 = E_2 \times 0.8$$

Taking $\frac{V_1}{V_2} = \frac{d_1 D_1}{\varepsilon_1} \times \frac{\varepsilon_2}{(d-d_1)D_2} = \frac{0.8}{\varepsilon_0} \times \frac{\varepsilon_0\times6.5}{0.2} = 26 \Longrightarrow V_1 = 26V_2$

$$V = V_1 + V_2 = 29KV \Rightarrow 26V_2 + V_2 = 29 \Rightarrow V_2 = 1.07Kv$$
$$\Rightarrow V_1 = 27.93KV$$

*when $V_1 = 27.93$ is applied across the air dielectric, dielectric stress or potential gradient

$$E_1 = \frac{V_1}{d_1} = \frac{27.93}{0.8} = 34.91\ KV/m$$

This stress exceeds the maximum dielectric strength of air (30 KV/Cm) so, the air breakdown $E_2 \frac{V_2}{d_2} = \frac{1.07}{0.2} = 5.35$----which is in limits of 290 kv/cm glass will not breakdown

22. IF $V = \frac{60sin\theta}{r^2}$ volts find V and E at (3, 60°, 25°) where V is the potential and E is the electric field intensity

Given:

$V = \frac{60sin\theta}{r^2}$ P (3,60°,25°)

$\rightarrow r = 3, \theta = 60°, \Phi = 25°$

$V = \frac{60sin60°}{3^2} = 5.77\ V$

For spherical co-ordinates

$\vec{E} = -\nabla V$

$= -[\frac{\partial v}{\partial r}\ \vec{a}r + \frac{1}{r}\frac{\partial v}{\partial \theta}\vec{a}v + \frac{1}{r\ sin\theta}\frac{\partial v}{\partial \emptyset}\vec{a}\emptyset]$

$\frac{\partial v}{\partial r} = \frac{\partial}{\partial r}\left(\frac{60sin\theta}{r^2}\right) = 60sin\theta(\frac{-2}{r^3}) = \frac{-120\ sin\theta}{r^3}$

$\frac{1}{r}\frac{\partial v}{\partial \theta} = \frac{60}{r^3}\frac{\partial}{\partial \theta}(sin\theta) = \frac{60cos\theta}{r^3}$

$\frac{\partial v}{\partial \emptyset}$=0

$\Rightarrow \vec{E} = -[\frac{-120sin\theta}{r^3}\vec{a}r + \frac{60cos\theta}{r^3}\vec{a}\theta]$

$\vec{E} = \frac{120sin\theta}{r^3}\vec{a}r + \frac{60cos\theta}{r^3}\vec{a}\theta$

$\vec{E}$ at P(3,60°,25°) is given as, $\vec{E} = [\frac{120sin60}{3^3}\vec{a}r - \frac{60cos\theta}{3^3}\vec{a}\theta]$

$\Rightarrow \vec{E} = 3.85\ \vec{a}_r - 1.11\ \vec{a}_\theta$

23.Derive an expression for capacitance of concentric spheres.(Nov/Dec 2015).

Consider a concentric sphere of inner radius aand outer radius b as shown in figure respectively. The inner shell carries a charge of +Q and outer shell carries a charge of –Q. The outer shell is earthed by a conducing wire. Conducing point P at a distance r from the axis of the shell at which the capacitance is determined.

The electric field intensity of a conducting sphere is given by

$$E = \frac{Q}{4\pi \in r^2}$$

The potential difference between the two spheres will be equal to the amount of work done in moving a unit positive charge from inner cylinder of radius aand outer cylinder of radius b.

$$V = V_A - V_B$$

$$V = -\int_b^a E dr$$

$$V = -\int_b^a \frac{Q}{4\pi \in r^2} dr$$

$$V = -\frac{Q}{4\pi \in}\int_b^a 1/r^2 dr$$

$$V = -\frac{Q}{4\pi \in}\left[\frac{r^{-2}+1}{-2+1}\right]$$

$$V = -\frac{Q}{4\pi \in}[-1/r]$$

$$V = \frac{Q}{4\pi \in}\left[\frac{1}{a} - 1/b\right]$$

But C=Q/V

Therefore,

$C = \frac{4\pi\in}{\left[\frac{1}{a}-1/b\right]} = \frac{4\pi\in}{[(b-a)/ab]} = \frac{4\pi\in(ab)}{[(b-a)]}$ Farad.

24. Derive an expression for polarization "P".(Nov/Dec 2015). .(Nov/Dec 2017) (Apl/May 2018)

Dielectric Polarization

Dipole may be described by its dipole moment P. If Q is the charge and l is the vector (distance) from the negative to the positive charge, the dipole moment is given by

$$P=Q.l$$

If there are n dipole per unit volume ΔV, then there are n ΔV dipoles, and the total dipole moment is given by

$$P_{total} = \sum_{i=1}^{n\,\Delta V} P_i$$

When electric field is applied across the dielectric, dipoles are formed in a particular direction due to separation of charges in the material, the dielectric is said to be polarized. Polarization is defined as dipole moment per unit volume.

$$P = \underset{\Delta V \to 0}{Lt} \frac{1}{\Delta V}\sum_{i=1}^{n\,\Delta V} P_i \; c/m^2$$

If the dipoles are aligned in random orientation, polarization P has zero value whereas if dipoles are aligned in same direction, polarization P has a significant value.

Consider a bound charge $Q_b$across a small element surface ds in dielectric containing non-polar molecules. Then

$$Q_b = -\oint_s^{\cdot} P.ds$$

If Q is the free charge enclosed by the surface s, the total enclosed charge is

$$Q_T = Q_b + Q$$

By Gauss's law, $Q_T = \oint_s^{\cdot} \varepsilon_0 \; E.ds$

The free charge enclosed,

$$Q = Q_T - Q_b$$

$$= \oint_s \varepsilon_0 \ E.ds + \oint_s P.ds = \oint_s (\varepsilon_0 \ E + P)ds$$

But $Q = \oint_s D.ds$

Equating these two equations,

$$D = \varepsilon_0 \ E + P$$

Polarization P can be Written as

$$P = x\varepsilon_0 \ E$$

Where, x is electric susceptitality.

$$D = \varepsilon_0 \ E + x\varepsilon_0 \ E = \varepsilon_0 \ E(1 + x) = \varepsilon_0 \ \varepsilon_R E$$

Where, $\varepsilon_R = x + 1$

25. Write Laplace's equation in Cartesian Co-ordinates. And obtain the solution when V is function of X only for the boundary condition $V = V_1$ at $x = x_1$ and $V = V_2$ at $x = x_2$.(April/May 2016)

Solution:

Laplace equation in Cartesian co-ordinates:

$$\boldsymbol{\Delta^2 V} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \frac{\partial^2 v}{\partial z^2} = 0$$

Since the potential V varies only with X, then

$$\frac{\partial^2 v}{\partial x^2} = 0$$

Integrate $\frac{\partial v}{\partial x} = A$

$$V = Ax + B$$

Where A and B are arbitrary constants.

Apply the boundary conditions,

$V = V_1$ at $X = X_1$

$V = V_2$ at $X = X_2$

Then,

$$V_1 = Ax_1 + B$$

$$V_2 = Ax_2 + B$$

$$A = \frac{V_1 - V_2}{X_1 - X_2}$$

$$B = \frac{V_2 X_1 - V_1 X_2}{X_1 - X_2}$$

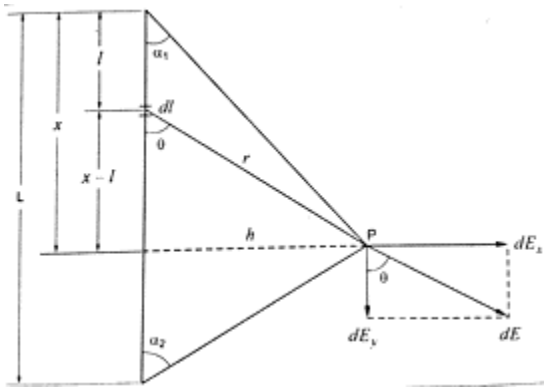
Substituting these Values (Constants)

$$V = \frac{V_1(X - X_2) - V_2(X - X_1)}{X_1 - X_2}$$

---

26. Explain the potential at a point in an electric field. Derive the electric field intensity at any point in a field due to a point charge.(April/May 2016)

Consider a uniformly charged line of length `L' whose linear charge density is **ρ** coulomb/metre. Consider a small element ***dl*** at a distance ***`l'*** from one end of the charged line as shown in fig 30.1. Let P be any point at a distance ***`r'***from the element ***dl.***



The electric field at a point P due to the charge element **ρ *dl***is given by

$$\mathbf{dE} = \frac{\rho dl}{4\pi\varepsilon r^2}$$

The x and y components of electric field **dE**are given by

$$dE_x = dE \sin\theta$$

$$dE_y = dE \cos\theta$$

$$\text{Then} \quad dE_x = \frac{\rho\, dl \sin\theta}{4\pi\varepsilon\, r^2}$$

From the figure 30.1

$$x - l = h \cot\theta$$

$$-dl = -h \operatorname{cosec}^2\theta \cdot d\theta$$

$$\text{and} \quad \frac{h}{r} = \sin\theta$$

$$r = h \operatorname{cosec}\theta$$

Substituting the values of dl and r in dE_x equation

$$dE_x = \frac{\rho\, (h \operatorname{cosec}^2\theta\, d\theta) \sin\theta}{4\pi\varepsilon\, (h \operatorname{cosec}\theta)^2}$$

$$dE_x = \frac{\rho \sin\theta\, d\theta}{4\pi\varepsilon\, h}$$

The electric field E_x due to the entire length of the line charge is given by

$$E_x = \int_{\alpha_1}^{\pi-\alpha_2} \frac{\rho \sin\theta \, d\theta}{4\pi\varepsilon h}$$

$$E_x = \frac{\rho}{4\pi\varepsilon h} \int_{\alpha_1}^{\pi-\alpha_2} \sin\theta \, d\theta$$

$$E_x = \frac{\rho}{4\pi\varepsilon h} \left[-\cos\theta \right]_{\alpha_1}^{\pi-\alpha_2}$$

$$E_x = \frac{\rho}{4\pi\varepsilon h} \left[\cos\alpha_1 + \cos\alpha_2 \right]$$

Similarly for y component of E,

$$dE_y = \frac{\rho}{4\pi\varepsilon h} \cos\theta \, d\theta$$

$$E_y = \frac{\rho}{4\pi\varepsilon h} \int_{\alpha_1}^{\pi-\alpha_2} (\cos\theta) \, d\theta$$

$$E_y = \frac{\rho}{4\pi\varepsilon h} \left[\sin\theta \right]_{\alpha_1}^{\pi-\alpha_2}$$

$$E_y = \frac{\rho}{4\pi\varepsilon h} \left[\sin\alpha_2 - \sin\alpha_1 \right]$$

Case (i) If the point P is at bisector of a line, then α_1 and $\alpha_2 = \alpha$

$$E_y = 0, \qquad E \text{ becomes } E_x$$

$$E = \frac{\rho}{2\pi\varepsilon h}$$

Case (ii) If the line is infinitely long, then $\alpha = 0$

$$E_y = 0, \qquad E \text{ becomes } E_x$$

$$E = \frac{\rho}{2\pi\varepsilon h}$$

If test charge q is moved from one point r_1 to another point r_2 in the electric field, there will be work done against the force

$$W = -\int_{r_1}^{r_2} q E \, dh = -q \int_{r_1}^{r_2} \frac{\rho_l}{2\pi\varepsilon h} \, dh$$

$$= -\frac{q\rho_l}{2\pi\varepsilon} \int_{r_1}^{r_2} \frac{1}{h} \, dh = -\frac{q\rho_l}{2\pi\varepsilon} \ln\frac{r_2}{r_1}$$

Work done per charge is potential

$$V = \frac{W}{q} = -\frac{\rho_l}{2\pi\varepsilon} ln \frac{r_2}{r_1}$$

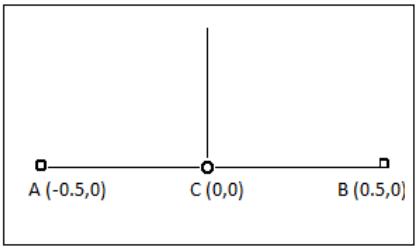
$$V = \frac{\rho_l}{2\pi\varepsilon} ln \left[\frac{r_1}{r_2}\right]$$

---

27. Calculate the potential at a point P (0, 0) m due to point charges Q1 and Q2. Q1 = 10^{-12} Coulomb is located at (0.5, 0) m and Q2 = -10^{-11} Coulomb is located at (-0.5, 0) m.(April/May 2016)

Solution:

Q1 = 10-12 C at (0.5, 0) m
Q2 = -10-11 C at (-0.5, 0) m.



Potential at C due to Charge Q_1

$$V_A = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$= \frac{10^{-12}}{4\times\pi\times8.854\times10^{-12}\times1^2}$$

= 0.089V

Potential at C due to Charge Q2

$$V_B = \frac{Q}{4\pi\varepsilon_o r^2}$$

$$= \frac{-10^{-11}}{4\times\pi\times8.854\times10^{-12}\times1^2}$$

= - 0.898V

Potential at C due to Charge Q2 at a point (Q1,Q2)

= 0.162V

---

28. Derive the electric potential due to an uniformly charged infinite line with uniform charge distribution.

Potential Difference due to Infinite Line Charge

Consider an infinite line charge along z-axis having uniform line charge density ρL C/m
The point B is at a radial distance rB while point A is at a radial distance rA from the charge, as shown in the Fig. 4.9.l.

The E due to infinite line charge along z-axis is known and given by,

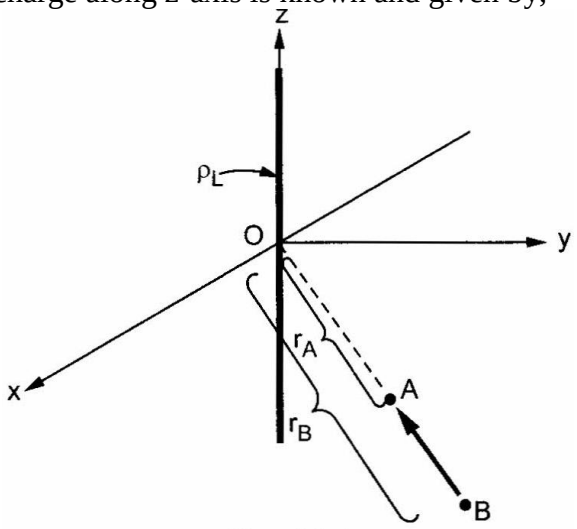


Fig. 4.9.1

$$\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\bar{a}_r$$

while $d\bar{L} = dr\,\bar{a}_r$

in cylindrical system in radial direction.

$$\therefore V_{AB} = -\int_B^A \bar{E}\bullet d\bar{L} = -\int_{r_B}^{r_A} \frac{\rho_L}{2\pi\varepsilon_0 r}\bar{a}_r \bullet dr\,\bar{a}_r$$

$$= -\int_{r_B}^{r_A} \frac{\rho_L}{2\pi\varepsilon_0 r}dr = \frac{-\rho_L}{2\pi\varepsilon_0}\int_{r_B}^{r_A}\frac{1}{r}dr = \frac{-\rho_L}{2\pi\varepsilon_0}\left[ln\ r\right]_{r_B}^{r_A} = \frac{-\rho_L}{2\pi\varepsilon_0}\left[ln\ r_A - ln\ r_B\right]$$

29. Obtain the electric potential due to electric dipole. (April/May 2017)

In spherical coordinates, the potential at point P due to the charge + Q is given by,

$$V_1 = \frac{+Q}{4\pi\varepsilon_0 r_1} \qquad \ldots (4.14.1)$$

The potential at P due to the charge – Q is given by,

$$V_2 = \frac{-Q}{4\pi\varepsilon_0 r_2} \qquad \ldots (4.14.2)$$

The total potential at point P is the algebraic sum of V_1 and V_2.

$$\therefore \quad V = V_1 + V_2$$

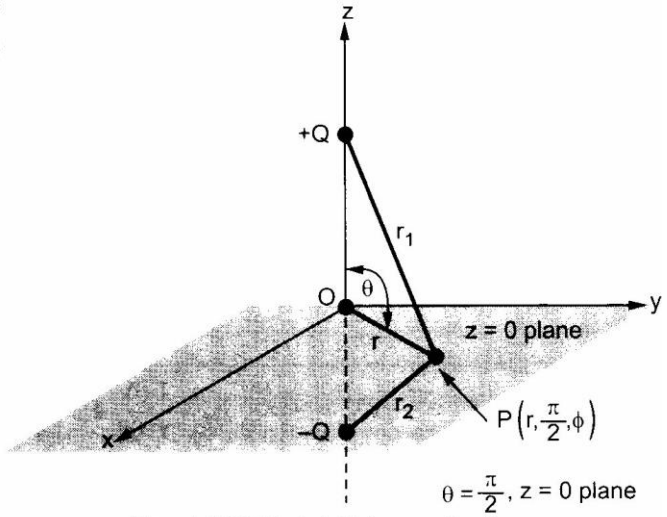


Fig. 4.14.2 Point P in z = 0 plane

$$= \frac{+Q}{4\pi\varepsilon_0 r_1} - \frac{Q}{4\pi\varepsilon_0 r_2} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{Q}{4\pi\varepsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right] \quad \ldots (4.14.3)$$

30. In spherical coordinates V=-25V on a conductor at r=2cm and V=150V at r=35cm. The space between the conductor is a dielectric of $\varepsilon_r = 3.12$. Find the surface charge density on the conductors.(Nov/ Dec 2016)

$$V = \frac{-A}{r} + B$$

The constants are determined from the boundary conditions

$$-25 = \frac{-A}{0.02} + B \qquad 150 = \frac{-A}{0.35} + B$$

giving $$v = \frac{-3.71}{r} + 160.61 \text{ (V)}$$

$$\mathbf{E} = -\nabla V = -\frac{d}{dr}\left(\frac{-3.71}{r} + 160.61\right)\mathbf{a}_r = \frac{-3.71}{r^2}\mathbf{a}_r \text{ (V/m)}$$

$$\mathbf{D} = \varepsilon_0\varepsilon_r\mathbf{E} = \frac{-0.103}{r^2}\mathbf{a}_r \text{ (nC/m}^2\text{)}$$

On a conductor surface, $D_n = \rho_s$.

at $r = 0.02$ m: $$\rho_s = \frac{-0.103}{(0.02)^2} = -256 \text{ nC/m}^2$$

at $r = 0.35$ m: $$\rho_s = \frac{+0.103}{(0.35)^2} = +0.837 \text{ nC/m}^2$$

31. Find the potential at r_A=5m with respect to r_B=15mdue to a point charge Q=500pC at the origin and zero reference at infinity (Nov/ Dec 2016) (Apl/May 2018)

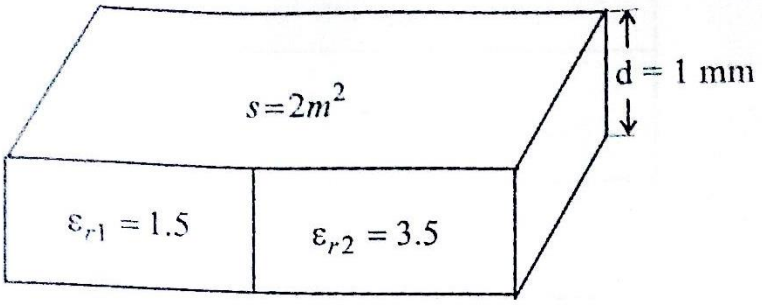
$$V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

$$V_{AB} = \frac{500 \times 10^{-12}}{4\pi \times 8.854 \times 10^{-12}}\left(\frac{1}{5} - \frac{1}{15}\right)$$

$$= 4.494(0.2 - 0.67)$$

$$V_{AB} = -2.112$$

32. Find the capacitance of a parallel plate capacitor with dielectric ε_{r1}=1.5, ε_{r2} =3.5 .Each occupy one half of the space between the plates of area 2m^2 and d=10^{-3}m. (Nov/ Dec 2016)



Here two capacitors are formed in parallel with s=2m2, $\varepsilon_{r1} = 1.5, \varepsilon_{r2} = 3.5$ and $d_1 = d_2 = d = 1 \times 10^{-3}m$

Each capacitor comprising one half of volume, ie $s = \frac{1}{2} \times 2 = 1m^2$

$$C_1 = \frac{\varepsilon_0 \varepsilon_{r1} s}{d}$$

$$= \frac{8.854\times10^{-12}\times1.5\times1}{1\times10^{-3}} = 13.28\times10^{-9}\text{F}$$

$$C_2 = \frac{\varepsilon_0 \varepsilon_{r2} s}{d}$$

$$= \frac{8.854\times10^{-12}\times3.5\times1}{1\times10^{-3}}$$

$$C_2 = 30.99\times10^{-9}\text{F}$$

Therefore $C_{equ} = C_1 + C_2$

$$= 13.28\times10^{-9} + 30.99\times10^{-9} = 44.27n\text{F}$$

33. Two point charges -4uC and 5uc are located at (2,-1,3) and (0,4,-2) . Find the potential at (1,0,1) assuming zero potential at infinity(Apl/May 2018)

Potential due to a point charge is given by $V_A = \frac{Q}{4\pi\varepsilon_0 r_a}$

$$Q_1 = -4\mu C$$

$$P_1 = (2,-1,3); \qquad P = (1,0,1)$$

$$\hat{r}_{01} = 1\hat{i} - 1\hat{j} + 2\hat{k} = 2.45$$

$$V_{10} = \frac{-4\times10^{-6}}{4\pi\varepsilon_0(2.45)} = -14.68\times10^3 \text{ V}$$

$$Q_2 = 5\mu C; \qquad P_2 = (0,4,-2); \qquad P = (1,0,1)$$

$$\hat{r}_{02} = -1\hat{i} + 4\hat{j} - 3\hat{k} = 5.1$$

$$V_{20} = \frac{5\times10^{-6}}{4\pi\varepsilon_0(5.1)} = 8.81\times10^3 \text{ V}$$

Total voltage, $V = -14.68\times10^3 + 8.81\times10^3$

$$V = -5.87\times10^3 \text{ V} = -5.87 \text{ kV}.$$

UNIVERSITY QUESTIONS
PART -A

1. Define electric potential and potential difference. (April 2010)
2. Write down Laplace's and Poisson's equations. (April 2010 & April 2011)
3. Define electric dipole and electric dipole moment. (Nov 2010)
4. Define polarization. (April 2011)
5. Define dielectric strength of materials and give its unit. (April 2010)
6. Draw the equipotential lines and electric field lines for a parallel plate capacitor.
8. What is the practical significance of dielectric strength? (April 2008)
9. Give examples for uniform and non-uniform electric fields? (May 2007)
10. Comment on the values of the relative permittivity.
11. State the properties of electric flux lines. (Nov/Dec 2014)
12. Give the significant physical differences between Poisson's and Laplace's equations. (Nov/Dec 2014)
14. What do you understand by linear, surface and volume charge densities?

15. What is the relation between intensity of electric field $\vec{E}$ and electric flux density $\vec{D}$ in free space?
16. State the principle of superposition of fields.
17. What is principle of conservation of charges?
18. Write the relation between potential and electric field. (Dec 2007)
19. Define electric scalar potential.
20. Define conduction and displacement current.
21. State boundary conditions for two different dielectric medium.
22. Write boundary conditions on a perfect conductor interface.
23. Why is the electrostatic potential continuous at a boundary?
24. Write the equation of continuity.
25. What is the practical significance of Lorentz's Force? (Apr/May 2015)
26. Find the electric field intensity in free space if D = 30 a_x C/m^3. (Apr/May 2015)

PART- B

1. Find the potential at any point along the axis of uniformly charged disc of ρ_sC/m^2. The disc has radius of 'a' m. (Dec 2010)

2. Determine the variation of field from point to point due to
   (a) Single spherical shell of charged with radius R_1,
   (b) Two concentric spherical shells of charge of radii R. (inner) and R2 (outer).
3. Derive Poisson's and Laplace's equation. (May 2010)

4. Derive the expression for boundary conditions at the charge interface of two dielectric media. (May 2010) (OR)
At an interface separating dielectric1(ε_{r1}) and dielectric2(ε_{r2}), show that the tangential component of E is continuous across the boundary, whereas the normal component of E is discontinuous at the boundary. (Nov/Dec 2014,2015)

5. Derive the expression for energy density in electrostatic fields. (Dec 2012,15)
6. Deduce an expression for the capacitance of a parallel plate capacitor having two dielectric media. (Dec 2010)
(OR)
A parallel plate capacitor has a plate separation t. The capacitance with air only between the plates is 'c' farads. When a slab of thickness 't' and relative permittivity ε_r is placed on one of the plates, the capacitance is 'C'. Show that $\frac{C'}{C} = \frac{\varepsilon_0 t}{t' + \varepsilon_r (t - t')}$
(OR)
Obtain an expression for the capacitance of a parallel plate capacitor with two dielectrics of relative permittivity ε_1 and ε_2 respectively interposed between the plates. (Apr/May 2015)

7. Derive the expression for capacitance between two co-axial cylinders of radii 'a'(inner) and 'b' (outer) respectively. (May 2010)

8. Find the force in Newton on charge Q_1=20 µc situated at (0,1,2) m due to charge Q_2 =-300µC situated at (2,0,0) m. (Dec 2012)

9. Determine the electric field intensity at P (-0.2, 0, -2.3)m due to a point charge of +5nc at Q(0.2, 0.1, -2.5)m in air. (May 2010)

10. A uniform line charge, infinite in extent with ρ_L = 20 nC/m lies along the axis. Find the $\vec{E}$ at (6, 8, 3)m

11. Determine the potential difference between the points A and B which are at a distance of 0.5 m and 0.1 m respectively from a negative charged of $20x10^{-10}$ C. Assume ε_0 = 8.854 p F/m.

12. Calculate the work done in moving a 4c charge from B (1,0,0) to A (0,2,0) along a straight path in the field $\vec{E} = 5\vec{a}_x$ V/m

13. The electric potential near the origin of a system of co-ordinates is V= $ax^2 + by^2 + cz^2$ Find the electric field at (1, 2, 3).

14. A uniform surface charge of $\sigma = 2\mu C/m^2$ situated at z = -2 plane. What is the value of flux density at (1, 1, 1) m?

15. Given field $\vec{E} = \left(-\frac{6y}{x^2}\right)\vec{I}_x + \left(\frac{6}{x}\right)\vec{I}_y + 5\vec{I}_z$ V/m find the potential difference V_{AB} given A(-7,2,1) and B (4, 1, 2).

16. Conducting spherical shells with radii a = 8cm and b = 20cm are maintained at a potential difference of 100V such that V(r = b) =0 and V(r =a) =70V. Determine V and E in the region between the shells. If ε_r = 2 in the region, determine the total charge induced on the shells and the capacitance of the capacitor. (May 2011)

17. A spherical volume charge density distribution is given by

$$\rho = \rho_0\left(1 - \frac{r^2}{a^2}\right); (r \le a)$$
$$= 0; (r > a)$$

(v) Calculate the total charge Q

(vi) Find the electric field intensity outside the charge distribution

(vii) Find the electric field intensity $\vec{E}$ inside

(viii) Show that the maximum value of $\vec{E}$ is at r= 0.745a

18. Two uniformly distributed line charges of $\lambda = 5n\ c/m$ each are parallel to the x axis, one at z = 0 and y = -2m and other at z = 0 and y = 4m determine the field at (4, 1, 3) m.

19. A point charge Q = 50nc is located at the origin. Find the electric density D at (2, 4,-5m) in Cartesian system of co-ordinates.

20. Determine the Capacitance of a capacitor consisting of two parallel metal plates 30cm x 30cm surface area, separated by 5mm in air. What is the total energy stored by the capacitor, if it

is charged to a potential difference of 500V? What is the energy density? (June 2013, Nov/Dec 2014))

21. A parallel plate capacitor with a separation of 1cm has 29kV applied, when air was used as dielectric. Assume that the dielectric strength of air as 30 kV/cm. A piece of glass with $\varepsilon_r = 6.5$ with a dielectric strength of 290 kV/cm with thickness 0.2cm is inserted. Find whether glass will breakdown or air? (Dec2010)

22. IF $V = \frac{60sin\theta}{r^2}$ volts find V and E at (3, 60°, 25°) where V is the potential and E is the electric field intensity

23. Derive an expression for capacitance of concentric spheres. (Nov/Dec 2015).

24. Derive an expression for polarization "P".(Nov/Dec 2015).

25.Explain the potential at a point in an electric field. Derive the electric field intensity at any point in a field due to a point charge.(April/May 2016)

26. Calculate the potential at a point P (0, 0) m due to point charges Q1 and Q2. Q1 = 10^{-12} Coulomb is located at (0.5, 0) m and Q2 = -10^{-11} Coulomb is located at (-0.5, 0) m.(April/May 2016

UNIT III – MAGNETOSTATICS

Lorentz Law of force, magnetic field intensity – Biot–savart Law - Ampere's Law – Magnetic field due to straight conductors, circular loop, infinite sheet of current – Magnetic flux density (B) – B in free space, conductor, magnetic materials – Magnetization – Magnetic field in multiple media – Boundary conditions – Scalar and vector potential – Magnetic force – Torque – Inductance – Energy density – Magnetic circuits.

Part A – Q & A

1. Define Scalar Magnetic Potential. (Apl/May 2018)

It is defined as dead quantity whose negative gradient gives the magnetic intensity if there is no current source present.

$\vec{H} = -\nabla V_m$ Only $\vec{J} = 0$, Where V_m is the magnetic scalar potential

$$V_m = -\int_b^a \vec{H}.d\vec{L}$$

2. Define Vector Magnetic Potential and state its unit. (May 2010)(April/May 2017) (Apl/May 2018)

It is defined as that quantity whose curl gives the magnetic flux density.

$\vec{B} = \nabla \times \vec{A}$, Where $\vec{A}$ is the magnetic vector potential and measured in Wb/m and given as,

$$\vec{A} = \frac{\mu}{4\pi} \iiint_V \frac{J}{r}\, dr\ Wb/m$$

3. State Laplace's equation for Scalar Magnetic Potential. (Dec 2010, May 2011)

$\nabla^2 V_m = 0$ for $\vec{J} = 0$, Where V_m is the scalar magnetic potential.

4. State Poisson's equation for Magnetic Field. (Dec 2010, May 2011)

$\nabla^2 \vec{A} = -\mu_0 \vec{J}$

This is the Poisson's equation for magneto static field, where $\vec{A}$ is the vector magnetic potential.

5. What is the relation between Magnetic Flux Density and Magnetic Field Intensity?

$\vec{B} = \mu \vec{H}$, Where $\vec{B} = \mu_0 \mu_r \vec{H}$ for any material medium and for free space $\vec{B} = \mu_0 \vec{H}$

For all non magnetic media $\mu_r = 1$, while for magnetic materials μ_r is greater than unity.

6. Define Biot–savart law? (Dec 2010)(April/May 2017)

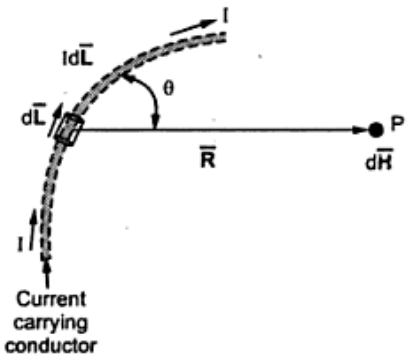
Biot-savart law is defined as, the magnetic field intensity $d\vec{H}$ produced at a point P, due to a differential current element $Id\vec{L}$ is

i. Proportional to the product of the current I and differential length dL
ii. Proportional to sine of the angle between element and line joining point P to the element.
iii. Inversely proportional to square of the distance R between point P and the element.

i.e., $d\vec{H} = \frac{KId\vec{L}\sin\theta}{R^2}$

(OR)

$$d\vec{H} = \frac{Id\vec{L}\times\vec{a}_R}{4\pi R^2} A/m$$



Where, $\vec{a}_R$ = unit vector in the direction from differential current element to point P.

7. Distinguish between Scalar and Vector Magnetic Potential. (Nov/Dec 2014,2015)

In scalar magnetic potential, the curl of gradient of scalar magnetic potential is zero.

$\nabla\times\nabla V_m = 0$

In vector magnetic potential the divergence of curl of a vector is always zero.

$\nabla.(\nabla\times\vec{A}) = 0$

8. A long straight line wire carries a current I = 1 amp. At what distance is the magnetic field H = 1 A/m. (June 2009)

Solution: I=1 A, H= 1 A/m

$$\vec{H} = \frac{I}{2\pi r}\vec{a}_\varphi A/m$$

$$=> |\vec{H}| = \frac{I}{2\pi r}$$

$$=> \ 1 = \frac{1}{2\pi r} \quad => r = \frac{1}{2\pi} = \mathbf{0.1592m}$$

At the perpendicular distance of **0.1592 m** from the wire, $\vec{H}$ will be 1 A/m.

9. State Ampere's Circuital law? (May/June 2014,2016)(Nov/Dec 2016) (Nov/Dec 2017)

The Ampere's Circuital law states that the line integral of Magnetic Field Intensity $\vec{H}$ around a closed path is exactly equal to the direct current enclosed by that path.

$\oint \vec{H}.d\vec{l} = \vec{I}$

The path enclosing the direct current I need not be circular and it may be any irregular shape. The law does not depend on the shape of the path but the path must enclose the entire current conductor once.

10. Can a static magnetic field exist in a good conductor? Explain.

A static magnetic field can exist in a good conductor when conductor carriescurrent and it produces the flux which can exist inside the conductor. Due to this flux, magnetic field and intensity exists at a point inside the good conductor.

11. Define under what condition will the E-field irrotational and solenoidal.

When curl of E field is zero,$\vec{E}$ field is irrotational.

$$(\nabla \times \vec{E}) = 0$$

When divergence of E field in zero, $\vec{E}$ field is solenoidal.

$$(\nabla . \vec{E}) = 0$$

12. For the configuration shown calculate the value of $\oint \vec{H}.\vec{dL}$

By Ampere's circuital law$\oint \vec{H}.d\vec{L} = I$ =current enclosed by closed path.

The current is +100 A (outwards) and -10 A (inwards), i.e., total 100-10=90 A.

$$\oint H.dL = 90 \text{ A}$$

The current shown outside the closed path has no effect on the integral.

13. List the application's of Ampere's circuital law. (Dec 2007)

(i) Hdue to infinitely long straight conductor.
(ii) Hdue to a co-axial cable.
(iii) Hdue to a infinite sheet of current.

14. State the expression of $\vec{H}$ due to infinite sheet of current. (Dec 2007)

$$\vec{H} = \frac{1}{2}\vec{K} \times \vec{a}_N$$

Where, $\vec{a}_N$ is the unit vector normal from the current sheet to the point at which $\vec{H}$ is to be obtained and $\vec{K}$ is the sheet current density in A/m in the direction of current.

15. What is $\vec{H}$ due to a long straight current carrying conductor? (May 2008)

$$\vec{H} = \frac{I}{2\pi r}\vec{a}_\phi \text{ A/m}$$

Where, $\vec{a}_\phi$ is the unit vector in the direction of field intensity to be obtained and $\vec{K}$ is the sheet charge density in the direction of current.

16. What is the force and its direction on a conductor in +Z axis carrying current I placed in a magnetic field $\vec{B}$ along X axis?

$$\vec{F} = \vec{Il} \times \vec{B}$$

$$=> \vec{F} = I(\vec{a}_z)l \times B(\vec{a}_x) \quad = \quad BIl(\vec{a}_y) \quad N/m$$

Where, $\vec{F}$ is the resultant force, I is the current in the conductor and B is the magnetic field density in which the conductor is placed.

17. Write Lorentz force equation. Give application of the same. (Nov 2005, May 2010)

The Lorentz force equation is given by $\vec{F} = m\vec{a} = Q(\vec{E} + \vec{v} \times \vec{B})$ Newton.

This equation relates the mechanical force with the electric force. The Lorentz force equation is generally used to determine electron in magnetron. It can also be used to determine proton path in cyclotron. It is extensively used in determining plasma characteristics in a magneto hydrodynamic generator. In general, motion of a charged particle in a combined electric and magnetic fields can be determined using the Lorentz force equation.

18. What is Magnetic Dipole Moment?

The magnetic dipole moment of a current loop is defined as the product of current through the loop (I) and area of the loop (S), directed normal to the current loop.

$$\vec{m} = (IS)\vec{a}_n \text{ A.m}^2$$

19. Define Magnetization.

Magnetization is defined as the magnetic dipole moment per unit volume. Its unit is A/m.

$$\vec{M} = \lim_{\Delta V \to 0} \frac{1}{\Delta v} \sum_{a=1}^{n} m\vec{a}$$

20. Define ferro magnetic materials.

The materials in which the atoms have large dipolemoment due to electron spin magnetic moments are called ferromagnetic materials.

21. Write down magnetic boundary conditions.(May/June 2016)

a. The normal components of flux density vector B is continuous across the boundary but the normal components of field intensity vector H is discontinuous at the boundary.

$$B_{N1} = B_{N2}$$

$$\frac{H_{N1}}{H_{N2}} = \frac{\mu_2}{\mu_1} = \frac{\mu_{r2}}{\mu_{r1}}$$

b. The tangential component of field intensity vector H is continuous across the boundary but the tangential components of flux density vector B are discontinuous at the boundary.

$$H_{t1} = H_{t2}$$

$$\frac{B_{t1}}{B_{t2}} = \frac{\mu_1}{\mu_2} = \frac{\mu_{r1}}{\mu_{r2}}$$

22. What is the expression for inductance of a toroid? (Nov/Dec 2017)

The inductance of a toroid is given

$$L = \frac{\mu N^2 I A}{(2\pi R)}$$

Where, A=Area of cross section of the toroid
N = Number of turns of the toroid
I = Current through the toroid
R = Mean radius of the toroid.

23. Define the terms: Magnetic Moment and Magnetic Permeability. (May 2010)

The magnetic moment of a current loop is defined as the product of current through the loop (I) and area of the loop (S), directed normal to the current loop.

$$\vec{m} = (IS)\vec{a}_n \text{ A.m}^2$$

Magnetic permeability is defined as the ratio of the magnetic flux density to the magnetic field intensity.

$$\mu = \frac{\vec{B}}{H}$$

24. What are major classifications of magnetic materials?

On the basis of the magnetic behavior, the magnetic materials is classified as diamagnetic, paramagnetic, anti-ferromagnetic, ferro magnetic and super magnetic.

25. Calculate inductance of a ring shaped coil having a mean diameter of 20cm wound on a wooden core of 20cm diameter. The winding is uniformly distributed and contain 200turns.

The inductance of the ring shaped coil is given by

$$L = \frac{\mu N^2 A}{2\pi r} = \frac{\mu_0 \mu_r N^2 A}{2\pi r}$$

For wood μ_r=0.9999
A=πr^2=0.0141 m^2

$$\therefore L = \frac{4\times\pi\times10^{-7}\times(200)^2\times0.03141}{2\times\pi\times1\times10^{-2}}$$

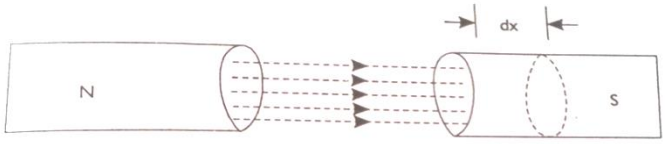
$\therefore$ L=**25.128 mH**

26. Write the expression for the magnetic force between an electromagnet and an armature to be attracted. (May 2013) (Nov/Dec 2017)

Increase in the stored energy in the magnetic field is equal to the work input

$$Fdx = \frac{1}{2}\frac{B^2 a}{\mu_0 \mu_R} dx$$

$$F = \frac{1}{2}\frac{B^2 a}{\mu_0 \mu_r} \text{ Newton}$$



Where a – area of cross section
B - magnetic field intensity

27. Write the expression for magnetic field 'H' at the centre of a circular coil carrying a current 'I' ampere. The radius of the coil is 'a' m. (Nov 2012)

$$H = \frac{I}{2\pi a}$$

28. Find the inductance per unit length of a long solenoid of N turns and having a length 'L' metre. Assume that it carries a current of 'I' amperes.(May 2012)(May/June 2014)

$$L = \frac{\mu N^2 A}{l} \text{ Henry}$$

29. State Ohm's law for magnetic circuits. (Nov 2012) (Nov/Dec 2014)

Consider a resistor, which is constructed from resistive material of conductivity σ, length l and cross sectional area A, connected to a battery volts.

Let I be the current at all the points in the circuit and E be the total field at the same points.

Then ohm's law is given by,

$$\frac{\vec{J}}{\sigma} = \vec{E}$$

Where, $\vec{E} = \vec{E}_C + \vec{E}_E$

$\vec{E}_C$is the static electric field due to charges

$\vec{E}_E$is field generated by other causes ,for example by a battery

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30. Determine the value of magnetic field 'H' at the center of a circular loop carrying a current of 10A. The radius of theloop is2 m. (Nov/Dec 2014)

$$H = \frac{I}{2\pi r}$$

$$= \frac{10}{2\pi * 2}$$

H = 0.795 A/m

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31. Find the maximum torque on an 100 turn rectangular coil,0.2m by 0.3m,carrying a current of 2A in the field of flux density 5W/m^2 (April/May2015)

Given:

N= 100

A=0.2×0.3=0.06m^2

I=2A

B=5W/m^2

T_{max} = NIAB

=100×2×0.06×5

=60N-m

---

32. Find the interaction between two charges $4*10^{-8}$ and $6*10^{-5}$ spaced 10cm apart in kerosene (ε_r = 2.0). (April/May 2015)

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_1 R_{12}^2}$$

$$= \frac{4 \times 10^{-8} \times 6 \times 10^{-5}}{4\pi \times 8.854 \times 10^{-2} \times 2 \times (10 \times 10^{-2})^2}$$

$$= 89.8876 \times 10^{-11}\text{N}$$

---

33. What is the mutual inductance of the two inductively coupled coils with self-inductance of 25mH and 100mH? (Nov/Dec 2015)

Given:

L_1=25mH

L_2=100mH

K=1

M=k$\sqrt{L_1 L_2}$=$\sqrt{25 \times 100}$

=50mH

34. A conductor 4 m long lies along the y axis with the current of 10 A in A_y direction if the field is B=0.05a_xTesla. calculate the force on the conductor(Nov/Dec 2016)

$$\overline{F} = I\overline{dl} \times \overline{B}$$

$$= (10\overline{a_y})(4) \times 0.05\overline{a_x}$$

$$Since, \quad \overline{a_y} \times \overline{a_x} = -\overline{a_z}$$

$$= -2\overline{a_z}$$

35. What is the total force acting on a moving charge Q in the presence of both electric and magnetic fields (Apl/May 2018)

$$F = Q(E + (v * B))$$

is called as lorentz force equation

Part B – Q & A

1. What is Biot-savart law? Find the magnetic flux density at a point Z on the axis of a circular loop of radius 'r' that carries a direct current I.(May 2010,May 2013)

(OR)

Find the magnetic field intensity $\vec{H}$ due to circular current conductor, carrying a current I, for the following cases:

(a) At a height 'z' from the center of the circular loop.

(b) At the center of the circular loop.(May 2016)

(OR)

Find the magnetic flux density at a point on the axis of a circular loop of a radius 'r' carrying a direct current I. (Nov/Dec 2017)

Obtain an expression for magnetic flux density and magnetic field intensity at any point along the axis of circular coil. (Nov/Dec 2015) (Apl/May 2018)

Develop an expression for the magnetic field intensity at any point on the line through the center at a distance `h' m from the center and perpendicular to the plane of a circular loop ( in XY plane) of radius `a'm and carrying a current I Ampere in the anti-clockwise direction.

Answer:

Biot-savart law is defined as, the magnetic field intensity $d\vec{H}$ produced at a point P, due to a differential current element $Id\vec{L}$ is

i. Proportional to the product of the current I and differential length dL
ii. Proportional to sine of the angle between element and line joining point P to the element.
iii. Inversely proportional to square of the distance R between point P and the element.

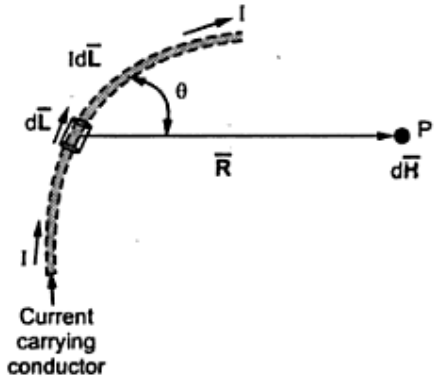
Mathematically, the Biot-savart law is given as:

$$d\vec{H} \propto \frac{IdL\sin\theta}{R^2}$$

$$\text{i.e., } d\vec{H} = \frac{KId\vec{L}\sin\theta}{R^2} \text{ --------- (1)}$$

(OR)

$$d\vec{H} = \frac{Id\vec{L}\times\vec{a}_R}{4\pi R^2} A/m$$



Where, $\vec{a}_R$ = unit vector in the direction from differential current element to point P.

From rule of cross product $d\vec{L}\times\vec{a}_R = dL|\vec{a}_R|\sin\theta = dL\sin\theta$ Since $|\vec{a}_R|=1$

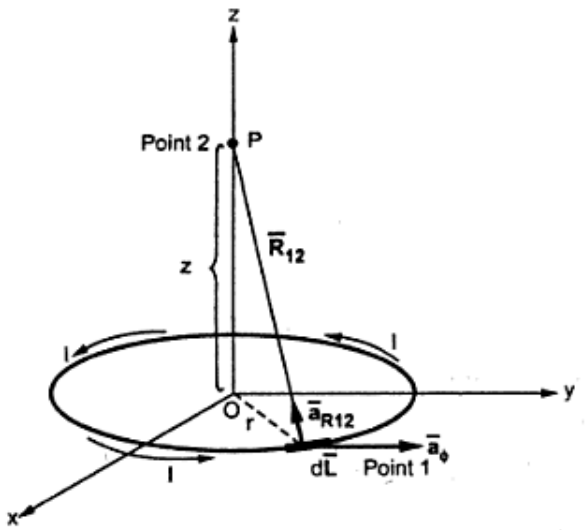
Replacing in equation (1)

$$d\vec{H} = \frac{Id\vec{L}\times\vec{a}_R}{4\pi R^2} A/m$$

$$\text{but } \vec{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{\vec{R}}{R}$$

$$d\vec{H}=\frac{Id\vec{L}\times\vec{R}}{4\pi R^3}\ A/m$$

$\vec{H}$ on the axis of a circular loop:



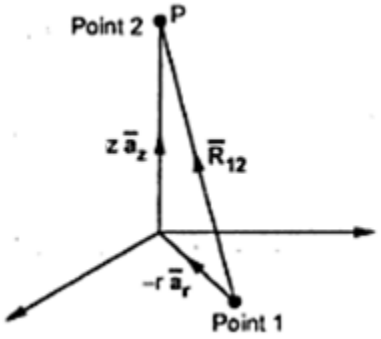
Consider a circular loop carrying a direct current I, placed in 'xy' plane. The magnetic field intensity $\vec{H}$at point P is to be obtained. The point P is at a distance 'z' from the plane of the circular loop along its axis.

In cylindrical co-ordinate system.

$$d\vec{L} = dr\vec{a}_r + rd\emptyset\vec{a}_\emptyset + dz\vec{a}_z$$

The $Id\vec{L}$ is tangential at a point in $\vec{a}_\emptyset$ direction.

$$\mathbf{Id\vec{L} = I\ rd\emptyset\vec{a}_\emptyset}$$



The unit vector $\vec{a}_{R12} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|}$

$\vec{R}_{12} = \vec{R}_2 - \vec{R}_1 = (z\vec{a}_z) + (-r\vec{a}_r) = -r\vec{a}_r + z\vec{a}_z$

$|\vec{R}_{12}| = \sqrt{(-r)^2 + (z)^2}$

$\vec{a}_{R12} = \frac{-r\vec{a}_r + z\vec{a}_z}{\sqrt{r^2+z^2}}$

$$d\vec{L}\times\vec{a}_{R12} = \begin{vmatrix} \vec{a}_r & \vec{a}_\emptyset & \vec{a}_z \\ 0 & rd\emptyset & 0 \\ \frac{-r}{\sqrt{r^2+z^2}} & 0 & \frac{z}{\sqrt{r^2+z^2}} \end{vmatrix} = \frac{1}{\sqrt{r^2+z^2}}\left[z\,rd\emptyset\vec{a}_r + r^2 d\emptyset\vec{a}_z\right]$$

According to Biot-savart Law

$$d\vec{H} = \frac{Id\vec{L}\times\vec{a}_{R12}}{4\pi R_{12}^2} = \frac{I\,[zr\,d\emptyset\vec{a}_r + r^2 d\emptyset\vec{a}_z]}{4\pi\,\sqrt{(r^2+z^2)}\left(\sqrt{(r^2+z^2)}\right)^2}$$

The total $\vec{H}$ is to be obtained by integrating $d\vec{H}$ over the circular loop with$\emptyset = 0$ to 2π

$$\vec{H} = \int_{\emptyset=0}^{2\pi} \frac{I[zr\,\vec{a}_r + r^2\vec{a}_z]d\emptyset}{4\pi(r^2+z^2)^{3/2}}$$

$$= \frac{I}{4\pi}\left\{\int_{\emptyset=0}^{2\pi} \frac{zrd\emptyset}{(r^2+z^2)^{3/2}}\vec{a}_r + \int_{\emptyset=0}^{2\pi} \frac{r^2\,d\emptyset}{(r^2+z^2)^{3/2}}\vec{a}_z\right\}$$

Consideringthe first integral and expressing the unit vector $\vec{a}_r$ in rectangular co-ordinate system,

$$\int_{\emptyset=0}^{2\pi} \frac{zrd\emptyset}{(r^2+z^2)^{3/2}}\vec{a}_r = \int_{\emptyset=0}^{2\pi} \frac{zrd\emptyset}{(r^2+z^2)^{3/2}}[cos\emptyset\vec{a}_x + sin\emptyset\vec{a}_y]$$

$$\int_{\emptyset=0}^{2\pi} cos\emptyset\, d\emptyset = [sin\emptyset]_{\emptyset=0}^{2\pi} = sin2\pi - sin0 = 0$$

$$\int_{\emptyset=0}^{2\pi} sin\emptyset d\emptyset = [-cos\emptyset]_{\emptyset=0}^{2\pi} = -cos2\pi[-cos0] = 0$$

$$\int_{\emptyset=0}^{2\pi} \frac{zrd\emptyset}{(r^2+z^2)^{3/2}}\vec{a}_r = 0$$

This proves that $\vec{H}$ at P cannot have any radial component.

$$\therefore\ H = \frac{I}{4\pi}\int_{\emptyset=0}^{2\pi} \frac{\mathrm{r}^2\, \mathrm{d}\emptyset}{(\mathrm{r}^2+\ \mathrm{z}^2)^{3/2}}\vec{\mathrm{a}}_\mathrm{z} = \frac{I\mathrm{r}^2\vec{\mathrm{a}}_\mathrm{z}}{4\pi(\mathrm{r}^2+\ \mathrm{z}^2)^{3/2}}\int_{\emptyset=0}^{2\pi} d\emptyset$$

$$= \frac{I\mathrm{r}^2\vec{\mathrm{a}}_\mathrm{z}[\emptyset]_0^{2\pi}}{4\pi(\mathrm{r}^2+\mathrm{z}^2)^{3/2}} = \frac{I\mathrm{r}^2\vec{\mathrm{a}}_\mathrm{z}[2\pi]}{4\pi(\mathrm{r}^2+\mathrm{z}^2)^{3/2}}$$

$$\Rightarrow \vec{\boldsymbol{H}} = \frac{\boldsymbol{Ir^2}}{\mathbf{2(r^2+z^2)^{3/2}}}\vec{\boldsymbol{a}}_\boldsymbol{z}\ \mathbf{A/m}$$

Where r= radius of the circular loop
Z=distance of point P along the x axis

If the point P is shifted to the center of the circular loop, then height z=0

$$\vec{H} = \frac{Ir^2}{2(r^2)3/2}\vec{a}_z$$

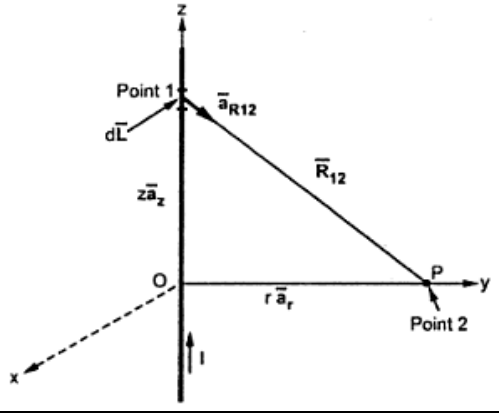
$$\vec{\boldsymbol{H}} = \frac{\boldsymbol{I}}{\boldsymbol{2r}}\vec{\boldsymbol{a}}_\boldsymbol{z}\ \mathbf{A/m} \quad \Rightarrow \vec{\boldsymbol{B}} = \boldsymbol{\mu}\ \frac{\boldsymbol{I}}{\boldsymbol{2r}}\vec{\boldsymbol{a}}_\boldsymbol{z}\ \mathbf{Wb/m^2}$$

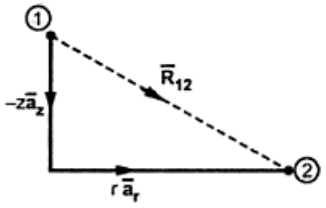
---

2. Obtain the flux density produced by an infinitely long straight wire carrying a current I at any point at a distant 'r' normal to the wire.
Derive the expression for magnetic field intensity due to infinitely long straight conductor carrying a current of I amps along Z- axis.(Nov/Dec 2016)

Consider an infinitely long straight conductor along z axis. The current passing through the conductor is a direct current I Amp. The field intensity $\vec{H}$ at a point P is to be calculated which is at a distance 'r' from z axis.

Considering a differential element at point 1, given as $\boldsymbol{Id\vec{L} = Idz\vec{a}_z}$





The distance vector joining point 1 and point 2

$$\vec{R}_{12} = \vec{R}_2 - \vec{R}_1 = (r\vec{a}_r) - (z\vec{a}_z)$$

$$= r\vec{a}_r - z\vec{a}_z$$

$$|\vec{R}_{12}| = \sqrt{(r)^2 + (-z)^2}$$

The unit vector $\vec{a}_{R12} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|} = \frac{r\vec{a}_r - z\vec{a}_z}{\sqrt{r^2+z^2}}$

$$d\vec{L} \times \vec{a}_{R12} = \begin{vmatrix} \vec{a}_r & \vec{a}_\emptyset & \vec{a}_z \\ 0 & 0 & dz \\ \frac{r}{\sqrt{r^2+z^2}} & 0 & \frac{-z}{\sqrt{r^2+z^2}} \end{vmatrix} = \frac{1}{\sqrt{r^2+z^2}} [\, rdz\, \vec{a}_\emptyset]$$

According to Biot-savart Law,

$$d\vec{H} = \frac{Id\vec{L}\times\vec{a}_{R12}}{4\pi R_{12}^2} = \frac{I\,[\, rdz\, \vec{a}_\emptyset]}{4\pi\, \sqrt{(r^2+z^2)}\left(\sqrt{(r^2+z^2)}\right)^2} = \frac{I\,[\, rdz\, \vec{a}_\emptyset]}{4\pi(r^2+z^2)^{3/2}}$$

Integrating dH for the full length of the conductor, we get

$$\Rightarrow \vec{H} = \int_{-\infty}^{\infty} \frac{I\,[\, rdz\, \vec{a}_\emptyset]}{4\pi(r^2 + z^2)^{3/2}}$$

Put z=r tanθ, dz = r sec$^2\theta\, d\theta and\ z^2 = r^2 tan^2\theta$

For $z = -\infty,\ \theta = -\frac{\pi}{2}$ and for $z = +\infty,\ \theta = +\frac{\pi}{2}$

$$\Rightarrow \vec{H} = \int_{\theta=-\pi/2}^{\pi/2} \frac{Ir(rsec^2\theta d\theta)\vec{a}_\emptyset}{4\pi(r^2 + r^2tan^2\theta)^{3/2}} = \int_{\theta=-\pi/2}^{\pi/2} \frac{Ir^2sec^2\theta d\theta \vec{a}_\emptyset}{4\pi(r^3sec^3\theta)}$$

$$= \int_{\theta=-\pi/2}^{\pi/2} \frac{I}{4\pi r}\frac{1}{sec\theta} d\theta \vec{a}_\emptyset = \frac{I}{4\pi r}[sin\theta]_{-\pi/2}^{\pi/2}\vec{a}_\emptyset$$

$$= \frac{I}{4\pi r}[sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)]\vec{a}_\emptyset$$

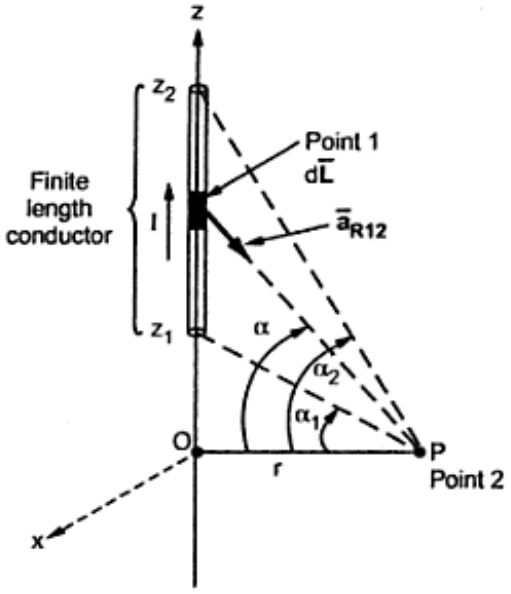
$$= \frac{I}{4\pi r}[1 - (-1)]\vec{a}_\emptyset = \frac{2I}{4\pi r}\vec{a}_\emptyset$$

$$\Rightarrow \vec{H} = \frac{I}{2\pi r}\vec{a}_\emptyset \ \ A/m$$

$$\Rightarrow \vec{B} = \mu\vec{H} = \frac{\mu I}{2\pi r}\vec{a}_\emptyset \ \ Wb/m^2$$

3.Find the magnetic field intensity at a point P due to a finite straight conductor, carrying a current I using Biot-savarts law.(Dec 2010) (April/May 2017)

Obtain an expression for magnetic flux density and magnetic field intensity at any point due to a finitelength conductor. (May 2016)



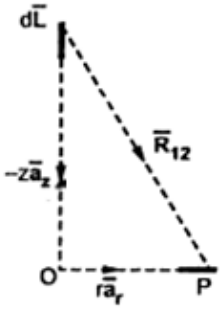
Consider the conductor of finite length placed along z axis. It carries a direct current I. The perpendicular distance of point P from z- axis is 'h' as shown in figure.Consider a differential element dLalong z axis, at a distance 'z' from the origin, which is given as$\boldsymbol{dL = dz\vec{a}_z}$

The distance vector from point P to the current element is given as,

$\vec{R}_{12} = \vec{R}_2 - \vec{R}_1 = (r\vec{a}_r) - (z\vec{a}_z) = r\vec{a}_r - z\vec{a}_z$

$|\vec{R}_{12}| = \sqrt{(r)^2 + (-z)^2}$

The unit vector$\vec{a}_{R12} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|} = \frac{r\vec{a}_r - z\vec{a}_z}{\sqrt{r^2+z^2}}$



$$d\vec{L} \times \vec{a}_{R12} = \begin{vmatrix} \vec{a}_r & \vec{a}_\emptyset & \vec{a}_z \\ 0 & 0 & dz \\ \frac{r}{\sqrt{r^2+z^2}} & 0 & \frac{-z}{\sqrt{r^2+z^2}} \end{vmatrix} = \frac{1}{\sqrt{r^2+z^2}}[\, rdz\, \vec{a}_\emptyset]$$

According to Biot-savart law $d\vec{H}$ at point P is,

$$d\vec{H} = \frac{Id\vec{L} \times \vec{a}_{R12}}{4\pi R_{12}^2}$$

$$= \frac{Irdz\vec{a}_\emptyset}{4\pi \sqrt{(r^2 + z^2)}\left(\sqrt{(r^2 + z^2)}\right)^2}$$

$$= \frac{Irdz\vec{a}_\emptyset}{4\pi(r^2 + z^2)^{3/2}}$$

Integrating $d\vec{H}$ over the full length of the conductor 'z',

$$\Rightarrow \vec{H} = \int_{z_1}^{z_2} \frac{\boldsymbol{Irdz\vec{a}_\emptyset}}{\boldsymbol{4\pi(r^2 + z^2)^{3/2}}}$$

$Putz = r\tan\alpha, z^2 = r^2 tan^2\alpha \quad and \quad dz = rsec^2\alpha\, d\alpha$

For $z=z_1$, $z_1 = r\tan\alpha_1$ and for $z=z_2$, $z_2 = r\tan\alpha_2$

$\alpha_1 = tan^{-1}\left(\frac{z_1}{r}\right)$ and $\alpha_2 = tan^{-1}\left(\frac{z_2}{r}\right)$

$$\vec{H} = \int_{\propto_1}^{\propto_2} \frac{Irrsec^2\alpha\, d\alpha \vec{a}_\emptyset}{4\pi[r^2 + r^2 tan^2\alpha]^{3/2}}$$

$$= \int_{\propto_1}^{\propto_2} \frac{Id\alpha\vec{a}_\emptyset}{4\pi(sec\alpha)r}$$

$$= \frac{I}{4\pi r}\int_{\propto_1}^{\propto_2} cos\alpha d\alpha \vec{a}_\emptyset$$

$$= \frac{I}{4\pi r}[sin\alpha]_{\propto_1}^{\propto_2}\vec{a}_\emptyset$$

$$\vec{H} = \frac{I}{4\pi r}[sin\alpha_2 - sin\alpha_1]\vec{a}_\emptyset \text{ A/m}$$

$$\vec{B} = \mu\vec{H} = \frac{\mu I}{4\pi r}[sin\alpha_2 - sin\alpha_1]\vec{a}_\emptyset \ Wb/m^2$$

Special case:

If the conductor is infinite in length then, $\alpha_2 = 90°\ and\ \alpha_1 = -90°$

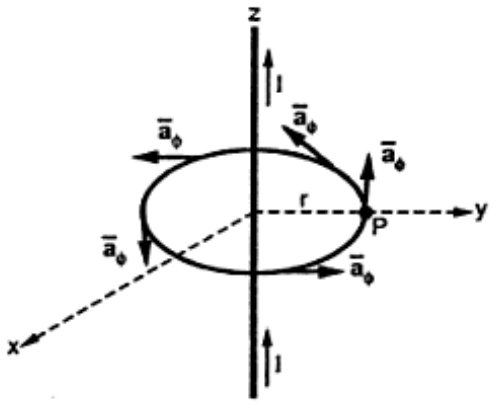
$$\vec{H} = \frac{I}{4\pi r}[sin(90°) \ - sin(-90°)]\vec{a}_\emptyset = \frac{I}{4\pi r}[1-(-1)]\vec{a}_\emptyset = \frac{I}{4\pi r}[2]\vec{a}_\emptyset = \frac{I}{2\pi r}\vec{a}_\emptyset \text{ A/m}$$

$$\vec{B} = \mu\vec{H} = \mu\frac{I}{2\pi r}\vec{a}_\emptyset \ Wb/m^2$$

---

4. State and explain Ampere's circuital law using mathematical expression for finding magnetic flux density due to current I.(May 2010,)(April/May 2017)

Ampere's circuital law:



It states that the lines integral of magnetic field intensity $\vec{H}$ around a closed path is exactly equal to the direct current enclosed by the path.

$$\oint \vec{H}.d\vec{L} = I$$

Proof of Ampere's circuital law:

Consider a long straight conductor carrying direct current I placed along z axis as show in figure. Consider a closed circular path of radius 'r' which encloses the straight conductor carrying direct current I. The point P is at perpendicular distance 'r' from the conductor. Consider differential element $d\vec{L}$ at point P which is in $\vec{a}_\emptyset$ direction tangential to circular path at point P.

$$d\vec{L} = rd\emptyset\vec{a}_\emptyset \text{----------(1)}$$

While $\vec{H}$ obtained at point P, from Biot-savart law due to infinitely long conductor is

$$\vec{\boldsymbol{H}} = \frac{\boldsymbol{I}}{\boldsymbol{2\pi r}}\vec{\boldsymbol{a}}_{\emptyset} \text{----------(2)}$$

Taking dot product of equation (1) and (2), we get

$$\vec{H}.d\vec{L} = \frac{I}{2\pi r}\vec{a}_{\emptyset}.rd\emptyset\vec{a}_{\emptyset}$$

$$= \frac{I}{2\pi r}rd\emptyset.$$

$$= \frac{I}{2\pi}d\emptyset$$

This proves that the integral $\vec{H}.d\vec{L}$ along the closed path gives the direct current enclosed by the closed path.

$$\oint \vec{H}.\vec{d}l = \int_{\emptyset=0}^{2\pi}\frac{I}{2\pi}[\emptyset]_0^{2\pi}$$

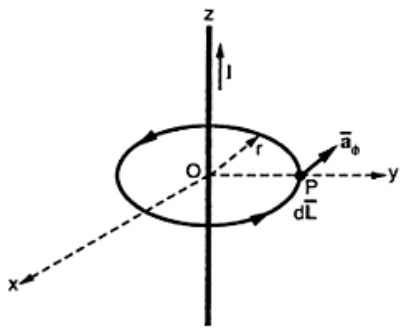
$$= \frac{I[2\pi]}{2\pi} \quad = I$$

Thus implies $\oint \vec{\boldsymbol{H}}.\boldsymbol{d\vec{L}} = \boldsymbol{I}$

---

5. Determine $\vec{\boldsymbol{H}}$ due to infinitely long straight conductor using Ampere's Law.

Consider an infinite long straight conductor placed along 'z' axis carrying a direct current I as shown in figure. Consider point 'P' on the closed path at which $\vec{H}$ is to be obtained. The magnitude of $\vec{H}$depends on 'r' and the direction is always tangential to the closed path and it has the only component$\vec{a}_{\emptyset}$.



$$\vec{H} = H_{\emptyset}\vec{a}_{\emptyset}, \qquad d\vec{L} = rd\emptyset\vec{a}_{\emptyset}$$

$$\Rightarrow \vec{H}.d\vec{L} = H_{\emptyset}\vec{a}_{\emptyset}.rd\emptyset\vec{a}_{\emptyset}$$

$$= H_{\emptyset}rd\emptyset$$

Ampere's circuital law

$$\oint \vec{\boldsymbol{H}}.\boldsymbol{d\vec{L}} = \boldsymbol{I}$$

$$\int_{\emptyset=0}^{2\pi} H_{\emptyset}rd\emptyset = I \quad \Rightarrow H_{\emptyset}r\int_{\emptyset=0}^{2\pi}d\emptyset = I \quad \Rightarrow H_{\emptyset}r(2\pi) = I \quad \Rightarrow H_{\emptyset} = \frac{\boldsymbol{I}}{\boldsymbol{2\pi r}}$$

Hence $\vec{H}$ at point P is given by $\vec{H} = H_{\emptyset}\vec{a}_{\emptyset}$

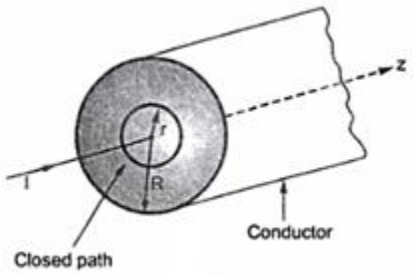
$$\vec{H} = \frac{I}{2\pi r}\vec{a}_{\emptyset} A/m$$

6. Obtain the expression for $\vec{H}$ in the regions of a cylindrical conductor carries a direct current I and its radius is 'R' m. Plot the variation of $\vec{H}$ against the distance R from the center of the conductor. (Jun 2007, May 2013) (Nov / Dec 2014) (Nov / Dec 2016)

Let the cylindrical conductor of radius R, carries a uniform direct current of 'I' A. It is placed along z axis and has infinite length. $\vec{H}$ is to be obtained considering two regions.

Region 1: Within the conductor r <R

- Consider the closed path of radius r within the conductor.
- As current I flows uniformly, it flows across the cross sectional area of R^2 .
- But the closed path encloses only part of the current which passes across the cross sectional area of πr^2
- Hence current enclosed by the path, $I_{enc} = I\frac{\pi r^2}{\pi R^2} = I\frac{r^2}{R^2}$
- H has only $\bar{a}_{\varphi}$ component and it is the function of r only hence,



$$\vec{H} = H_{\varphi}\vec{a}_{\varphi} \text{ and } d\vec{L} = rd\varphi\vec{a}_{\varphi} \text{ in } \vec{a}_{\varphi} \text{ direction}$$

$$\vec{H}.d\bar{L} = H_{\varphi}\vec{a}_{\varphi}.rd\varphi\vec{a}_{\varphi} = H_{\varphi}rd\varphi$$

According to ampere's circuital law,

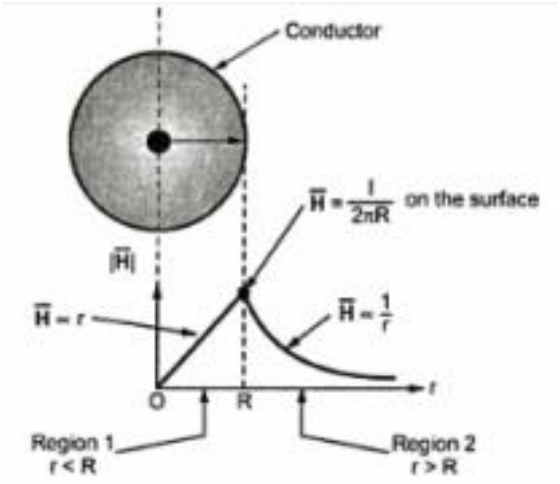
$$\oint \vec{H}.d\bar{L} = I_{enc}$$

$$\int_{\varphi=0}^{2\pi} H_{\varphi}rd\varphi = I\frac{r^2}{R^2}$$

$$H_{\varphi}r(2\pi) = I\frac{r^2}{R^2}$$

$$H_{\varphi} = \frac{I}{2\pi r} \times \frac{r^2}{R^2} = \frac{Ir}{2\pi R^2}$$

$$\vec{H} = \frac{Ir}{2\pi R^2}\vec{a}_{\varphi}\, A/m$$



Inside the conductor $\vec{H} \propto$ r and on the surface r=R hence $\vec{H} = \frac{I}{2\pi R}\vec{a}_{\varphi}$ **on the surface** of conductor.

Region 2: Outside the conductor r> R

The conductor is infinite length along z axis carrying direct current I hence using the result earlier,

$$\vec{H} = \frac{I}{2\pi r}\vec{a}_{\varphi} \textbf{for r>R}$$

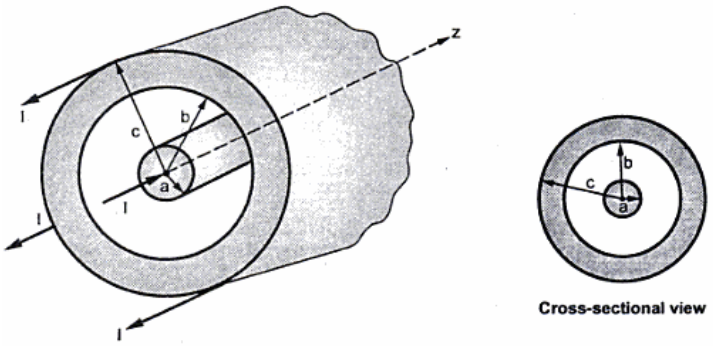
So outside the conductor, $\vec{H} \propto \frac{1}{r}$

Variation of H w.r.t to 'r'

7. Derive $\vec{H}$ due to a Co-axial Cable. (May 2012)

Derive the expression for the magnetic field intensity inside and outside a coaxial conductor of inner radius 'a', outer radius 'b', and carrying a current of "I" amperes in the inner and outer conductors. (May / June 2014)

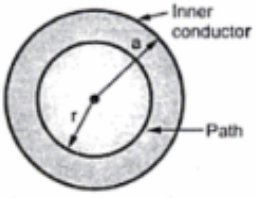
Consider a co-axial cable as shown in figure. Its inner conductor is solid with radius a, carrying direct current I. The outer conductor is in the form of concentric cylinder whose inner radius is b and outer radius is c. This cable is placed along z axis. The current I is uniformly distributed in the inner conductor. While –Iis uniformly distributed in outer conductor.



The space between inner and outer conductor is filled with dielectric say air. The calculation of $\vec{H}$ is divided corresponding to various regions of the cable.

Region 1: Within the inner conductor, $r < a$

Consider a closed path having radius $r < a$.As in the figure it encloses only part of the conductor.The area of cross section enclosed is $\pi r^2 m^2$. The total current flowing is I through the area πa^2. Hence the current enclosed by closed path is



$$I' = \frac{\pi r^2}{\pi a^2} I = \frac{r^2}{a^2} I$$

The $\vec{H}$ is again only in $\vec{a}_\varphi$ direction and depends only on 'r'.

$$\vec{H} = H_\varphi \vec{a}_\varphi$$

So consider $d\vec{L}$ in the $\vec{a}_\varphi$ direction which is r $d\varphi$

$$d\vec{L} = \mathrm{r}d\varphi\vec{a}_\varphi$$

$$\vec{H}.d\vec{L} = H_\varphi \vec{a}_\varphi . r d\varphi \vec{a}_\varphi = H_\varphi r d\varphi$$

According to Amperes circuital law,

$\oint \vec{H}.\mathrm{d}\vec{L} = I'$

$$\oint H_\varphi \, r d\varphi = \frac{r^2}{a^2} I$$

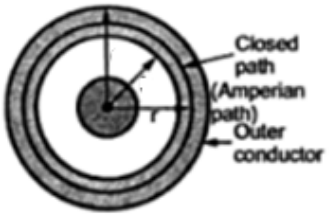
$$\int_{\varphi=0}^{2\pi} H_\varphi \, r d\varphi = \frac{r^2}{a^2} I$$

$$H_\varphi r[2\pi] = \frac{r^2}{a^2} I$$

$$H_\varphi = \frac{r^2}{2\pi r a^2} I$$

$\overrightarrow{\boldsymbol{H}} = \frac{\boldsymbol{Ir}}{\boldsymbol{2\pi a^2}} \vec{\boldsymbol{a}}_{\boldsymbol{\varphi}}$ **A/m**

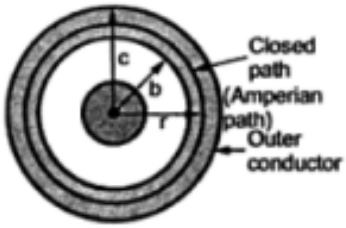
Region 2: Within $a < r < b$



Consider a circular path which encloses the inner conductor carrying direct current I. this case of infinitely long conductor along z axis. Hence $\vec{H}$in this region is

$\overrightarrow{\boldsymbol{H}} = \frac{\boldsymbol{I}}{\boldsymbol{2\pi r}} \vec{\boldsymbol{a}}_{\boldsymbol{\varphi}}$ **A/m**

Region 3: Within outer conductor $b < r < c$



Consider the closed path .the current enclosed by the closed path is only part of the current –I, in the outer conductor. The total current –I is flowing through the cross section$\pi(c^2 - b^2)$while the closed path encloses the cross section $\pi(r^2 - b^2)$.

Hence the current enclosed by the closed path of outer conductor is $I'' = \frac{\pi(r^2 - b^2)}{\pi(c^2 - b^2)}(-I) = -\frac{(r^2 - b^2)}{(c^2 - b^2)} I$

Where, I'' = Current in the inner conductor enclosed

Total current enclosed by the closed path is,

$$I_{enc} = I' + I'' = -\frac{(r^2 - b^2)}{(c^2 - b^2)} I + I$$

$$= I\left[1 - \frac{(r^2 - b^2)}{(c^2 - b^2)}\right] = I\left[\frac{c^2 - r^2}{c^2 - b^2}\right]$$

According to Amperes circuital law,

$$\oint H.dL = I_{enc}$$

Now$\vec{H}$is again in $\vec{a}_\varphi$direction only and is a function of 'r' only

$\vec{H}.d\vec{L} = H_\varphi \vec{a}_\varphi .\mathrm{r}d\varphi \vec{a}_\varphi = H_\varphi \mathrm{r}d\varphi$

$$\int_{\varphi=0}^{2\pi} H_\varphi \, r d\varphi = I_{enc}$$

$$H_\varphi r\lfloor 2\pi \rfloor = I\left[\frac{c^2 - r^2}{c^2 - r^2}\right]$$

$$H_\varphi = \frac{I}{2\pi r}\left[\frac{c^2 - r^2}{c^2 - b^2}\right]$$

$$\vec{H} = H_\varphi \vec{a}_\varphi = \frac{I}{2\pi r}\left[\frac{c^2 - r^2}{c^2 - b^2}\right] \vec{a}_\varphi \, {}^{A}/_{m}$$

Region 4: Outside the cable r > c

Consider the closed path r>c such that it encloses both the conductors.

The total current enclosed is

$$I_{enc} = +I - I = 0A$$

$$\oint H.dL = 0$$

$$\vec{H} = 0 \ {}^{A}/_{m}$$

Thus magnetic field does not exist outside the cable.

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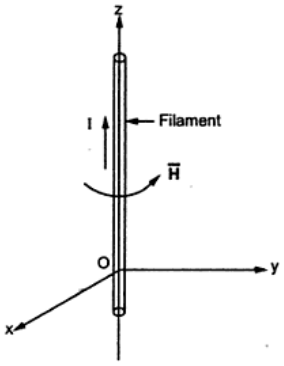
8. Obtain an expression for magnetic vector potential in the region surrounding an infinitely long straight filamentary current I. (May 2012) or
Derive the expression for magnetic vector potential in the cases of an infinitely long, straight conductor in free space. (May/ June 2014)



Consider an infinitely long filament carrying direct current I placed along z axis as shown in figure

The magnetic field intensity due to such filament given by

$$\vec{H} = \frac{I}{2\pi r}\vec{a}_{\emptyset}$$

If it is placed in free space

$$\vec{B} = \mu_0 \vec{H}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}\vec{a}_{\emptyset}$$

Assuming cylindrical co-ordinate system

$$\vec{B} = \nabla \times \vec{A}$$

$$\frac{\mu_0 I}{2\pi r}\vec{a}_{\emptyset} = \left[\frac{1}{r}\frac{\partial A_z}{\partial \emptyset} - \frac{\partial A_{\emptyset}}{\partial z}\right]\vec{a}_r + \left[\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial \emptyset}\right]\vec{a}_{\emptyset} + \frac{1}{r}\left[\frac{\partial (rA_{\emptyset})}{\partial r} - \frac{\partial A_r}{\partial \emptyset}\right]\vec{a}_z$$

Equating the co-efficient of $\vec{a}_{\emptyset}$

$$\vec{B} = \frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} = \frac{\mu_0 I}{2\pi r}$$

$\vec{B}$is a function of ‘r’ only , the $\vec{A}$ will also change with ‘r’ only and will be constant with respect to ‘z’

$$\frac{\partial A_r}{\partial z} = 0$$

$$\frac{\partial A_z}{\partial r} = -\frac{\mu_0 I}{2\pi r}$$

$$A_z = -\int \frac{\mu_0 I}{2\pi r}\, dr + C_1 \; C_1\text{=constant}$$

$$A_z = \frac{\mu_0 I}{2\pi} Ln[r] + C_1$$

To find C_1: When A_z= 0, r=r_0

$$0 = \frac{\mu_0 I}{2\pi} Ln[0] + C_1$$

$$C_1 = \frac{\mu_0 I}{2\pi r} \ln[r_0]$$

$$A_z = \frac{\mu_0 I}{2\pi} \ln[r] + \frac{\mu_0 I}{2\pi} \ln[r_0] \quad => A_z = \frac{\mu_0 I}{2\pi} \ln[\frac{r_0}{r}]$$

$$\vec{A} = A_z \vec{a}_z = \frac{\mu_{0I}}{2\pi} \ln\left[\frac{r_0}{r}\right] \vec{a}_z \; Wb/m$$

This is the required vector magneticpotential.

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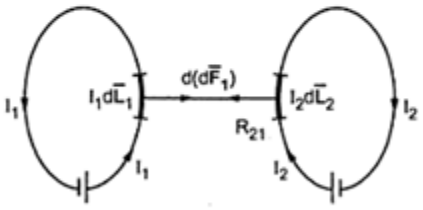
9. Derive the magnetic force between two parallel conductors carrying equal current in (i) same direction (ii) opposite direction.(May 2010)

- Let us now consider two current elements$I_1 d\vec{L}_1$ and$I_2 d\vec{L}_2$ as shown in figure.
- Assuming that the direction ofI_1 and $I_2$are same, both the current element producesits own magnetic fields.
- As the currents are flowing in the same direction through the elements, the force d(d$\vec{F}_1$) exerted on element $I_1 d\vec{L}_1$due to the magnetic field d$\vec{B}_2$ produced by other element $I_2 d\vec{L}_2$ is the force of attraction.

Force between two parallel current carrying conductors



Force between two current elements

From the equation of force, the force exerted on a differential current element is given by

$$\boldsymbol{d(d\vec{F}_1) = I_1 d\vec{L}_1 \times d\vec{B}_2} \text{----------(1)}$$

According to Biot-savart law, the magnetic field produced by current element$I_2 d\vec{L}_2$is givenfor free space, by:

$$\boldsymbol{d\vec{B}_2 = \mu_0 d\vec{H}_2 = \mu_0 \left[\frac{I_2 d\vec{L}_2 \times \vec{a}_{R21}}{4\pi R_{21}^2}\right]} \text{-------- (2)}$$

Sub equation (2) in (1)

$$\boldsymbol{d(d\vec{F}_1) = \mu_0 \frac{I_1 d\vec{L}_1 \times (I_2 d\vec{L}_2 \times \vec{a}_{R21})}{4\pi R_{21}^2}} \text{---------(3)}$$

The equation (3) represents force between two current elements.By integrating $d(d\vec{F}_2)$ twice, $\vec{F}_1$ on current element 1 due to current element 2 is given by

$$\boldsymbol{\vec{F}_1 = \frac{\mu_0 I_1 I_2}{4\pi} \oint\oint_{L_1 L_2} \frac{d\vec{L}_1 \times (d\vec{L}_2 \times \vec{a}_{R21})}{R_{21}^2}}$$

Similarly $\vec{F}_2$exerted on the current element 2 due to the magnetic field $\vec{B}_1$produced by the current element1. Thus,

$$\boldsymbol{\vec{F}_2 = \frac{\mu_0 I_2 I_1}{4\pi} \oint\oint_{L_1 L_2} \frac{d\vec{L}_2 \times (d\vec{L}_1 \times \vec{a}_{R12})}{R_{12}^2}}$$

$$\vec{F}_2 = -\vec{F}_1$$

The force F_1 and F_2 obey Newton's third law that for every action there is equal and opposite reaction. For the two current carrying conductors of length 'l' each, the force exerted in given by

$$F = \frac{\mu I_1 I_2 l}{2\pi d}$$

Where I_1 and I_2 are the currents flowing through conductor 1 and conductor 2 and'd' is the distance of separation between two conductors.

If the two current flowing the same direction, the current carrying conductors attract each other. While if, the two current flows in opposite direction to each other, the current carrying conductors repel each other.

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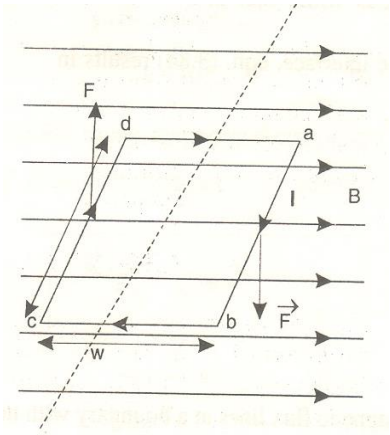
10. Derive the torque equation on a closed loop conductor. (Or)
Derive an expression for torque in a rectangular loop (am×bm) which is carrying a current of "I" amperes and is situated in a uniform field 'B' Wb/m^2. (Nov / Dec 2014)(May/June 2016)
Find the torque about y axis for two conductors of length l carrying current in opposite direction separated by a fixed distance w in an uniform magnetic field in x direction. (April/May 2017) (Nov/Dec 2017)

The concept of current loop experiencing a torque in magnetic field is important in understanding the principle of DC motor. The torque is defined as the vector product of $\vec{F}$ and the moment of arm $\vec{r}$. $\vec{T}$has the unit of Newton-meter



The torque about the axis is given by,

$$\vec{T} = \vec{r} \times \vec{F}$$

$\vec{r}$ → Radius of the moment arm

$\vec{F}$ →Force on the conductor

Consider the single turn coil in the Z=0 plane of width 'w' in the x direction and length 'l' along y direction. The field is uniform and in the +x direction.

Force acting on a current element placed in the field, is given as

$$\vec{F} = Il \times \vec{B}$$

$$\vec{F} = \boldsymbol{BIl \sin \alpha}$$

Where,α is the angle made by the current element with$\vec{B}$.

Now the force exerted on side 'ab' carrying current from a to b is given as,

Force on conductor 'ab':

$\vec{F}_{ab} = Il(\vec{a}_y) \times B\vec{a}_x = -BIl\vec{a}_z$

Force on conductor 'cd':

$\vec{F}_{cd} = lI(-\vec{a}_y) \times B(\vec{a}_x) = BIl\vec{a}_z$

Since the sides 'bc' and 'da' are parallel with B, no force is exerted on them. Equal force act on length 'l' only at different point in opposite direction.

The direction of force on 'ab' is such that it will tend to move downwards where as'cd' will tend to move upwards.

$\therefore$The torque about y axis from the left current element requires a lever arm$r = \left(-w/2\right)\vec{a}_x$.
The sign will change for lever arm to right side$(ab)\ as\ r = w/2\ \vec{a}_x$
$\therefore$The torque from both elements is

$$\vec{T} = \left[\frac{-w}{2}(\vec{a}_x) \times IlB(-\vec{a}_z)\right] + \left[\frac{w}{2}\vec{a}_x \times IlB(\vec{a}_z)\right]$$

$$\vec{T} = IwlB\left(-\vec{a}_y\right)$$

$$\boldsymbol{\vec{T} = IAB\left(-\vec{a}_y\right)}\textbf{N-m}$$

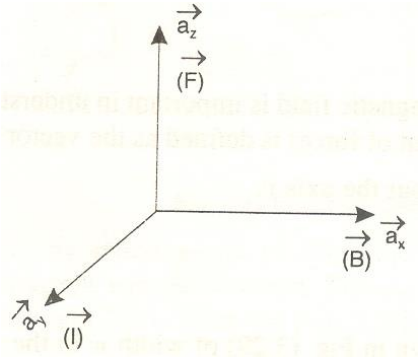
Where, A is the area of the coil.
If the coil is not parallel with $\vec{B}$ but is normal makes an angle $\alpha$the force will be$\vec{F} = BIl\sin\alpha$

$$\vec{T} = I\vec{A} \times B\left(-\vec{a}_y\right)$$

$$\vec{T} = I\vec{A} \times \vec{B}$$

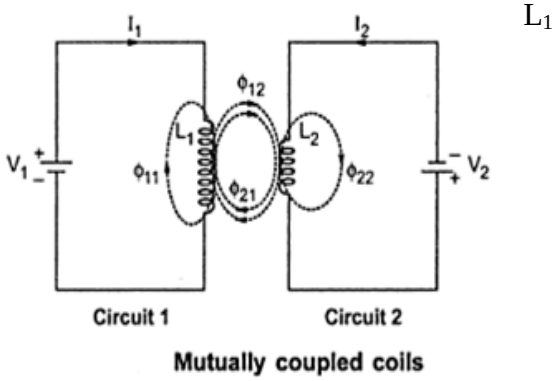
$$\boldsymbol{T = \vec{m} \times \vec{B}}$$

Where $\vec{m} = IA\vec{a}_N$
$\vec{m} \rightarrow magneticdipolemoment$
I= Current through the loop, A=Area of the loop



11. Derive the expression for coefficient of coupling in term of mutual and self in direction.(May 2010)

Consider two coils having self-inductance L_1 and L_2 respectively with number of turns in the coilsbeing N_1 and N_2 respectively. When two such coil are placed very close to each other, ideally all the flux produced by each coil links with other coil with no leakage.



Mutually coupled coils

Considering the figure, the total flux produced by current $I_1$is a combination of leakage flux$\emptyset_{11}$ and linkage flux $\emptyset_{12}$

The mutually induced emf in coil2 is given by

$$V = N_2\frac{d\emptyset_{12}}{dt}\text{----------(1)}$$

The same mutually induced e.m.f can be expressed in term of mutual inductance as

$$V = M\frac{di_1}{dt}\text{----------(2)}$$

Comparingequation (1) and (2)$\boldsymbol{M = N_2\frac{d\emptyset_{12}}{di_1}}$----------(3)

The total flux resulted by current i_2 flowing through L_2 is $\emptyset_2$ and it is combination of leakage flux $\emptyset_{22}$ and linking flux$\emptyset_{21}$.
The mutually induced e.m.f in coil 1 due to linkage flux $\emptyset_{21}$is given by

$$V = N_{12}\frac{d\emptyset_{21}}{dt}\text{----------(4)}$$

The same mutually induced e.m.f can be expressed in term of mutual inductance as.

$V = M\frac{di_2}{dt}$----------(5)

Comparingequation (4) and (5) $\boldsymbol{M = N_1\frac{d\emptyset_{21}}{di_2}}$ ---------- (6)

The coefficients of coupling 'K' is defined as the ratio of linking flux to total flux.

$$K = \frac{\emptyset_{12}}{\emptyset_1} = \frac{\emptyset_{21}}{\emptyset_2}\text{------ (7)}$$

Multiplying equations (3) and (6)

$$\boldsymbol{M^2 = \left(N_2\frac{d\emptyset_{12}}{di_1}\right)\left(N_1\frac{d\emptyset_{21}}{di_2}\right)}\text{----------(8)}$$

$$\emptyset_{12} = K\emptyset_1 \; and \; \emptyset_{21} = K\emptyset_2$$

Putting values of $\emptyset_{12}$ and $\emptyset_{21}$ in equation (8)

$$M^2 = \left[N_2\frac{d(K\emptyset_1)}{di_1}\right]\left[N_1\frac{d(K\emptyset_2)}{di_2}\right]$$

$$=> M^2 = K^2\left[N_1\frac{d\emptyset_1}{di_1}\right]\left[N_2\frac{d\emptyset_2}{di_2}\right} \qquad =>M^2 = K^2L_1L_2 \qquad =>M = K\sqrt{L_1L_2}$$

$$=>\boldsymbol{K = \frac{M}{\sqrt{L_1L_2}}}$$

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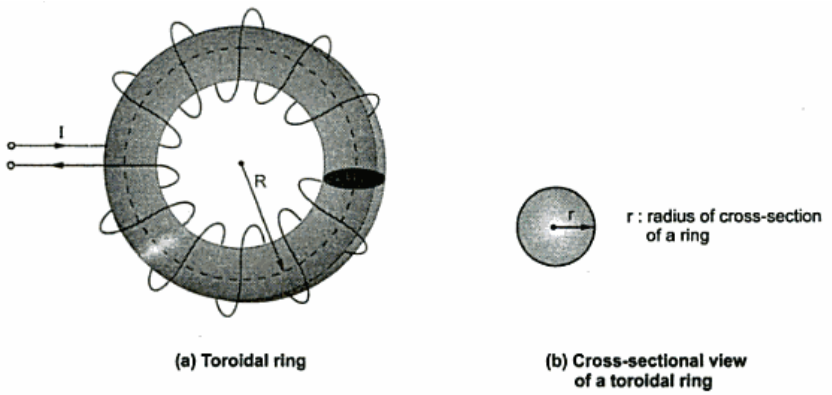
12. Derive the expression for inductance of a toroid.

Consider a toroidal ring with N turns and carrying current I. Let the radius of the toroid be'R' as shown in figure.



(a) Toroidal ring (b) Cross-sectional view of a toroidal ring

The magnetic flux density inside a toroidal ring is given by

$$B = \frac{\mu NI}{2\pi R}\text{---------- (1)}$$

The total flux linkage of a toroidal ring having N turns is given by

Total flux linkage = $N\emptyset$

Where A = $\pi r^2 m^2$ = Area of cross section of a toroidal ring and $\emptyset = (B)(A)$

Total flux linkage = $N(B)(A) = N(\frac{\mu NI}{(2\pi R)}(A) = \frac{\mu N^2 IA}{(2\pi R)}$

The inductance of a toroid is given by

$$L = \frac{Total\ flux\ linkage}{Total\ current}$$

$$=> L = \frac{\mu N^2 IA}{(2\pi R)(I)}$$

$$I = \frac{\mu N^2 A}{2\pi R}\ H$$

For a toroid with N number of turns h as the height of toroid with r1 as inner radius and r2 as outer radius, the inductance is given by.

$$\boldsymbol{L = \frac{\mu N^2 h}{2\pi}\ ln\left[\frac{r_2}{r_1}\right]} \qquad H$$

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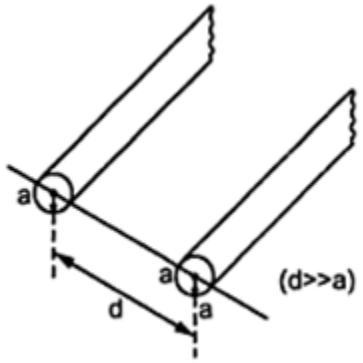
13. Calculate the internal and external inductances per unit length of a transmission line consisting of two long parallel conducting wires of radius 'a' that carry currents in opposite directions. The axes of the wires are separated by a distance'd', which is much longer than 'a'.

Consider transmission line made up of two long parallelconductors of radius 'a' carrying currents in opposite directions with'd'distance of separation.



Energy stored is given by

$$W_m = \frac{1}{2} LI^2$$

$$L = \frac{2W_m}{I^2}$$

But $W_m = \int B.H\ dv = \int \frac{B^2}{2\mu} dv$--------(1)

Substituting W_m in equation (1)

$$L_{in} = \frac{2}{I^2} \int \frac{B^2}{2\mu}\ dv$$

$$= \frac{I}{I^2 \mu} \int_{z=0}^{l} dz \int_{\emptyset=0}^{2\pi} d\emptyset \int_{\rho=0}^{a} \frac{\mu^2 I^2 \rho^2}{4R^2 a^4} \rho d\rho$$

$$= \frac{\mu}{4\pi^2 a^4} \int_0^l dz \int_0^{2\pi} d\emptyset \int_0^a \rho^2 dp$$

$$= \frac{\mu}{4\pi^2 a^4} [l][2\pi] \left[\frac{a^4}{4}\right] = \frac{\mu l}{8\mu}$$

For outer conductor,

$$L_{out} = \frac{2}{I^2}\int \frac{B^2}{2\mu} dv \;= \frac{1}{I^2\mu}\iiint \frac{\mu^2 I^2}{4\pi^2\rho^2}$$

$$= \frac{\mu}{4\pi^2}\int_0^l dz \int_0^{2\pi} d\emptyset \int_a^{(d-a)} \frac{1}{\rho} dp$$

$$= \frac{\mu}{4\pi^2}[l][2\pi][ln\,\rho]_a^{(d-a)}$$

$$= \frac{\mu l}{2\pi} ln\left[\frac{d-a}{a}\right]$$

Hence total inductanceof the two symmetrical conductors, is

$$L = 2[L_{int} + L_{out}] = \frac{\mu l}{4\pi} + \frac{\mu l}{\pi} ln\left[\frac{d-a}{a}\right] = \frac{\mu l}{\pi}\left[\frac{1}{4} + ln\left(\frac{d-a}{a}\right)\right]$$

$Since\ d \gg a\ ,\ \ we\ can\ consider\ \ d - a \approx d$

The total inductance in given by

$$\boldsymbol{L = \frac{\mu l}{\pi}\left[\frac{1}{4} + ln\left(\frac{d}{a}\right)\right]}\ \ Henry$$

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14. Derive the expression for energy stored in magnetic field and energy density in the magnetic field.(May 2013,Dec 2012)

When a current is passing through a inductor, energy is stored in the form of magnetic field. If the current is increasing from 0 to I with p.d. across the inductor equal to V, then the source is supplying power equal to VI.

Energy supplied by the source in time 'dt' is VIdt.

i.e, $dW = VIdt$

If the current is increased upto I in time 't',

$$W = \int VI\, dt = \int LI\frac{dI}{dt}dt = L\int_0^I IdI$$

$\boldsymbol{W = \frac{1}{2}LI^2}$is the Energy density in the magnetic field

The inductance of the solenoid is given by

$$\boldsymbol{L = \frac{\mu_0\mu_r N^2 A}{l} I^2}$$

$Where\ l_m$=mean circumference

$A = area\ of\ the\ solenoid$

$N = No.\ of\ tums$

$$\text{Energy stored} = \frac{1}{2}LI^2 = \frac{1}{2}\left(\frac{\mu_0\mu_r N^2 A}{l}\right)I^2 = \frac{1}{2}\mu_0\mu_r\left(\frac{NI}{l}\right)^2(lA)$$

But $H = \frac{NI}{l}$ and $B = \mu_0\mu_r H$, lA = Volume of the space inside the coil

∴ Energy stored per unit volume is given as,

$$W = \frac{1}{2}\mu H^2$$

$$W = \frac{1}{2} BH \quad J/m^3$$

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15. Derive the boundary conditions at an interface between two magnetic media. (May'10)

or

Consider the boundarybetween two media. Show that the angles between the normal to the boundary and the magnetic flux densities oneither side of the boundary satisfy the relation

$$\frac{tan\,\theta 1}{tan\,\theta 2} = \frac{\mu 1}{\mu 2}$$

Where μ_1 and μ_2 are the permeability's of the respective media and θ_1 and θ_2 are the angles(May / June 2014) (Nov/Dec 2017)

State and prove magnetic boundary conditions. (May/June 2015)(April/May 2017)

Boundary conditions:

The conditions of magnetic field existing at the boundary of the two media when the magnetic field passes from one medium to other are called boundary conditions for the magnetic fields or simply magnetic boundary conditions.

When we consider magnetic boundary conditions, the conditions of $\vec{B}$ and $\vec{H}$ are studied at the boundary. The boundary between two different magnetic materials is considered to study conditions of$\vec{B}$ and $\vec{H}$ at the boundary. Consider boundary between two isotropic, homogeneous linear materials with different permeability's$\mu_1 and \mu_2$.

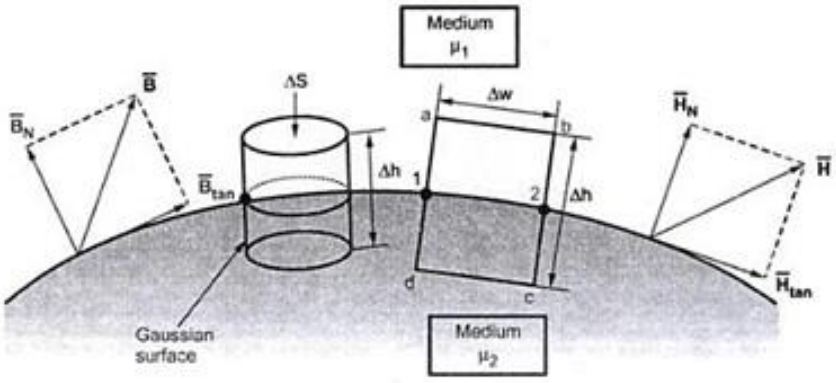
Both the vectors are resolved into two components;

a) **Tangential to boundary**
b) **Normal (perpendicular) to boundary**

To determine the boundary conditions let us use the closed path and the Gaussian surface.

Boundary conditions for normal component:

To find the normal component of$\vec{B}$, select a closed Gaussian surface in the form of a right circular cylinder. Let the height beΔh and be placed in such a way that $\Delta h/2$ is in medium 2. Also the axis of cylinder is in the normal direction surface.



Boundary between two magnetic materials of different permeabilities

According to the gauss's law for magnetic field

$$\oint_s \vec{B}.\vec{dS} = 0 \text{ ----->(1)}$$

The surface integral must be evaluated over three surfaces

(1) Top

(2) Bottom

(3) Lateral

$$\oint_{top} \vec{B}.d\vec{S} + \oint_{bottom} \vec{B}.d\vec{S} + \oint_{lateral} \vec{B}.d\vec{S} = 0 \text{ ----->(2)}$$

- As we are very much interested in the boundary conditions, reduce Δh to zero.
- As $\Delta h \rightarrow 0$, the cylinder tends to boundary and only top and bottom surfaces contribute in the surface integral.
- Thus surface integral are calculated for top and bottom surfaces only. The surface is very small.
- Let the magnitude of normal component of $\vec{B}$ be B_{N1} and B_{N2} in medium 1 and medium 2 respectively.
- As both the surfaces are very small, we can assume B_{N1} and B_{N2} constant over the surface.
- Let the area of the top and bottoms are same, equal to ΔS.

For top surface:

$\oint_{top} \bar{B}.d\bar{S} = B_{N1} \oint_{top} d\bar{S}$=$B_{N1}\Delta S$-------->(3)

For bottom surface:

$\oint_{Bottom} \bar{B}.d\bar{S} = BN_2 \oint_{top} d\bar{S}$= $B_{N2}\Delta S$-------->(4)

For lateral surface:

$\oint_{lateral} \bar{B}.d\bar{S} = O$ -------->(5)

Putting values of surface integrals in equation we get

$B_{N1}\Delta S - B_{N2}\Delta S = 0$ -------->(6)

Note that the negative sign is used for one of the surface integral because normal component in medium 2 is entering the surface while medium 1 the component is leaving the surface. Hence B_{N1} and B_{N2} are in opposite direction.

From equation (6) we can write: $B_{N1}\Delta S - B_{B2}\Delta S = 0$

$$\mathbf{B_{N1}=B_{N2}}\text{-------->(7)}$$

Thus the normal component of $\bar{B}$ is continuous at the boundary.

As the magnetic flux density and the magnetic field intensity are related by

$$\vec{B} = \mu\vec{H}$$

Equation (7) can be written as:$\mu_1 H_1 = \mu_2 H_2$

$$\frac{H_{N1}}{H_{N2}} = \frac{\mu_2}{\mu_1} = \frac{\mu_{r1}}{\mu_{r2}} \text{-------> (8)}$$

The normal component of $\bar{H}$ is not continuous at the boundary. The field strength in two media are inversely proportional to their relative permeability's

Boundary conditions for tangential component:

According to Amperes circuital law,

$$\oint \vec{H}.d\vec{L} = I \text{ -------- >(9)}$$

Consider a rectangular closed path a-b-c-d-a. It is traced in clockwise direction. This closed path is placed in plane normal to the boundary surface. Hence $\oint \vec{H}.d\vec{L}$ can be divided into 6 parts.

$$\oint \vec{H}.d\vec{L} = \int_a^b \vec{H}.d\vec{L} + \int_b^1 \vec{H}.d\vec{L} + \int_1^c \vec{H}.d\vec{L} + \int_c^d \vec{H}.d\vec{L} + \int_d^2 \vec{H}.d\vec{L} + \int_2^a \vec{H}.d\vec{L} \text{-------> (10)}$$

- From the figure, it is clear that the closed path is placed in such a way that, the two sides a-b and e-d are parallel to the tangential direction to the surface while other two sides are normal to the surface at the boundary.
- This closed path is placed is placed in such a way that half of its portion is in medium 1 and remaining is in medium 2.
- The rectangular path is an elementary rectangular path with elementary height Δh and elementary width Δw.
- Thus over small $width \Delta w$, $\vec{H}$ can be assume constant say H_{tan1} in medium 1 and H_{tan2} in medium 2.similarly over a small height $\frac{\Delta h}{2}$, $\vec{H}$ can be assumed constant say H_{N1} in medium 1 and H_{N2} in medium 2.
- Now assume $\vec{K}$ is the surface current normal to the path. Normal and tangential components are opposite.

$$K.dw = H_{tan1}(\Delta w) + H_{N1}\left(\frac{\Delta h}{2}\right) + H_{N1}\left(\frac{\Delta h}{2}\right) - H_{tan2}(\Delta w) - H_{N2}\left(\frac{\Delta h}{2}\right) - H_{N1}\left(\frac{\Delta h}{2}\right) \text{----->(11)}$$

To get conditions at boundary, $\Delta h \to 0$. Thus,

$$K.dw = H_{tan1}(\Delta w) - H_{tan2}(\Delta w)$$

$$H_{tan1} - H_{tan2} = \vec{K} \text{---------> (12)}$$

In vector form

$$\bar{H}_{tan1} - \bar{H}_{tan2} = \bar{a}_{N12} \times \bar{K} \text{---------> (13),}$$ where $\bar{a}_{N12}$ is the unit vector.

$\bar{B}$ is tangential components

$$\frac{B_{tan1}}{\mu_1} - \frac{B_{tan2}}{\mu_2} = K \text{--------> (14)}$$

Consider the special case that the is free for current so K=0

$$H_{tan1} - H_{tan2} = 0 \text{ (or) } H_{tan1} = H_{tan2} \text{---------> (15)}$$

For tangential components $\bar{B}$

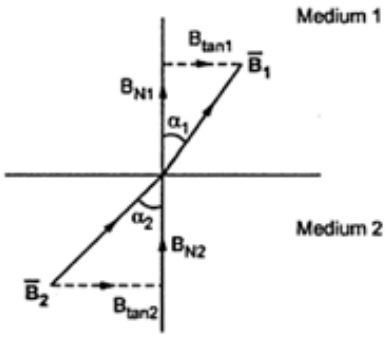
$$\frac{B_{tan1}}{\mu_1} - \frac{B_{tan2}}{\mu_2} = 0 \quad => \frac{B_{tan1}}{\mu_1} = \frac{B_{tan2}}{\mu_2}$$

$$\frac{B_{tan1}}{B_{tan2}} = \frac{\mu_1}{\mu_2} = \frac{\mu_{r1}}{\mu_{r2}} \text{--------> (16)}$$

From the equations (15) and (16) it is clear that tangential component $\vec{H}$ are continuous, while tangential component $\vec{B}$ are discontinuous at the boundary. Boundary is current free.

Reflection and refraction at the boundary:

Let the fields make angles $\alpha_1 and \alpha_2$ normal to the interface.



Component of $\bar{B}$ at boundary

In terms of angles $\alpha_1 and \alpha_2$ we can write relationship between normal and tangent components
In medium 1

$$\tan \alpha_1 = \frac{B_{tan1}}{B_{N1}}$$

Similarly in Medium 2,

$$\tan \alpha_2 = \frac{B_{tan2}}{B_{N2}}$$

Dividing medium 1 and medium 2

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{B_{tan1}}{B_{N1}} \cdot \frac{B_{N2}}{B_{tan2}}$$

We know that $B_{N1}=B_{N2,}$

$$\frac{\tan \alpha_1}{\tan \alpha_2} = \frac{B_{tan1}}{B_{tan2}} = \frac{\mu_{r1}}{\mu_{r2}}$$

Consider an interface between air and soft iron. Air $\mu_r = 1, soft iron \mu_r = 7000$

$$\frac{B_{tan1}}{B_{tan2}} = \frac{tan\ \alpha_1}{tan\ \alpha_2} = \frac{1}{7000}$$

$\alpha_2 = 85^\circ, \alpha_1 = 0.093^\circ$ and $B_{tan1}=0$

Thus practically when field crosses a medium of high μ_r to a medium of lowμ_r,then the magnetic fields B and H are always perpendicular to the boundary.

16. Derive the Scalar and Vector magnetic potential.(May 2012) (Nov/Dec 2017)

Magnetic scalar and vector potentials:

In electrostatics it seen that there exist a scalar electric potential v which is related to the electric field intensity $\vec{E}$ as $\vec{E} = -\Delta V$

In case of magnetic fields there are two types of potential which can be defined as

1. The scalar magnetic potential denoted as V_m
2. Thevector potential denoted as $\bar{A}$

To define scalar and vector potential, let us two vector identities which are listed as properties of curl

$\nabla \times \nabla V = 0, V = Scalar$----------> (1)

$\nabla.\left(\nabla \times \vec{A}\right) = O, \vec{A}$= vector----------> (2)

Scalar magnetic potential:

If V_m is the scalar magnetic potential then it must satisfy

$\nabla \times \nabla V_m = 0$----------> (3)

But scalar magnetic potential is related to the magnetic field intensity $\bar{H}$

$\boldsymbol{\vec{H} = -\nabla V_m}$----------> (4)

Using in equation 3

$\boldsymbol{\nabla \times (-H) = 0}$----------> (5)

$\boldsymbol{\nabla \times \vec{H} = \vec{J}}$----------> (6)

Thus the scalar magnetic potential V_m can be defined source free region where current density is zero.

$\boldsymbol{\vec{H} = -\nabla V_m}$----------> (7)

Similar to the relation between $\bar{E}$ and electric scalar potential,magnetic scalar potential is expressed as

$$\boldsymbol{V_{m(a,b)} = -\int_b^a \vec{H}.d\vec{L}}$$

Laplace's equation for scalar magnetic potential

It is known that as monopole of magnetic field is Non existing

$\oint \vec{B}.d\vec{S}$=0----------> (8)

Using divergence theorem

$\oint \vec{B}.d\vec{S} = \iiint(\nabla.\bar{B})dv$=0----------> (9)

$\left(\nabla.\vec{B}\right)$=0----------> (10)

$\nabla.\left(\mu_0\vec{H}\right)$=0----------> (11)

$(\nabla.\vec{H})$=0----------> (12)

$\nabla.-\nabla V_m$=0----------> (13)

$\nabla^2 V_m$=0----------> (14)

This is Laplace equation for scalar magnetic potential. Similar scalar electric potential ∇^2V=0

Vector Magnetic Potential

The vector magnetic potential is denoted as $\bar{A}$ and measured in Wb/m.it has satisfy equation (2) the divergence of curl of vector is always zero

$$\nabla.(\nabla \times \vec{A}) = O$$

$(\nabla.\vec{B})$=0

$\vec{B} = \nabla \times \bar{A}$----------> (15)

Thus curl of vector magnetic potential is the flux density.

Now$\boldsymbol{\nabla \times \vec{H} = \vec{J}}$

$$\nabla \times \frac{\vec{B}}{\mu_0} = \vec{J}$$

$\nabla \times \vec{B} = \mu_0 \vec{J}$----------> (16)

$$\nabla \times \nabla \times \vec{A} = \mu_0 \vec{J}$$

Using vector identity to express right and left hand side

$$\nabla(\nabla.\vec{A}) - \nabla^2 \vec{A} = \mu_0 \vec{J}$$

$\vec{J} = \frac{1}{\mu_0}[\nabla \times \nabla \times \vec{A}] = \frac{1}{\mu_0}[\nabla(\nabla.\vec{A}) - \nabla^2 \vec{A}]$----------> (17)

Thus if vector magnetic potential is known then the current density can be obtained.

Poisson's equation for magnetic field:

In vectoralgebra, a vector can be fully defined if its curl and divergence are defined

For a vector magnetic potential $\bar{A}$ is curl defined $\boldsymbol{\nabla \times \vec{H} = \vec{B}}$ **which is known.**

But to completely define $\bar{A}$ its divergence must be known assume that$\boldsymbol{\nabla \times A}$**,the divergence of** $\vec{A}$is zero.

$$\therefore \vec{J} = \frac{1}{\mu_0}[-\nabla^2 \vec{A}]$$

$$\nabla^2 \bar{\vec{A}} = -\mu_0 \vec{J}\text{----------> (18)}$$

17. Write short notes on classification of magnetic material. (Apl/May 2018) (May 2011)
Describe the classification of magnetic material and draw atypical magnetization (B-H) curve. (Nov/Dec 2017) (Nov / Dec 2014,2015)

Magnetic materials grouped into threetypes

1. Diamagnetic
2. Paramagnetic
3. Ferromagnetic

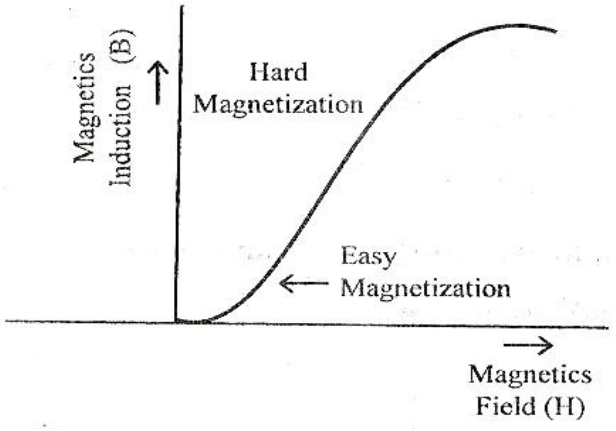
Diamagnetism occurs in materials where the magnetic fields due to electrons motions orbiting and spinning completely cancel each other.Thus the permanent magnetic moment of each atom is zero and the materials easily affected by a magnetic field.
Examples are bismuth, lead, and copper. For diamagnetic material, the values of $\mu_r \leq 1$.

Materials whose atoms have Nonzero permanent magnetic moment may be paramagnetic or ferromagnetic.paramagnetism occurs in materials where the magnetic fields produced by orbital orspinning electrons do not cancel completely.
Examples Platinum, Tungsten.Paramagnetic material, the values of $\mu_r \leq 1$.

Ferro magnetism occurs in material whose atoms have relatively large permanent magnetic moment examples Cobalt, Iron.

B-H Curve for classifying Magnetic Materials:

If an alternating magnetic field is applied to a soft magnetic material, the magnetic induction changes with the magnetic field.



18. State and explain ampere's circuital law and show that the field strength at the end of long solenoid is one half that at the center. (Dec 2010)

(OR)

Derive the general expression for the field B at any point along the axis of a solenoid (uniform cylindrical coil wound on a non-magnetic frame)

(OR)

Calculate the self-inductance of infinitely long solenoid. (May 2012)

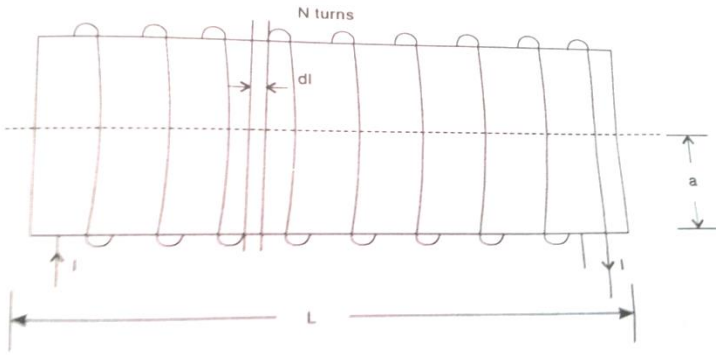
Obtain an expression for inductance and toque on a long solenoid coil. April/May 2015)

Ampere's circuital law:

It states that the lines integral of magnetic field intensity $\vec{H}$ around a closed path is exactly equal to the direct current enclosed by the path.

$$\oint \vec{H}.d\vec{L} = I$$

Field intensity of long solenoid:



Consider the cross section of the solenoid. Since the solenoid consists of circular loops, the result applied here. The contribution to the magnetic field H at the p by an element of the solenoid of length dx

Consider the small length dx at the distance x m from one end of solenoid. Let point 'P' be at a distanceof L_1 m and the angles made by the two ends of the solenoid at P is α_1 and α_2 magnetic field due to one circular loop of radius 'a' is

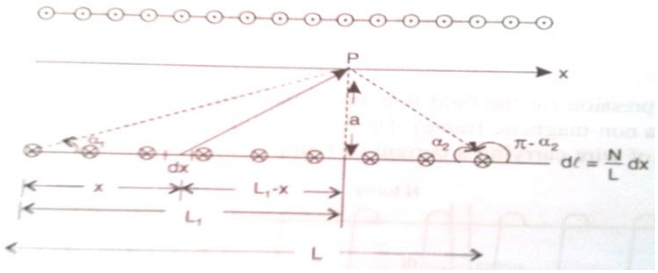
$$\vec{H} = \frac{Ia}{2[a^2 + h^2]^{3/2}}$$

Now due to differential length 'dl' of the solenoid is

$$d\vec{H}_x = \frac{\vec{I}dla^2}{2[a^2 + (l_1 - x)^2]^{3/2}}$$

Where $dl = \frac{N}{L}dx$ and $h=L_1-x$

$$d\vec{H}_x = \frac{I\frac{N}{L}dxa^2}{2[a^2 + (l_1 - x)^2]^{3/2}}$$



L_1-x=$a\cot\theta$ and -dx= $-a\,cosec^2\theta d\theta$

Integrating H between the limits $\theta_1 = \alpha_1\ to \theta_2 = \pi - \alpha_2$

$$d\vec{H}_x = \frac{\frac{NI}{L}a^2(acosec^2\theta)d\theta}{2(a^3 cosec^3\theta)}$$

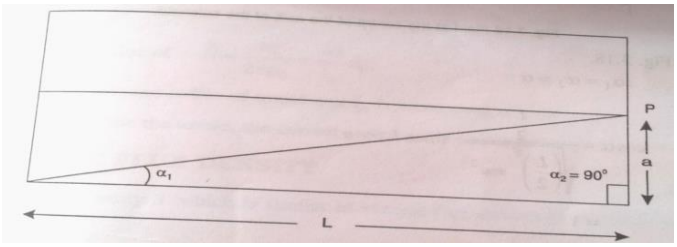
$$d\vec{H}_x = \frac{NI}{2L}\sin\theta d\theta$$

$$H_x = \int_{\theta=\alpha_1}^{\theta=\pi-\alpha_2} \frac{NI}{2L}\sin\theta d\theta$$

$$= \frac{NI}{2L}[-\cos\theta]_{\alpha_1}^{\pi-\alpha_2}$$

$$\vec{H}_x = \frac{NI}{2L}[\cos\alpha_1 + cos\alpha_2]\vec{a}_x$$

Case 1: Let the point P be at the one of the end of solenoid



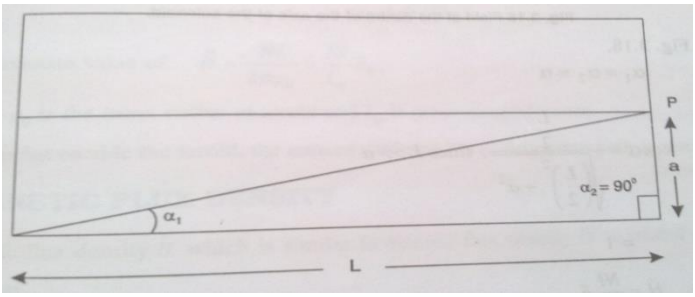
$\alpha_2 = 90^\circ$

$$cos\alpha_1 = \frac{L}{\sqrt{L^2 + a^2}}$$

$$\vec{H}_x = \frac{NI}{2L}\cos\alpha_1 = \frac{NI}{2L}\frac{L}{\sqrt{L^2+a^2}}$$

Since L>>a, $\vec{H} = \frac{NI}{2L}\vec{a}_x$

Case 2:Let 'P' at the center $\alpha_1 = \alpha_2 = \alpha$



$$\cos\alpha = \frac{\frac{L}{2}}{\sqrt{\left(\frac{L}{2}\right)^2 + a^2}} = 1$$

$$\vec{H} = \frac{NI}{L}\vec{a}_x$$

$$\vec{B} = \mu\vec{H} = \frac{\mu NI}{L}\vec{a}_x$$

Self Inductance of the solenoid:

If A is the cross sectional area in m^2 of the solenoid, The total flux through the section

$$\varphi = BA = \frac{\mu NIA}{L}$$

Flux linkage in the solenoid having N turns $N\varphi = \frac{\mu N^2 IA}{L}$ Henry

The inductance of the solenoid is

$$L = \frac{Total\ flux\ linkages}{total\ current}$$

$$L = \frac{\mu N^2 IA}{LI}$$

$$L = \frac{\mu N^2 A}{L} \text{Henry}$$

Torque

T=m B

Where m= Maximum torque

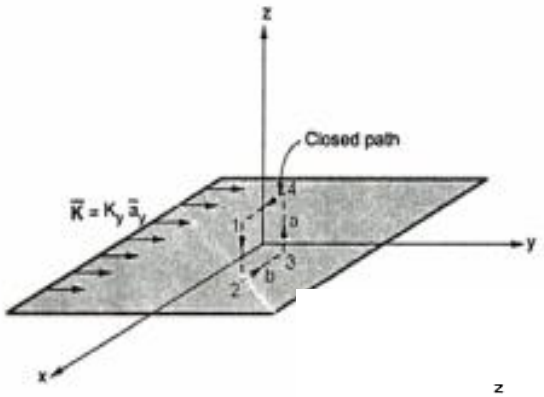
B = Magnetic flux density

---

19. For an infinite current sheet of uniform current density 'K' A/m, derive the expression for the magnetic field intensity. (May 2013)

Consider an infinite sheet of current in the z=0.the surface current density is $\vec{K}$ the current flowing in positive y direction hence $= k_y a_y$ this is shown in fig

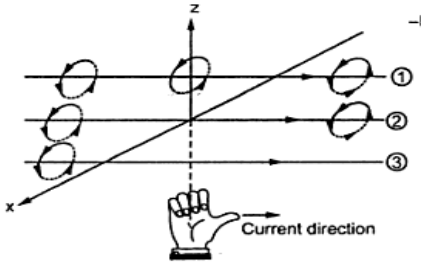
Consider the closed path 1-2-3-4 .the width of the path is b while the height is a. It is perpendicular to the direction of current in xz plane.



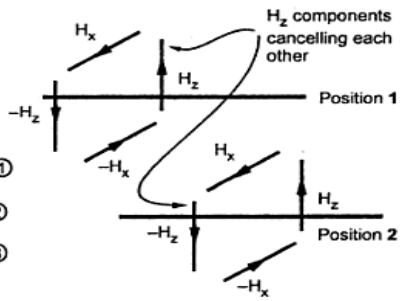
The current flowing across the distance b is given by $k_y b$.

$$I_{enc} = k_y b \text{----------} > (1)$$

Consider the magnetic lines of force due to the current in $\vec{a}_y$direction, according to the right hand thumb rule. Shown in figure.



(a)



(b)

It is clear that in between two very closely spaced conductors, the component of $\vec{H}$ in z direction are oppositely directed.

As current is flowing in y direction,$\vec{H}$ cannot have component in y direction

So $\vec{H}$ has component in x direction

$$\vec{H}=H_x\vec{a}_x \text{ for z>0 ----------> (2)}$$
$$=- H_x\vec{a}_x \text{forz<0----------> (3)}$$

Applying ampere's circuit law

$$\oint \vec{H}.d\vec{L} = I \text{ --------->(3)}$$

Evaluate the integral along the path 1-2-3-4-1

For path 1-2 $d\vec{L} = dz\vec{a}_z$

For path 3-4 $d\vec{L} = dz\vec{a}_z$

But $\vec{H}$ is in x direction while$\vec{a}_z.\vec{a}_x$=0

Along path 3-4path 1-2 the integral 2-3 along $\oint \vec{H}.d\vec{L} = 0$

Consider the closed path$d\vec{L} = dx\overrightarrow{a_x}$

$$\int_2^3 H\, d\vec{L} = \int_2^3 (-H_x a_x).(dx\overrightarrow{a_x}) = H_x\int_2^3 dx = bH_x$$

The path 2-3 lying in z<0 region for which $\vec{H}.is\ -H_x a_x$. and the limits from 2 to 3,positive to negative x hence effective sign of integral is positive.

Consider path 4-1 $d\vec{L} = dx\vec{a}_x$ z>0$\vec{H} = H_x a_x$

$$\int_4^1 \overrightarrow{H}.d\vec{L} = \int_4^1 (H_x a_x).(dx\overrightarrow{a_x}) = H_x\int_4^1 dx = bH_x$$
$$\oint \vec{H}.d\vec{L} = bH_x + bH_x = 2bH_x \text{--------->(4)}$$

Equating this to I_{enc}n equation 1

$$H_x = \frac{1}{2}k_y$$
$$2bH_x = k_y b$$

Hence

$$\vec{H} = \tfrac{1}{2}k_y\vec{a}_x \text{for z>0}$$
$$\vec{H} = -\tfrac{1}{2}k_y\vec{a}_x \text{for z<0}$$

Finite sheet of current density $\vec{K}$ A/m we can write,

$$\vec{H} = \tfrac{1}{2}\vec{K}\times\vec{a}_N \quad \text{A/m}$$

20. A very long solenoid with 2×2 cm cross –section has an iron core (μ_r= 1000 and4000 turns/meter. If it carries current of500mA, find

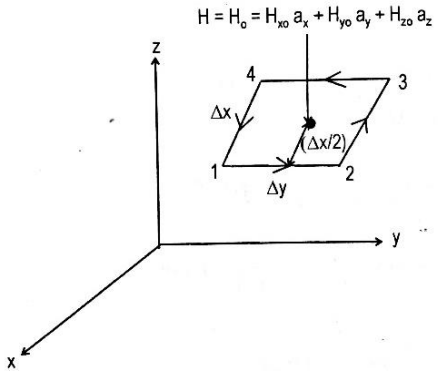
a) Its self-inductance per meter

b) The energy per meter stored in field. **(Nov/Dec 2014)**

Solution:

a) The inductance per unit length of the solenoid is given by
$L' = \mu N^2 AN/m$
Where A= Area of cross section,
N= Number of turns
$= (1000\times4\times\pi\times10^{-7})\ (4000)^2(2\times10^{-2}\times2\times10^{-2})$
$= 8.042H/m$

b) The energy stored by inductor is given by,
$W_m = \frac{1}{2} LI^2$
$= \frac{1}{2}\ (8.042)\ (500\times10^{-3})^2$
$=1.005J/m$



21. Deduce the point form of ampere's circuital law.(May 2016)

Consider a closed path 1,2,3,4 in Cartesian co-ordinate system. Magnetic field intensity at the centre is given by,

$$H = H_o = H_{xo}\, a_x + H_{yo} a_y + H_{zo} a_z.$$

We have four paths viz.,1-2,2-3,3-4,4-1.

$$(H \cdot \Delta L)_{1-2} = H_{y1-2}\,\Delta y$$

$$= \left[H_{yo} + \frac{\partial H_y}{\partial x}\left(\frac{\Delta x}{2}\right)\right]\Delta y$$

$$(H \cdot \Delta L)_{2-3} = H_{x\,2-3}\,(-\Delta x)$$

$$= \left[H_{xo} + \frac{\partial H_x}{\partial y}\left(\frac{\Delta y}{2}\right)\right](-\Delta x)$$

$$(H \cdot \Delta L)_{3-4} = H_{y\,3-4}\,(-\Delta y)$$

$$= \left[H_{yo} + \frac{\partial H_y}{\partial x}\left(\frac{-\Delta x}{2}\right)\right](-\Delta y)$$

$$(H \cdot \Delta L)_{4-1} = H_{x4-1}\,(\Delta x)$$

$$= \left[H_{xo} + \frac{\partial H_x}{\partial y}\left(\frac{-\Delta y}{2}\right)\right](\Delta x)$$

Adding (1),(2),(3) and (4),

$$\oint H \cdot dl = \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right](\Delta x \Delta y).$$

By Ampere's circuital law, the above equation is

$$\oint H \cdot dl = \Delta I$$

$$\Delta I = \Delta x \Delta y \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] = \oint H \cdot dl$$

i.e., $$\Delta x \Delta y Jz = \Delta x \Delta y \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] = \oint H.dl$$

Where

$$\Delta I = \Delta x \Delta y Jz.$$

$$J_z = \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] = \frac{\oint H \cdot dl}{\Delta x \Delta y}.$$

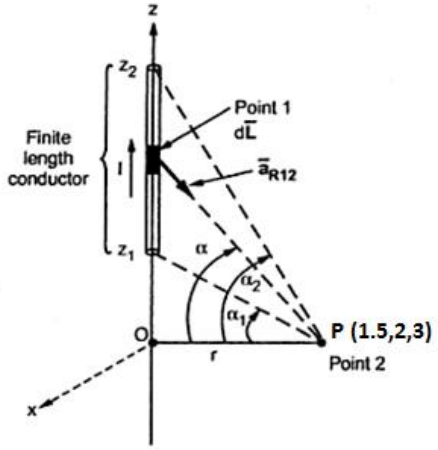
As the closed path shrinks to zero,

$$\lim_{\Delta x \Delta y \to 0} \frac{\oint H \cdot dl}{\Delta x \Delta y} = \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right] = J_z$$

Similarly,

$$\underset{\Delta y \Delta z \to 0}{Lt} \frac{\oint H \cdot dl}{\Delta y \Delta z} = \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right] = J_x$$

$$\underset{\Delta z \Delta x \to 0}{Lt} \frac{\oint H \cdot dl}{\Delta z \Delta x} = \left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right] = J_y.$$



$$\text{Curl H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) a_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) a_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial}{} \right.$$

$$\nabla \times H = \begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = J.$$

i.e., $\nabla \times H = J$.

This is point form or differential form of ampere's circuital law.

22. Find the magnetic field intensity at point P(1.5,2,3) caused by a current filament of 24 amphere in tha a_z direction on the Z axis and extending from Z = 0 and Z = 6. (May 2016)

Solution:

I = 24 A

$r = \sqrt{1.5^2 + 2^2 + (3-6)^2}$

r = 3.905

For z=0, $z_1 = r\tan\alpha_1$ and for z=6, $z_2 = r\tan\alpha_2$

$\alpha_1 = tan^{-1}\left(\frac{0}{3.905}\right)$ and $\alpha_2 = tan^{-1}\left(\frac{6}{r3.905}\right)$

$\alpha_1 = 0$ $\alpha_2 = 56.94$

$$\vec{H} = \frac{I}{4\pi r}[sin\alpha_2 - sin\alpha_1]\vec{a}_\emptyset \text{ A/m}$$

$$= \frac{24}{4\pi \times 3.905}[sin\ 56.54 - sin0]$$

$$= \frac{24}{4\pi \times 3.905}[0.838]$$

$$= 0.489 \times 0.838$$

H = 0.4097 A/m

23.An Air co-axial transmission line has a solid inner conductor of radius 'b',Determine the inductance per unit length of the line.(Nov/Dec2015)

Consider a co-axial cable with solid inner conducor of radius `a' and outer radius `b'. Let I be the current in solid conductor and I be the current in outer conductor.

The flux density within the solid conductor at a distance r from axis of the cable is

$$\boldsymbol{B = \frac{\mu_o \mu_r I' r}{2\pi a^2}} \; ; \qquad \mathbf{0<r<a}$$

The current flowing in solid inner conductor between a to r is

$$\mathbf{I' = \frac{I}{\pi a^2}.\pi r^2 = \frac{I}{a^2} r^2}$$

$$\mathbf{Now\ B = \frac{\mu_o \mu_r I' r}{2\pi a^2}.\ \frac{I r^2}{a^2} = = \ \frac{\mu_o \mu_r I' r^3}{2\pi a^4}}$$

The total flux linkage per unit length between 0 and a.

$$\boldsymbol{\Phi = \int_0^a \frac{\mu_o \mu_r I' r^3}{2\pi a^4} . dr}$$

$$\boldsymbol{= \frac{\mu_o \mu_r I'}{2\pi a^4} \left[\frac{r^4}{4}\right]}$$

$$\boldsymbol{= \frac{\mu_o \mu_r I'}{2\pi a^4} \left[\frac{a^4}{4}\right]}$$

$$\boldsymbol{= \frac{\mu_o \mu_r I}{8\pi}}$$

Inductance of solid conductor between 0 and a.

$$\mathrm{L} = \frac{\phi}{I}$$

$$L_1 = \frac{\mu_o \mu_r}{8\pi}$$

Inductance of co-axial cable per unit length between a and b

$$\boldsymbol{L_2 = \frac{\mu_0}{2\pi} \ln \left[\frac{b}{a}\right]}$$

Inductance of co-axial cable per unit length is

$$\mathbf{L = L_1 + L_2}$$

$$= \frac{\mu_o \mu_r}{8\pi} + \frac{\mu_0}{2\pi} \ln \left[\frac{b}{a}\right]$$

$$\mathbf{L} = \frac{\mu_o}{4\pi}\left[\frac{1}{2}\mu_r + 2\ \mathbf{ln}\left[\frac{\boldsymbol{b}}{\boldsymbol{a}}\right]\right]\ \mathbf{H/m}$$

---

24. Calculate the inductance of a ring shaped coil of mean diameter 20cm wound on a wooden core of 2 cm diameter containing 200 turns (nov/Dec 2016)

$$Mean\ radius\ R\ = \frac{20}{2} = 10\ \times 10^{-2} m$$

$$\mathrm{N} = 200$$

$$Radius\ of\ cross-section\ r\ = \frac{2}{2} = 1\ \times 10^{-2} m$$

$$\boldsymbol{L = \frac{\mu_0 \mu_r N^2 A}{2\pi R}} = \frac{4\pi \times 10^{-7} \times 1 \times 200^2 \times \pi \times (1\ \times 10^{-2})^2}{2\pi \times 10\ \times 10^{-2}} = \mathbf{1.4}\ \boldsymbol{H}$$

25. An iron ring 0.2m in diameter and 10cm^2 sectional area of the core, is uniformly wound with 250 turns of wire. The wire carries a current of 4A. The relative permeability of iron is 500. Determine the value of self inductance and the stored energy(Apl/May 2018)

$$L = \frac{\mu_0 \mu_r N^2 A}{2\pi R}$$

R=d/2=0.1m

$$L = \frac{\mu_0 \mu_r N^2 A}{2\pi R}$$

$$L = \frac{4\pi * 10^{-7} * 500 * 250 * 250 * 10 * 10^{-4}}{2\pi * 0.1}$$

$$= 0.039$$

stored energy=LI^2

$$= 16 * 0.039 = 0.624J$$

26. What is the maximum torque on a square loop of 1000 turns in a field of uniform flux density of 1 tesla? The loop has 10cm sides and carries a current of 3A.What is the magnetic moment of the loop. (Apl/May 2018)

Flux density = B tesla

Area A = $10 \times 10\ cm^2 = 100\ cm^2 = 100 \times 10^{-4}\ m^2$

Current I = 3A

Magnetic moment, "m" = $\frac{T_{max}}{B}$

T_{max} = BIA =B × 3× 100 × 10^{-4} N.m

"m" = $\frac{T_{max}}{B} = 300 \times 10^{-4} = 30 \times 10^{-3}$.

PART A

1. Define Scalar Magnetic Potential.
2. Define Vector Magnetic Potential and state its unit. (May 2010)
3. State Laplace's equation for Scalar Magnetic Potential. (Dec 2010, May 2011)
4. State Poisson's equation for Magnetic Field. (Dec 2010, May 2011)
5. What is the relation between Magnetic Flux Density and Magnetic Field Intensity?
6. Define Biot–savart law? (Dec 2010)
7. Distinguish between Scalar and Vector Magnetic Potential. (Nov/Dec 2014,15)
8. A long straight line wire carries a current I = 1 amp. At what distance is the magnetic field H = 1 A/m. (June 2009)
9. State Ampere's Circuital law? (May/June 2014)
10. Can a static magnetic field exist in a good conductor? Explain.
11. Define under what condition will the E-field irrotational and solenoidal.
12. For the configuration shown calculate the value of $\oint \overrightarrow{H}.\overrightarrow{dL}$
13. List the application's of Ampere's circuital law. (Dec 2007)
14. State the expression of $\vec{H}$ due to infinite sheet of current. (Dec 2007)
15. What is $\vec{H}$ due to a long straight current carrying conductor? (May 2008)
16. What is the force and its direction on a conductor in +Z axis carrying current I placed in a magnetic field $\vec{B}$ along X axis?
17. Write Lorentz force equation. Give application of the same. (Nov 2005, May 2010)
18. What is Magnetic Dipole Moment?
19. Define Magnetization.
20. Define ferro magnetic materials.
21. Write down magnetic boundary conditions.
22. What is the expression for inductance of a toroid?
23. Define the terms: Magnetic Moment and Magnetic Permeability. (May 2010)
24. What are major classifications of magnetic materials?
25. Calculate inductance of a ring shaped coil having a mean diameter of 20cm wound on a wooden core of 20cm diameter. The winding is uniformly distributed and contain 200turns.
26. Write the expression for the magnetic force between an electromagnet and an armature to be attracted. (May 2013)
27. Write the expression for magnetic field 'H' at the centre of a circular coil carrying a current 'I' ampere. The radius of the coil is 'a' m. (Nov 2012)
28. Find the inductance per unit length of a long solenoid of N turns and having a length 'L' metre. Assume that it carries a current of 'I' amperes.(May 2012)(May/June 2014)

29. State Ohm's law for magnetic circuits. (Nov 2012) (Nov/Dec 2014)
30. Determine the value of magnetic field 'H' at the centre of a circular loop carrying a current of 10 A. The radius of the loop is 2 m. (Nov/Dec 2014)

PART B

1. What is Biot-savart law? Find the magnetic flux density at a point Z on the axis of a circular loop of radius 'r' that carries a direct current I.(May 2010,May 2013)

(OR)

Find the magnetic field intensity $\vec{H}$ due to circular current conductor, carrying a current I, for the following cases:

(c) At a height 'z' from the centre of the circular loop.
(d) At the centre of the circular loop.

(OR)

Find the magnetic flux density at a point on the axis of a circular loop of a radius 'r' carrying a direct current I.

2. Obtain the flux density produced by an infinitely long straight wire carrying a current I at any point at a distant 'r' normal to the wire
3. In the magnetic field intensity at a point P due to a finite straight conductor, carrying a current I.(Dec 2010)
4. Obtain an expression for magnetic flux density and magnetic field intensity at any point due to a finite length conductor. (April/May 2015)
5. Obtain an expression for magnetic flux density and magnetic field intensity at any point along the axis of a circular coil (Dec 2015)
6. State and explain Ampere's circuital law using mathematical expression for finding magnetic flux density due to current I.(May 2010)
7. Determine $\vec{H}$ due to infinitely long straight conductor using Ampere's Law.
8. Obtain the expression for $\vec{H}$ in the regions of a cylindrical conductor carries a direct current I and its radius is 'R' m. Plot the variation of $\vec{H}$ against the distance R from the center of the conductor. (Jun 2007, May 2013) (Nov / Dec 2014)
9. Derive $\vec{H}$ due to a Co-axial Cable. (May 2012)

Derive the expression for the magnetic field intensity inside and outside a coaxial conductor of inner radius 'a', outer radius 'b', and carrying a current of "I" amperes in the inner and outer conductors. / June 2014)

10. Obtain an expression for magnetic vector potential in the region surrounding an infinitely long straight filamentary current I. (May 2012) or
Derive the expression for magnetic vector potential in the cases of an infinitely long, straight conductor in free space. (May/ June 2014)

11. Derive the magnetic force between two parallel conductors carrying equal current in (i) same direction (ii) opposite direction.(May 2010)

12. Derive the torque equation on a closed loop conductor. (Or)

Derive an expression for torque in a rectangular loop which is carrying a current of "I" amperes and is situated in a uniform field 'B' Wb/m^2. (Nov / Dec 2014)

13. Derive the expression for coefficient of coupling in term of mutual and self in direction.(May 2010)
14. Derive the expression for inductance of a toroid.
15. Calculate the internal and external inductances per unit length of a transmission line consisting of two long parallel conducting wires of radius 'a' that carry currents in opposite directions. The axes of the wires are separated by a distance'd', which is much longer than 'a'.
16. Derive the expression for energy stored in magnetic field and energy density in the magnetic field.(May 2013,Dec 2012)
17. Derive the boundary conditions at an interface between two magnetic media. (May'10) or (May / June 2014)
State and prove magnetic boundary conditions. (May/June 2015)
18. Derive the Scalar and Vector magnetic potential. (May 2012)
19. Write short notes on classification of magnetic material. (May 2011)
Describe the classification of magnetic material and draw a typical magnetization (B-H) curve. (Nov / Dec 2014,15)
20. State and explain ampere's circuital law and show that the field strength at the end of long solenoid is one half that at the centre. (Dec 2010)
(OR)
21. Derive the general expression for the field B at any point along the axis of a solenoid (uniform cylindrical coil wound on a non-magnetic frame)
(OR)
22. Calculate the self-inductance of infinitely long solenoid. (May 2012)
Obtain an expression for inductance and toque on a long solenoid coil. April/May 2015)
23. For an infinite current sheet of uniform current density 'K' A/m, derive the expression for the magnetic field intensity. (May 2013)

UNIT IV - ELECTRODYNAMIC FIELDS

Magnetic circuits- Faraday's laws – Transformer and motional EMF - Displacement current- Maxwell's equations (differential and integral forms) -Relation between field theory and circuit theory- applications.

PART -A

1. State Faraday's law of electromagnetic induction. (May 2011, 2016,Dec 2010)(Nov/Dec 2016)

Faraday's law states that, in time varying magnetic fields, the induced emf is equal to the time rate of change of magnetic flux linking with the closed circuit.

$$e = \frac{-Nd\emptyset}{dt}\ volts$$

Where N = number of turns in the circuit
e = induced emf
Φ= flux linked with the circuit

2. State Lenz's law (May 2010, June 2007, and Dec 2006)

According to Faraday's law of electromagnetic induction

$$e = \frac{-Nd\emptyset}{dt}$$

The minus sign indicates the direction of the induced emf which acts to produce field opposite to original field. This is called as Lenz's law.

3. What are displacement current density and its significance?(May 2011, May 2010, Dec 2012, Nov 2007, Dec 2012, June 2012)(Nov/Dec 2017)

The term $\overrightarrow{J_d} = \frac{\partial \vec{d}}{\partial t}$ is known as displacement current density and it is a result of time-varying electric field. A typical example of such current is the current through a capacitor when an alternating voltage source is applied to its plates.

Significance: Without displacement current, the electromagnetic wave propagation is Impossible.

4. What is motional emf? (Or) Write down the expression for the emf induced in the moving loop in static B field. (May 2008, Nov 2006)

The induced emf obtained by a stationary fields and a moving circuit is called motional emf or generator emf. The above action is similar to generator action.

$$\oint \vec{E}_m . d\vec{l} = \int \left(\vec{v} \times \vec{B}\right) . \overrightarrow{dl}$$

(or)

$$e = BLv \sin 90°$$
$$e = BLv$$

5. What is transformer emf?

The induced emf obtained by keeping the circuit stationary and field carrying sinusoid ally is called transformer emf. The above action is similar to transformer action.

$$\overrightarrow{\oint \vec{E} . \overrightarrow{dl}} = -\int_s \overrightarrow{\frac{\partial B}{\partial t}} . \overrightarrow{ds}$$

6. What is the equation for total emf due to motion (or) transformer action? Give one practical case.

$$\overrightarrow{\oint \vec{E} . \overrightarrow{dl}} = -\int_s \overrightarrow{\frac{\partial B}{\partial t}} . \overrightarrow{ds} + \int \left(\vec{v} \times \vec{B}\right) . \overrightarrow{dl}$$

Example: A DC machine suffering from armature reaction.

7. Write about conduction current density.

The current through a conductor or resistor is called conduction current and its ratio with cross – sectional area of the conductor or resistor. $J_c = \frac{Ic}{A}$ iscalled as conduction current density

8. Write down the differential form of Maxwell's equations (or) Write point from of Maxwell's equations.

(i) From faraday's law

$$\nabla \times \vec{E} = -\frac{\overrightarrow{\partial B}}{\partial t}$$

(ii) From Ampere's law

$$\nabla \times \vec{H} = \vec{J} + \frac{\overrightarrow{\partial D}}{\partial t}$$

(iii) From Gauss's law

$$\nabla . \vec{D} = \rho v$$

(iv) No isolated magnetic charges

$$\nabla . \vec{B} = 0$$

9. Write down the integral form of Maxwell's equation. (Nov 2007)

(i) From faraday's law

$$\oint \vec{E} . \overrightarrow{dl} = -\int_s \frac{\overrightarrow{\partial B}}{\partial t} . \overrightarrow{ds}$$

(ii) From Ampere's law

$$\oint \vec{H}.\overrightarrow{dl} = I + \int_s \frac{\overrightarrow{\partial D}}{\partial t}.\overrightarrow{ds}$$

(iii) From Gauss's law

$$\oint \vec{D}.\overrightarrow{ds} = \int_v \rho_v d_v$$

(iv) No isolated magnetic charges

$$\int_s \vec{B}.\overrightarrow{ds} = 0$$

10. State Maxwell's I and II equations (Dec 2010)

Maxwell's I equation

$$\oint \vec{H}.\overrightarrow{dl} = I + \int_s \frac{\overrightarrow{\partial D}}{\partial t}.\overrightarrow{ds}$$

Maxwell' II equation

$$\oint \vec{E}.\overrightarrow{dl} = -\int_s \frac{\overrightarrow{\partial B}}{\partial t}.\overrightarrow{ds}$$

11. Compare field theory and circuit theory.(May 2010, June 2006, May 2006)(Nov/Dec 2017)

FIELD THEORY	CIRCUIT THEORY
1. The analysis of the transmission line from the field point of view is carried out starting with Maxwell's equation based on Faraday's law and Ampere circuit law with conduction current $\vec{J} = 0$	The analysis of the transmission line from the circuit point of view is carried out by considering of it as a four terminal network. Out of these four terminals two are at sending end while other two are at receiving end.
2. In field theory, current density $\overrightarrow{J}$ is used	In circuit theory, the current is used as a variable.
3.Ohm's law is given by $\oint \vec{E}.\overrightarrow{dl} = z\int \ \vec{H}.\overrightarrow{dl}$	Ohm's law is given by $V = IZ$

12. Mention the limitations of circuit theory. (May 2008)

1. Circuit theory is applied for conductor only and not for any other media.
2. It use only lumped parameters
3. It cannot be used to analyze the wave propagation in a medium.

13. Give the energy relation in an electromagnetic field.

$$w = \frac{1}{2} BH \; J/m^3$$

$$= \frac{1}{2} \mu H^2 \; J/m^3$$

$$= \frac{1}{2} B^2/\mu \; J/m^2$$

Where w- energy density in magnetic field.

14. A conductor of 1m length is moved with a velocity of 100 m/sec. perpendicularly to a field of 1 Tesla. What is the value of emf induced? (Dec 2012)

Sol:

L=1 m, V=100 m/sec, B= 1 tesla θ = 90 (since perpendicular to a field)

E=Blv sinθ =1x1x100xsin 90
= 100 volts.

15. State continuity equations. (June 2012)

Continuity equation state that

$$\vec{J}_T = \vec{J}_c + \vec{J}_D$$

Where $\vec{J}_T$ is the total current density
Jc is conduction current density
Jd is displacement current density

16. Define mutual and self inductance. (April/ May 2015)

Self-inductance :

The ratio of total flux linkage to the current producing that flux is called inductance denoted by L
It is measured in Henry (H)

$$\mathbf{L} = \frac{N\emptyset}{I} \; \mathbf{H}$$

Mutual inductance:

With respect to the flux linkage, the mutual inductance between the two coils is defined as the ratio of the flux linkage of one coil to the current in other coil. Hence we can write,

$M_{12} = \frac{N_2\phi_{12}}{I_1}$ and $M_{21} = \frac{N_1\phi_{21}}{I_2}$. For linear medium, $M_{12} = M_{21} = M$

---

17. Distinguish between transformer emf and motional emf. (April/May 2017)

Motional emf

Motional emf is the emf induced due to the relative motion between magnetic flux lines and a conductor, like that which is induced in a generator.

Transformer emf

The transformer emf is the induced emf by electromagnetic coupling between two objects, which don't move relative to each other, due to time- varying current like that in the transformer.

18. Give the two important equations that provide a connection between field and circuit theory.(Nov/Dec 2014)

- Faraday's law
- Ohm's law

20.A parallel plate capacitor with plate area of $5Cm^2$ and plate sepanatian of 3mm has a voltage $50 \sin 10^3 t\ V$ applied to its plates. Calculate the displacement current assuming$\varepsilon = 2\ \varepsilon_0$. (April/May 2014)(Nov/Dec 2015)

Sol:

$D = \varepsilon\epsilon = \varepsilon\frac{v}{d}$ Area A=$5Cm^2 = 5 \times 10^{-4}\ m^2$

$$J_d = \frac{\partial D}{\partial t} = \frac{\epsilon}{d}\frac{\partial v}{\partial t} d = 3mm = 3 \times 10^{-3} m$$

$J_d = \frac{Id}{A}$

$Id = J_d . A = \frac{\epsilon A}{d}\frac{\partial v}{\partial t} = c\frac{\partial v}{\partial t}$

$Id = 2 \times 8.854 \times 10^{-12} \times \frac{5 \times 10^{-4}}{3 \times 10^{-3}} \times \frac{d}{dt}(50\ sin 10^3 t)$

$= 147.4 cos 10^3 t\ n\ A$

---

21. Find the emf induced in a conductor of length 1 meter moving with a velocity of 100 m/s perpendicular to a field of 1tesla?(April/May 2017)

$$e = Blv\ sin\theta$$

As the field and the direction of motion are [perpendicular to each other\, $\theta = 90°$, that means

$sin\theta$=sin 90 = 1

$$\therefore e = (1)(1)(100)\ (1)\ =\ 100\,Volts$$

22. Moist soil has a conductivity of 10^{-3} S/m and ε_r=2.5. Determine the conduction and displacement current density if E =6.0 10^{-6}sin 910^9tv/m (Nov/Dec 2016)

Conduction current density, $J_C = \sigma E = 10^{-3} \times 9 \times 10^9 sin9 \times 10^9$

$$J_C = 9 \times 10^6 sin9 \times 10^9$$

Displacement current density,

$$J_D = \varepsilon \frac{\partial E}{\partial t} = \varepsilon_0 \varepsilon_r \frac{\partial E}{\partial t}$$

$$= (8.854 \times 10^{-12} \times 2.5) \times 6 \times 10^{-6} \times 9 \times 10^9 cos9 \times 10^9 t$$

$$= 1.19 \times 10^{-6} cos9 \times 10^9 t$$

23. Define reluctance and permeability?(April/May 2018)

Permeability: The ability of a material to support the formation of a magnetic field within itself. Basically, tells us how much a material allows itself to be magnetized by a magnetic field. Highly magnetic materials like soft iron have a high relative permeability and magnetic susceptibility

Magnetic reluctance: Analogous to resistance in an electric circuit, it is used to analyse a "magnetic circuit". Electric current always follows the path of least resistance. Similarly, a magnetic field causes magnetic flux to follow the path of least reluctance.

24. Compare and explain in detail conduction and displacement current. (April/May 2015)(Nov/Dec 2015)(April/May 2018)

Conduction current	Displacement current
◆ It is developed by metal or a semiconductor on application of Electric field 'E'.	◆ It is current flows in the capacitor.
◆ Conduction current $J = \sigma E$	◆ Displacement current density $J_D = \varepsilon_0 \frac{\partial E}{\partial t} Amp/m^2$
◆ It is strong	◆ Displacement current is negligible relative to conduction current.

UNIT –IV (PART –B)

1. State and explain Faraday's law of electromagnetic induction. Hence derive the expressions for statically and dynamically induced emf. (May 2006, May 2010)(May/June 2014)
Explain the concept of emf induction in static and time varying magnetic field.(Nov/Dec 2016)(Nov/Dec 2017)(April/May 2018)

A major advancement in Electro – magnetic theory was made by Alexander Michael faraday. He discovered that a current was induced by a conducting loop for a changing magnetic flux.

Faraday's law:

Faraday's law states that the emf produced around a closed path is proportional to the rate of change of the magnetic flux through any surface bounded by that path.

$e = -\frac{d\emptyset}{dt}$

Where e=electric motive face (emf)

$$\emptyset = \text{magnetic flux in webers}$$

For 'N' number of turns, $e = -N\frac{d\emptyset}{dt}$

The principle is used in operation of generators, transformers, and most other electrical machines.

Lenz's law:

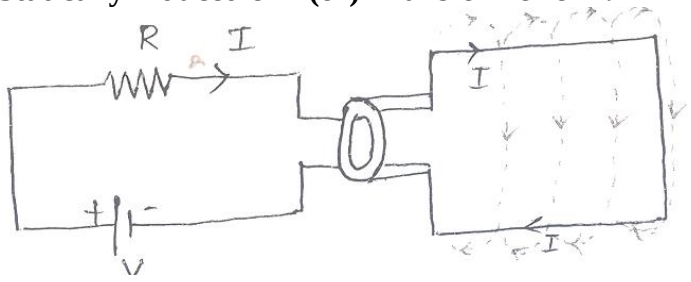
"The emf induced in an electric circuit always act in such a direction that the current it drives around the circuit opposes the change in magnetic flux which produces the emf "as per Lenz's law.

Induced emf

The emf induced in a circuit (or) coil by two ways' through which they can be classified in to two categories.

(i) Statically induced emf (or) Transformer emf
(ii) Dynamically induced emf (or) Motional emf

Statically induced emf (or) Transformer emf.



Consider a stationary coil shown in figure when a supply of 'V' voltage is applied, it feeds 'I' current through rectangular loop or coil. Whenever a current is flowing through the coil or inductor it is getting magnetized and magnetic field will be set up in the form of flux lines with a density of 'B'. If there is a charge in the value of current through the loop, there will be a change in flux.

Because as per Ampere's law $\oint \vec{H}.\overrightarrow{dL} = I$

We know $\vec{B} \propto \vec{H}$ and $\vec{B} = \mu\vec{H}$
Also $B = \frac{\emptyset}{A}$ or $\emptyset = \oint_s \vec{B}.\overrightarrow{ds}$
As per faraday's law

$$e = -\frac{d\emptyset}{dt} = -N\frac{d\emptyset}{dt} = V$$

Where N= N number of turns.

e- Statically induced emf or transformer emf. In a statically induced emf or transformer emf, the conductor loop is stationary and the magnetic field is changing.

We know that

$$\oint \vec{E}.\overrightarrow{dL} = V$$

$$\Rightarrow \oint \vec{E}.\overrightarrow{dL} = V = -\frac{d\emptyset}{dt}$$

$\Rightarrow v = \oint \vec{E}.\overrightarrow{dL} = -\frac{-d(\oint \vec{B}.\overrightarrow{ds})}{dt}$---------- (1)
As per stokes theorem

$$\oint \vec{E}.\overrightarrow{dL} = \oint_s \nabla \times \vec{E}.\overrightarrow{ds}$$---------- -(2)

Equating 1 and 2 we get

$$\oint_s \nabla \times \vec{E}.\overrightarrow{ds} = \frac{-d(\oint \vec{B}.\overrightarrow{ds})}{dt}$$

Interchanging integration and differentiation in RHS

$$\oint_s \nabla \times \vec{E}.\overrightarrow{ds} = -\oint_s \frac{\partial \vec{B}}{\partial t}.\overrightarrow{ds}$$

$\Rightarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$--------(3)

$$\Rightarrow \nabla \times \vec{E} = -\mu\frac{\partial \vec{H}}{\partial t}$$

Equation (3) is one of Maxwell's equations.

Dynamically induced emf (or) Motional emf

In system, where the magnetic field remains stationary and the conductor or coil is moving, an dynamically induced emf (or) motional emf is produced.

A charged particle (or) loop (or) coil in motion in a magnetic filed($\overrightarrow{B}$) experiences a force ($\overrightarrow{F}$)

$\vec{F} = Q(v \times \vec{B}) \rightarrow Lorentz\ force$

$\vec{F} = QvB\ sin\theta$ ---------- (4)

But we know $\vec{F} = Q\vec{E} \Rightarrow E\frac{\vec{F}}{Q}$---------- (5)

Sub, eqn (4) in (5),

$$\vec{E} = \frac{Q(\vec{v} \times \vec{B})}{Q} = \vec{v} \times \vec{B}$$

$$but\ V = \oint \vec{E}.\overrightarrow{dL} = \oint(\vec{v} \times \vec{B}).\overrightarrow{dL}$$

This emf is called motional emf or flux cutting emf,

Applying stokes theorem

$$\int \nabla \times \vec{E}.\overrightarrow{ds} = \int_s \nabla \times (\vec{v} \times \vec{B}).ds$$

$\nabla \times \vec{E} = \nabla(\vec{v} \times \vec{B})$---------- (6)

Combined transformer and motional emf:-

Assuming a time varying magnetic field, in which a conducting loop or coil is moving, here both the transformer emf and motional emf are induced.

Hence the total emf induced is

$$V = \oint \vec{E}.\overrightarrow{dL} = -\int_s \frac{\partial B}{\partial t}ds + \int_L (\vec{v} \times \vec{B})dL$$

$$\Rightarrow \int_s \nabla \times E.ds = -\int_s \frac{\partial \vec{B}}{\partial t}\overrightarrow{ds} + \int_s \nabla \times (\vec{v} \times \vec{B}).ds$$

$$= \int_s \left[-\frac{\partial \vec{B}}{\partial t} + (\nabla(\vec{v} \times \vec{B}))\right]ds$$

$\Rightarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} + \nabla \times (\vec{v} \times \vec{B})$---------- (7)

2. Derive and explain Maxwell's equations both in integral and point forms.(Nov/Dec 2017) (Dec 2012,May 2007, May 2006,Dec 2008 , Dec 2005,May 2003,June 2012,May 2010) (May/June 2014, 2016)(Nov/Dec 2014) (April/May 2017)(Nov/Dec 2016)
Derive and explain Maxwell's equations in integral form from fundamental laws for a good conductor. (April/May 2015)(April/May 2018)
Derive the set of Maxwell's equations with solutions in integral form from fundamental laws for a free space. (Nov/Dec 2015)

Maxwell's equations are a set of four partial differential equation that describe the properties of the electric and magnetic fields and relate them to their sources, charge density and current density.

These equation are used to show that, light is an electromagnetic wave.
Basically there are four Maxwell's equations. They are

Maxwell's equation obtained from Ampere's law.
Maxwell's equation obtained from Faraday's law.
Maxwell's equation obtained from Gauss law for Electric field.
Maxwell's equation obtained from Gauss law for Magnetic field.

(I) Maxwell's equation obtained from Ampere's law.

Ampere's law states that, the line integral of magnetic field intensity vector around any closed path is equal to the direct current enclosed by the path.

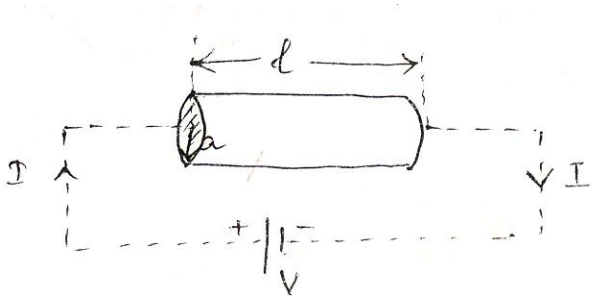
$$\oint \vec{H}.\overrightarrow{dL} = I$$

$$\Rightarrow \oint \vec{H}.\overrightarrow{dL} = I = \oint_s \vec{J}.\overrightarrow{ds} \text{---------- (1)}$$

Where $\vec{J}$ current density in A/m^2
Where
Where $J_c \rightarrow$ conduction current density
$J_D \rightarrow$Displacement current density

To derive J_c

Current flowing through a resistive element is called conduction current and the density is conduction current density.



From ohm's law $V = IR = I_cR$
or

$$I_c = {}^{V}/_{R}$$

Where R→ resistance of the element

$$R = \rho\frac{l}{a} = \frac{\sigma l}{a}$$

Where ρ-resistivity of the element

L-length of the element

A-area of cross-section of the element

σ-conduvtiuity of the material

$\Rightarrow I_c = V.\frac{\sigma a}{l}$

But we know, V=-$\int \vec{E}.\overrightarrow{dl}$

$\Rightarrow v = E(l)$

$\Rightarrow I_c = E.l.\frac{\sigma a}{l} E\sigma a$

$$J_c = \frac{I_c}{a} = \frac{E\sigma a}{a} = E\sigma$$

$$J_c = E\sigma$$

$J_c = \sigma E \rightarrow$ "Ohm'slaw in point forms"--------(3)

To derive J_d

The displacement of current taking place in a capacitive element is called displacement current. When a potential is applied across the plates of a parallel plate capacitor, the current is displaced as positive and negative charge across the plate. It is given as.

$$I_D = \frac{dQ}{dt} = \frac{d}{dt}(cv) = \frac{C\,dv}{dt}$$

$$\Rightarrow I_D\left(\frac{A\varepsilon}{d}\right)\frac{dv}{dt}$$

\where A- c/s area of the parallel mate capacitor

D-distancebetween the plates

But we know

$$v = Ed$$

$$\Rightarrow I_D = \frac{A\varepsilon}{d} \times \frac{d}{dt}(Ed)$$

$$= \frac{A\varepsilon}{d} \times d \times \frac{dE}{dt}$$

$$I_D = \frac{\varepsilon A\, dE}{dt}$$

$$\Rightarrow J_D = \frac{I_D}{A} = \frac{\varepsilon A\,\frac{dE}{dt}}{A} = \frac{\varepsilon\, dE}{dt}$$

$\Rightarrow \overrightarrow{J_D} = \frac{\varepsilon\,\partial \vec{D}}{\partial t}$ [Considering of partialdifferentiation]

$\Rightarrow \overrightarrow{J_D} = \frac{\partial \vec{D}}{\partial t}$---------- (4)

Now, sub eqn,(3) and (4) in (1)

$$\Rightarrow \oint \vec{H}.\overrightarrow{dL} = \iint_s (J_c + J_D)\, ds$$

$$\Rightarrow \oint \vec{H}.\overrightarrow{dL} = \iint_s (\sigma\vec{E} + \frac{\varepsilon\, d\vec{E}}{dt}) ds$$

$\Rightarrow \oint \vec{H}.\overrightarrow{dL} = \iint_s (\vec{J} + \frac{d\vec{D}}{dt}) ds$ Maxwell's equation

"The above equation is Maxwell's equation I in Integral form"
Appling stokes theorem

$$\oint \vec{H}.\overrightarrow{dL} = \iint_s (\nabla \times \vec{H}).\overrightarrow{ds}$$

$$\iint_s (\nabla \times \vec{H}).\overrightarrow{ds} = \iint_s (\vec{J} + \frac{d\vec{D}}{dt})\overrightarrow{ds}$$

$\nabla \times \vec{H} = \vec{J} + \frac{d\vec{D}}{dt}$ - Maxwell's I equation in point form

It states that, "The magnetic field intensity$\vec{H}$around only closed path is equal to sum of the conduction and displacement current densities enclosed by that path".

(II) Maxwell's equation obtained from Faraday's law:
From Faraday's law,

$$e = -\frac{d\emptyset}{dt}$$

$$e = -\frac{d}{dt}(\int \vec{B}.\overrightarrow{ds})$$

$$\Rightarrow V = \oint \vec{E}.\overrightarrow{dL} = \frac{-d}{dt}\int_s \vec{B}.\overrightarrow{ds}$$

$\Rightarrow \oint \vec{E}.\overrightarrow{dL} = -\iint_s \frac{\partial B}{\partial t}\overrightarrow{ds}$ - Maxwell's II equation in Integral form

$$\Rightarrow \oint \vec{\mathrm{E}}.\overrightarrow{\mathrm{dl}} = -\mu \iint_s \frac{\partial \mathrm{H}}{\partial \mathrm{t}}\overrightarrow{\mathrm{ds}}$$

Appling stokes theorem,

$\iint_s \nabla \times \vec{E}.\overrightarrow{ds} = -\mu \iint_s \frac{\partial \vec{H}}{\partial t}.\overrightarrow{ds}.$

$\Rightarrow \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$- Maxwell's II equation in point form

(III) Maxwell's equation obtained from Gauss's law for Electric field.

The Gauss's law states that, the electric flux passing through any closed surface is equal to the total charge enclosed by that pat.

$$\oiint_s \vec{D}.\overrightarrow{ds} = Q = \iiint_v \rho_v d_v$$

$\Rightarrow \oiint_s \vec{D}.\overrightarrow{ds} = \iiint_v \rho_v d_v$ →Maxwell's equation in integral from as obtained from gauss's law

From Divergence theorem, we know the

$$\oiint_s \vec{D}.\overrightarrow{ds} = \iiint_v \nabla.\vec{D}d_v$$

$$\iiint_v \nabla.\vec{D}\,dv = \iiint_v \rho_v d_v$$

$\Rightarrow \nabla.D = \rho_v$ →Maxwell'sequation in point from as obtained from electric gauss's law

(IV) Maxwell's equation obtained from Gauss's law for magnetic field.

The magnetic Gauss's law states that the net magnetic flux of lines passing through any closed surface is zero.

i.e.$\emptyset = 0$

$$but\ \emptyset = B \times A = \oiint_s \vec{B}.\overrightarrow{ds} = 0$$

$\oiint_s \vec{B}.\overrightarrow{ds} = 0$ -Maxwell's equation in integral from obtained from magnetic gauss law

Applying divergence theorem,

$$\oiint_s \vec{B}.\overrightarrow{ds} = \iiint \nabla \times \vec{B}\,dv$$

$\Rightarrow \nabla.B = 0$ →Maxwell'sequation in point from obtained from magneticgauss law

i.e"No isolated magnetic changes can exist"

Summary of Maxwell's equations in general form

Point form	Integral form
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E}.\overrightarrow{dl} = -\iint_s \frac{\partial \vec{B}}{\partial t}.\overrightarrow{ds}$
$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$	$\oint \vec{H}.\overrightarrow{dL} = \vec{I} + \iint_s \frac{\partial \vec{D}}{\partial t}.\overrightarrow{ds}$
$\nabla.\vec{D} = \rho_v$	$\oiint_s \vec{D}.\overrightarrow{ds} = \iiint_v \rho_v d_v$
$\nabla.\vec{B} = 0$	$\oiint_s \vec{B}.\overrightarrow{ds} = 0$

Summary of Maxwell's equations in free space

Point form	Integral form
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E}.\overrightarrow{dL} = -\iint_S \frac{\partial \vec{B}}{\partial t}.\overrightarrow{ds}$
$\nabla \times H = \frac{\partial \vec{D}}{\partial t}$	$\oint \vec{H}.\overrightarrow{dL} = \iint_S \frac{\partial \vec{D}}{\partial t}.\overrightarrow{ds}$
$\nabla.\vec{D} = 0$	$\oiint_S \vec{D}.\overrightarrow{ds} = 0$
$\nabla.\vec{B} = 0$	$\oiint_S \vec{B}.\overrightarrow{ds} = 0$

Summary Maxwell's equation for good conductor

Point form	Integral form
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint \vec{E}.\overrightarrow{dL} = -\iint_S \frac{\partial \vec{B}}{\partial t}.\overrightarrow{ds}$
$\nabla \times \vec{H} = J$	$\oint \vec{H}.\overrightarrow{dL} = \iint_S \vec{J}.\overrightarrow{ds}$
$\nabla.\vec{D} = 0$	$\oiint_S \vec{D}.\overrightarrow{ds} = 0$
$\nabla.\vec{B} = 0$	$\oiint_S \vec{B}.\overrightarrow{ds} = 0$

3. Derive general field relations for time – varying electric and magnetic field using Maxwell's equations. (Dec 2007, May 2005, May 2013)

The basic relation between an electric and magnetic field, starting from Faraday's law is given by,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

We know that $\vec{B} = \nabla \times \vec{A}$ where $\vec{A}$ – vector maganatic potential

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}(\nabla \times \vec{A})$$

Interchanging operators at R.H.S of above equation, we get

$$\nabla \times \vec{E} = -\nabla \text{x} \frac{\partial \vec{A}}{\partial t}$$

$$\nabla \times \vec{E} + \nabla \times \frac{\partial \vec{A}}{\partial t} = 0$$

$$\nabla \times \left[\vec{E} + \frac{\partial \vec{A}}{\partial t}\right] = 0 \rightarrow \quad (1)$$

But according to vector identity curl of a gradient of a scalar is always zero

$$\overrightarrow{E} + \frac{\partial \vec{A}}{\partial t} = \nabla A \quad (2)$$

So that $\nabla \times \nabla = 0$

$\Rightarrow E = -\nabla V - \frac{\partial \vec{A}}{\partial t}$ [Potential gradient in general is '–ve']

This is the general field equation for time varying fields.

Now it the field is static, $\frac{\partial \vec{A}}{\partial t} = 0$

$$\Rightarrow E = -\nabla V$$

By equation of continuity, we can write $I = -\frac{dQ}{dt}$

Where Q_i -internal charge

$$I = -\frac{d}{dt}\left[\int_v \rho_v d_v\right] = -\frac{d\rho_v}{dt} d_v$$

But we know $I = \int_v \vec{J}.\overrightarrow{ds}$

$\Rightarrow \int_s \vec{J}.\overrightarrow{ds}$=-$\int_v \frac{d\rho_v}{dt} d_v$

Appling divergence theorem

$$\Rightarrow \int_v \nabla.\vec{J}.\overrightarrow{ds} = \int_v \frac{d\rho_v}{dt} d_v$$

$$\Rightarrow \nabla.\vec{J} = \frac{-d\rho_v}{dt} = -\frac{\partial \rho_v}{\partial t}$$

$$\Rightarrow \nabla.\vec{J} + \frac{\partial l_v}{\partial x} = 0 (3)$$

Now for time varying fields we can write,

$$\nabla.\left(\nabla \times \overrightarrow{\text{H}}\right) = \nabla.\vec{J} + \frac{\partial \rho_v}{\partial x}$$

[Because $\nabla \times \overrightarrow{\text{H}} = \vec{J}$ [$from\ maxwell's equation\ taking\ divergence\ on\ both\ sides$

$$\nabla.\nabla \times \vec{H} = \nabla.\vec{\text{J}} = 0 \rightarrow \text{-}(4)$$

Comparing (3) and (4) we get equation (5)

$$\nabla.\left(\nabla \times \overrightarrow{\text{H}}\right) = \nabla.\vec{J} + \frac{\partial \rho_v}{\partial x} \text{---}(5)$$

We know that $\nabla.\overrightarrow{\text{D}} = \rho_v$[from point from of gauss law]

$$\nabla.\left(\nabla \times \overrightarrow{\text{H}}\right) = \nabla.\vec{\text{J}} + \frac{\partial}{\partial \text{t}}[\nabla.\overrightarrow{\text{D}}]$$

Interchanging operators of R.H.S

$$\nabla.\left(\nabla \times \vec{H}\right) = \nabla.\vec{J} + \nabla\frac{\partial \vec{D}}{\partial t}$$

(or)

$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$Amperes circuit law tor time varying fields

$$\nabla \times \vec{H} = \vec{J}_c + \vec{J}_D$$

Maxwell's equations for harmonically varying fields

Assuming that the electric and magnetic fields are varying harmonically with time we can write

$$\vec{D} = \vec{D}_0 e^{jwt}$$
$$\vec{B} = \vec{B}_0 e^{jwt}$$

Taking partial derivative with respect time

$$\frac{\partial \vec{D}}{\partial t} = \boldsymbol{jw}\vec{D}_0 e^{jwt=} jw\vec{D} = jw\varepsilon\vec{E}$$

$$\frac{\partial \vec{B}}{\partial t} = \boldsymbol{jw}\vec{B}_0 e^{jwt=} jw\vec{B} = jw\varepsilon\vec{H}$$

(A) Point form

$$\nabla \times \vec{H} == j\omega\mu\vec{H}$$
$$\nabla \times \vec{H} = \vec{J} + j\omega\vec{D} = (\sigma + j\omega\varepsilon)\vec{E}$$
$$\nabla.\vec{D} = \rho_v$$
$$\nabla.\vec{B} = 0$$

But according to vector identity 'curl' of a gradient of a scalar is always zero.

(B) Integral form:

(i) $\oint \vec{E}.\overrightarrow{dL} = -\iint_s j\omega\vec{B}.\overrightarrow{ds} = -\iint_s j\omega\mu\vec{H}.\overrightarrow{ds}$

$$\oint \vec{E}.\overrightarrow{dL} = -j\omega\mu \iint_s \vec{H}.\overrightarrow{ds}$$

(ii) $\oint \vec{H}.\overrightarrow{dL} = \mathrm{I} + \iint_s j\omega\vec{D}.\overrightarrow{ds}$

$$= \iint_s \vec{J}.\overrightarrow{ds} + \iint_s j\omega\varepsilon\vec{E}.\overrightarrow{ds}$$

$$= \iint_s \sigma\vec{E}.\overrightarrow{ds} + \iint_s j\omega\varepsilon\vec{E}.\overrightarrow{ds}$$

$$= (\sigma + j\omega\varepsilon)\iint_s \vec{E}.\overrightarrow{ds}$$

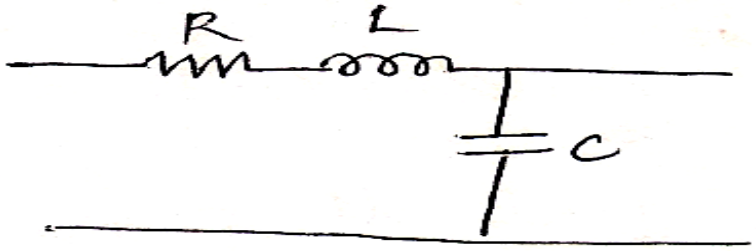
(iii) $\oiint_s \vec{D}.\overrightarrow{ds} = \oint_v \rho_v dv$

(iv) $\iint_s \vec{B}.\overrightarrow{ds} = 0$

Point form	Integral form
$\nabla \times \vec{E} = -j\omega\mu\vec{H}$	$\oint \vec{E}.\overrightarrow{dL} = -j\omega\mu \iint_s \vec{H}.\overrightarrow{ds}$
$\nabla \times \vec{H} = (\sigma + j\omega\varepsilon)\vec{E}$	$\oint \vec{H}.\overrightarrow{dL} = (\sigma + j\omega\varepsilon) + \iint_s \vec{E}.\overrightarrow{ds}$
$\nabla \times \vec{D} = \rho_v$	$\oiint_s \vec{D}.\overrightarrow{ds} = \iiint_v \rho_v dv$
$\nabla \times \vec{B} = 0$	$\oiint_s \vec{B}.\overrightarrow{ds} = 0$

4.Explain the relationship between the field theory and circuit theory using a simple RLC series circuit. (June 2007, Dec 2005, May 2010,2016)(Nov/Dec 2017)
Explain how the circuit equation for series RLC circuit is derived from the field relation. (Nov/Dec 2014)(April/May 2018)

Let us consider the transmission line RLC series circuit



Resistance was introduced in the study of current flowing in the material fields.

Capacitance was introduced in the discussion of electrostatic fields.
Inductance was introduced in the discussion of magnetic fields.

To make the analysis of relationship between circuit theory and field theory, a general expression for the electric field in a region of space which compared with the wavelength of the excitation that produces the fields, is considered.

From the generalized field equation, we can write

$$E = -\nabla V - \frac{\partial A}{\partial t} \text{---------- (1)}$$

$-\nabla V = E_c =$ field due to static charge distribution

$-\frac{\partial A}{\partial t} = E_a$ =Field due to time –changing maganatic field
From point form ohm's law

$$E_t = \frac{J}{\sigma} (2)$$

Where E_t-total electric field intensity at a point inside a conductor

$$E_t = E_c + E_a$$

$$E_a = E_t - E_c = \frac{J}{\sigma} + \frac{\partial A}{\partial t} + \nabla V \text{---------- (3)}$$

Consider the application of this field relation to the stationary series circuit as taken above.

Case (i) if no capacitor is present

The field relationintegratedaround a closed path that follows the conduction circuit we obtain eqn (3) as

$\oint \overrightarrow{E_a}.\overrightarrow{dL} = \oint \frac{j}{\sigma}.dL + \oint \frac{\partial A}{\partial t}.dL + \oint \nabla V.dL$---------- (4)

But scalar potential due to closed path is zero

$\oint \overrightarrow{E_a}.\overrightarrow{dL} = Vi$ ---------- (5)

Vi-emf applying by generator

$\oint \frac{j}{\sigma}.dL = IR$ Drop around the circuit ---------- (6)

Total resistance R of the circuit is assumed to be lumped in a single resistor

$\oint \frac{\partial A}{\partial t}.dL$ is expressed as $\frac{\partial}{\partial t} \oint \overrightarrow{A}.\overrightarrow{dL}$

$$= \frac{\partial}{\partial t} \iint_s (\nabla \times \vec{A}).\overrightarrow{ds}$$

$$= \frac{\partial}{\partial t} \iint_s \vec{B}.\overrightarrow{ds}$$

$$= \frac{d\lambda}{dt} = \frac{LdI}{dt} \text{---------- (7)}$$

Now eqn (4) can be written as

$\oint \vec{E}_a.dL = Vi = IR + \frac{LdI}{dt}$ →This equation similar to circuit theory equation

This circuit relationship is obtained by starting the analysis from their field relationships

Case (ii) if capacitance is included

If we consider the eqn (4) for this case ,

$$\oint \vec{E}_a.\overrightarrow{dL} = \frac{J}{\sigma}\overrightarrow{dL} + \oint \frac{\partial A}{\partial t}.\overrightarrow{dL} + \oint \nabla V.\overrightarrow{dL}$$

The term involving J/σ becomes indeterminate

Because the capacitor J and σ approaches zero

∴ We get

$$Vi = IR + \frac{LdI}{dt} + Vc$$

V_c-represent the instantaneous potential difference between the plate of capacitor

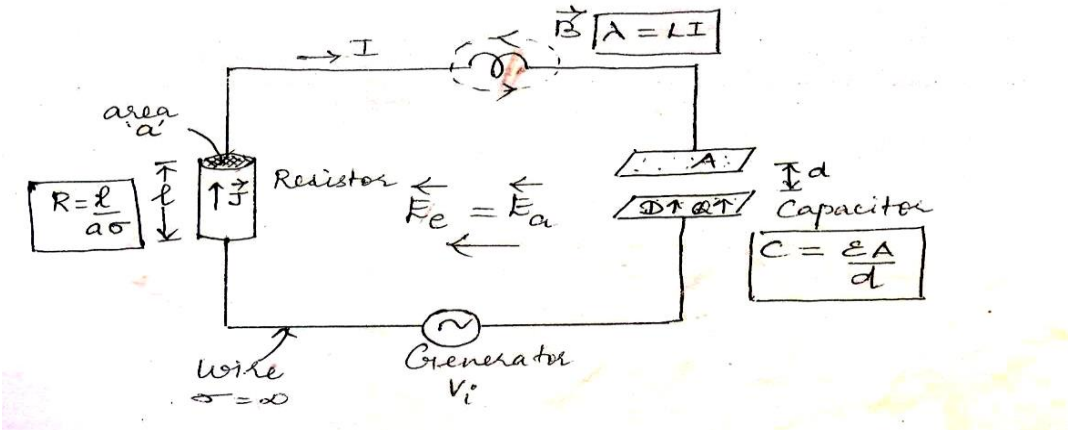
The potential across the capacitance expressed in terms of the change Q on theplates and the capacitance C $V_c = \frac{Q}{c} = \frac{1}{C}\int I\, dt$

$$\therefore V_i = IR + \frac{LdI}{dt} + \frac{1}{C}\int I\, dt$$

If we include variation with time the eqn is

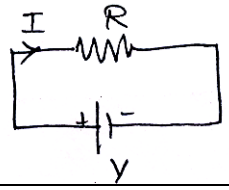
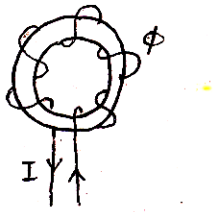
$$V_i = IR + j\omega LI + \frac{1}{j\omega C}$$

Again the circute theory equation is obtaned form the field theory eqn



5. Compare the field theory and circuit theory (Dec 2012,Dec 2008, Dec2007, May 2005, May 2003, June 2012)(May/June 2014)(April/May 2015,2016)(April/May 2017)

	Circuit theory	Field theory
1	This originated as a result of the phenomenal terms like current voltage and power	This evolved from transmission line theory
2	Inner electrical properties of there conductors are analyzed like voltage drop, power loss etc,	Outer magnetic properties of the conductor are analyzed like electric field intensity ($\vec{E}$) ,magnetic fiueld intensity ($\vec{H}$) etc,

3	Circuit model 	
4	Ohm's law for electric field is current= $\frac{emf}{resistance}$	Ohm's law of the magnetic field Flux=$\frac{mmf}{reluctance}$
5	Parameters of the medium is not involved	Parameters of the medium like permittivity and permeability is involved
6	Frequency is used as reference	Wavelength is used as reference
7	Lumped components are used	Distributed components are involved
8	Two dimensionalapproach (x,y)	Three dimensional approach (x,y,z)
9	Laplace transform is employed for analysis	Maxwell's equation is employed for analysis
10	Applicable only for portion of RF range	Applicable beyond RF range (microwave)

6. Drive the boundary condition for electromagneticfields.

In electromagnetic boundary condition the combined analysis based on the boundary condition pertaining toelectric field and magnetic field are discussed

Boundary condition for electric field:

1. The tangential component of electric field intensity in medium is equal to the tangential component of electric field intensity in medium2

$$E_{t1} = E_{t2}$$

2. The normal component of electric flux density in medium, is equal to the normal component of electric flux density in medium 2

$$D_{n1} = D_{n2}$$

Boundary conditions for magneticfield

3. The tangential component of magneticfield intensity in medium 1 is equal to tangential component of magnetic field intensity in medium 2

$$H_{t1} = H_{t2}$$

4. The normal component of magnetic flux density in medium 1 is equal to the normal component of magnetic flex density in medium 2

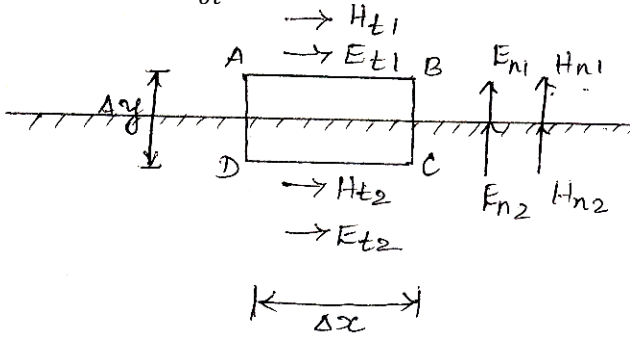
$$B_{n1} = B_{n2}$$

Condition I and III:

(a) From integral of Maxwell's equation form faraday's law,

$$\oint \vec{E}.\overrightarrow{dL} = -\iint_s \frac{\partial \vec{B}}{\partial t}.\overrightarrow{ds}$$

For static fields $\frac{\partial \vec{B}}{\partial t} = 0 \Rightarrow \oint \vec{E}.\overrightarrow{dL} = 0$



$$\oint \vec{E}.\overrightarrow{dL} = \left[\int_{AB} + \int_{BC} + \int_{CD} + \int_{DA}\right] E.dL = 0$$

$$\Rightarrow \oint \vec{E}.\overrightarrow{dL} = E_{t1}\Delta x - \frac{E_{n1}\Delta y}{2} - \frac{E_{n2}\Delta y}{2} - E_{t2}\Delta x = 0$$

At the boundary $\Delta y \to 0$

$$\oint \vec{E}.\overrightarrow{dL} = E_{t1}\Delta x - E_{t2}\Delta x = 0$$

$$\Rightarrow E_{t1} = E_{t2}$$

(b)from integral form of maxwells equation for amperes′slaw

$$\oint \vec{E}.\overrightarrow{dL} = \iint_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right).\overrightarrow{ds}$$

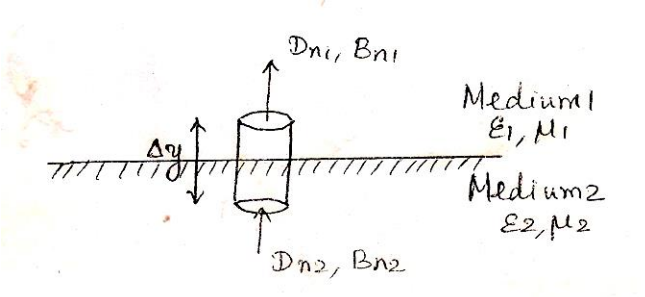
$$= \iint_s \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)\Delta x \Delta y = 0$$

At the boundary $\Delta y \to 0$

$$\oint \vec{E}.\overrightarrow{dL} = H_{t1}\Delta x - H_{t2}\Delta x = 0$$

$$\Rightarrow H_{t1} = H_{t2}$$

Condition II and IV



Applying Gauss's law

$\oiint_s \vec{D}.\overrightarrow{ds} = Q = 0$ [Charge and free]

$$\iint_{\substack{S\\(top)}} \overrightarrow{D_{n1}}.\overrightarrow{ds} + \iint_{\substack{S\\(bottom)}} -\overrightarrow{D_{n2}}.\overrightarrow{ds} + \iint_{S} lateral\ surface = 0$$

$$as\ \Delta y \to 0\ at\ the\ bonndary$$

$$\int D_{n1}ds - \int D_{n2}ds = 0$$

$$D_{n1} - D_{n2} = 0$$

$$D_{n1} = D_{n2}$$

(a) From integral from of gauss's law for magnetic fields

(b) $\oiint_S \vec{D}.\overrightarrow{ds} = 0$

$$\int B_{n1}ds - \int B_{n2}ds = 0$$

$$\int (B_{n1} - B_{n2})ds = 0$$

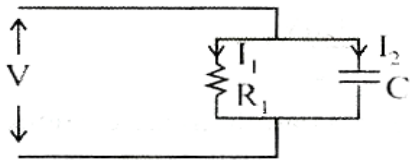
$$B_{n1} = B_{n2}$$

These are the form boundary conditions derived for electromaganatic fields

7. Compare and explain in detail conduction and displacement current. (April/May 2015)(Nov/Dec 2015)(April/May 2018)

Conduction current	Displacement current
◆ It is developed by metal or a semiconductor on application of Electric field 'E'.	◆ It is current flows in the capacitor.
◆ Conduction current $J = \sigma E$	◆ Displacement current density $J_D = \varepsilon_0 \frac{\partial E}{\partial t} Amp/m^2$
◆ It is strong	◆ Displacement current is negligible relative to conduction current.

Consider a resistor (R) and capacitor (C) are connected in parallel with voltage source (V) as shown in figure.



Let current flow through 'R' be "I_1' and current flow through 'C' be 'I_2

The nature of the current flowing through the resistor and capacitor is entirely different. In the case of resistor the current is flowing because of the actual movement of charges . Even though a constant voltage is applied across the resistor the current flows through the resistor.

Here current $I_1 = \frac{V}{R}$ .....(1)

The current obtained is due to the actual movement of charges, it is called as "conduction current" it is denoted by I_C.

$$I_C = I_1 = \frac{V}{R} \quad .....(2)$$

The density of this current is known as "conduction current density". It is defined as the ratio between conduction current and cross sectional area

$$J_C = \frac{I_C}{A} \quad .....(3)$$

where

J_C = Conductor current density
I_C = Conduction current
A = Cross sectional area

The current passing through the capacitor 'I_2' is not flowing when the applied voltage 'V' is constant. It is following only when the applied voltage is changing

$$I_2 = \frac{dQ}{dt} \quad .....(4)$$

But $Q = CV$ .....(5)

Sub (5)in equation (4)

$$I_2 = \frac{d}{dt}(CV)$$

$$I_2 = C\frac{dV}{dt} \quad \text{.....(6)}$$

where C = capacitance V = applied voltage.

In the case of capacitor the current is not passing because of the actual motion of charges through the plates of capacitor. But the current flowing out from one plate is equal to the current flowing in the other plate . This current is called as "displacements current" It is denoted by I_d

$$\therefore I_d = I_2 = \frac{CdV}{dt} \quad \text{.....(7)}$$

We know that, capacitance of a parallel plate capacitor $C = \frac{\varepsilon A}{d}$ .....(8)

where

C = Capacitance

ε = permittivity of the medium

A = Area of the plates

Sub (8) in (7)

$$I_d = \frac{\varepsilon A}{d}\frac{dV}{dt} \quad \text{.....(9)}$$

It is evident that

$$V = Ed \quad \text{(10)}$$

$$\left[E = \frac{V}{d}\right]$$

Substitute the equation (9) in (8)

$$I_d = \frac{\varepsilon A}{d}\frac{d}{dt}(Ed)$$

$$= \frac{\varepsilon \cdot A}{d}.d\frac{dE}{dt}.$$

$$= \varepsilon A\frac{dE}{dt}$$

$$I_d = A\frac{d(\varepsilon E)}{dt} \quad [\because \varepsilon E = D]$$

$$I_d = A\frac{dD}{dt} \quad \text{.....(11)}$$

The density of this current is known as "Displacement current density". It is denoted by J_d. It is given by

$$J_d = \frac{I_d}{A} = \frac{dD}{dt}$$

$$\therefore J_d = \frac{dD}{dt} \quad \text{.....(12)}$$

Total current density is given by

$$J = J_C + J_D$$

The vector form, J is

$$\vec{J} = \vec{J_C} + \vec{J_D} \quad \text{.....(13)}$$

8. Calculate the induced emf at $t = 10$ sec *when* the flux through each turns of a 200 turns coil is $t(t-1) mWb$ (May 2007)

Sol:

$$\emptyset = t(t-1) = t^2 - t \quad mWb$$

$$e = \frac{N\, d\emptyset}{dt}$$

$$= 200\,(2t-1)$$

$$At\ t = 10sec$$

$$emf = 200(2 \times 10 - 1) = 3800\ mV = 3.8\ V$$

9. In a material for which $\sigma = 5.0\ \frac{s}{m}$ *and* $\varepsilon_r = 1$ *and* $\vec{E} = 250 \sin 10^{10} t$ (V/m). Find the conduction and displacement current densities

Sol:

$$J_c = \sigma E = 5[250 sin 10^{10} t]$$

$$J_c = 1250\ sin 10^{10} t\ A/m^2$$

$$J_D = \frac{\partial \vec{D}}{\partial t} = \frac{\partial}{\partial t}\left[\varepsilon \vec{E}\right] = (\varepsilon_0 \varepsilon_r)\frac{\partial \vec{E}}{\partial t}$$

$$= (8.854 \times 10^{-12})(1)\frac{d}{dt}[250 sin 10^{10} t]$$

$$= 8.854 \times 10^{-12} \times 250 \times 10^{10} \times cos 10^{10} t$$

$$J_D = 22.135\ cos 10^{10} t\ A/m^2$$

10. A Circular cross section conductor of radius 2mm carries a current $i_c = 2.5\sin(5 \times 10^8 t)\mu A$. What is the amplitude of the displacement current density if $\sigma = 35\frac{\mu s}{m}$ and $\varepsilon_r = 1$.

Sol:

The magnitude of conduction current density is given by

$$J_c = \frac{I_c}{A} = \frac{2.5 \times 10^{-6}}{(\pi r^2)} = \frac{2.5 \times 10^{-6}}{\pi(2 \times 10^{-3})^2} = 0.1987$$

$$\frac{J_c}{J_D} = \frac{\sigma}{\omega\epsilon}$$

$$\therefore J_D = (J_c)\left(\frac{\omega\varepsilon}{\sigma}\right) = 0.1987 \times \left[\frac{5 \times 10^8 \times 8.8542 \times 10^{-12} \times 1}{35 \times 10^6}\right]$$

$\therefore \omega = 5 \times 10^8$

$= 25.164\ p\ A/m^2$

11.The conduction current ofpassing through a wire with conductivity $\sigma = 3 \times 10^7 s/m$ and relative permeability $\varepsilon_r = 1$ is given by $I_c = 3\sin \omega t\ \ mA\ if\ \omega = 10^8$ rad/s find the displacement current in wire.

Sol:

$$J_c = \frac{I_c}{A}\sigma E$$

$$I_c = \sigma E A$$

$$E = \frac{I_c}{\sigma A} = \frac{3 \times 10^{-3} sin\omega t}{3 \times 10^7 A} = \frac{1 \times 10^{-10}}{A}\ sin\omega t$$

$$J_D = \frac{Id}{A} = \frac{\partial D}{\partial t}$$

$$\therefore Id = J_D \times A$$

$$= \frac{\partial D}{\partial t} \times A$$

$$= \epsilon\frac{\partial D}{\partial t} \times A$$

$$= 8.854 \times 10^{-12} \times 4 \times \frac{\partial}{\partial t}\left(\frac{1 \times 10^{-10}}{4} sin\omega t\right)$$

$$= 8.854 \times 10^{-12} \times 1 \times 10^{-10} \times \frac{\partial}{\partial t}(sin\omega t)$$

$$= 8.854 \times 10^{-12} \times 1 \times 10^{-10}\omega\,.\cos\omega t$$

$$= 8.854 \times 10^{-12} \times 1 \times 10^{-10} \times 10^8 \omega s\ \ 10^8\ t$$

Id$= 8.854 \times 10^{-14}\omega s\ \ 10^8\ t$

12.A straight conductor of length 40 cm moves perpendicularly to its axis at a velocity of 50 meter per second is a uniform magnetic field of flux density 1.2 (tesla) . Evaluate the e.m.f induced in the conductor if the direction of motionis.

(c) Normal to the field

(d) Parallel to the field

(e) As an angle 60° to the orientation of the field

Sol:

The motional e.m.f is given by

$$e = VBl\ sin\theta$$

a) $\theta = 90°$

$e = 50 \times 1.2 \times 0.4 \times 1 = 24$ volts

b) As the motions is parallel to the field $\theta = 0$

$$e = 0$$

No emf is included as there is no effective flux linkage

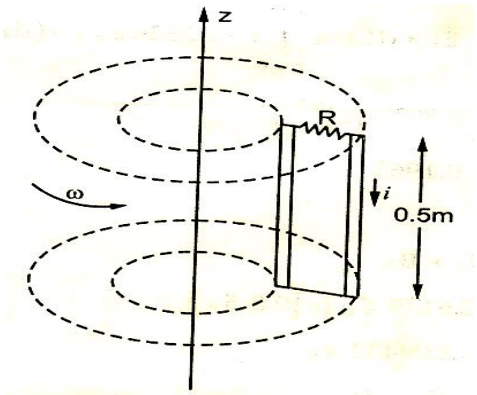
c) $\theta = 60°$

$$e = VBl\ sin60°$$

$$e = 24 \times 0.866$$

$$= 20.8\ volts$$

13. A rectangular conducting loop with a resistance of 0.2 Ω rotates at 500 r.p.m the vertical conductor at $r_1 = 0.03$ m is in the field $\vec{B}_1 = 0.25\ \vec{a}_r$ T and other conductor is at



$r_2 = 0.05\ m$ and in the field $\vec{B}_2 = 0.8\ \vec{a}_r$ T .Find current flowing in the loop.

Sol:

The inner conductor which is at $r_1 = 0.03$ m rotates at 500 r.p.m. thus inner conductor rotates with $\frac{500}{60}$ revolutions per second. As in one second the distance covered is $(2\pi r)$ meter, for the inner conductor the distance covered is $\left(\frac{500}{60}\right)(2\pi r)$ meters,

Then the linear velocity for inner conductor are given by

$\vec{v}_1 = \left(\frac{500}{60}\right)(2\pi)(0.03)\ \vec{a}_\phi$ m/s
$= 1.5707\ \vec{a}_\phi$m/s

Similarly for outer conductor, linear velocity is given by

$\vec{v}_2 = \left(\frac{500}{60}\right)(2\pi)(0.05)\ \vec{a}_\phi$ m/s
$= 5.6179\ \vec{a}_\phi$m/s

Here $\vec{B}$ is not varying with time, it is constant in $\vec{a}_r$ direction. Thus under such condition, the induced e.m.f. is given by,

$$e.m.f. = \int \overrightarrow{E}.d\vec{L}$$

Where $\vec{E} = \vec{v} \times \vec{B}$
For inner conductor,

$$\vec{E} = \overrightarrow{v_1} \times \vec{B}_1$$
$$= \left[1.5707\vec{a}_\phi\right] \times [0.25\ \vec{a}_r]$$
$$= -0.3926\ \vec{a}_z$$

Both the conductors are vertical .let us assume that length of each conductor be 0.5 m.

$$d\overrightarrow{L_1}. = dz\vec{a}_z$$

e.m.f.1 $= \int \overrightarrow{E_1}.d\overrightarrow{L_1}$

$$= \int_{z=0}^{0.5\ m} (-0.3926\vec{a}_z).(dz\vec{a}_z)$$
$$= -0.3925[z]_0^{0.5}$$
$$= -0.1963\ V$$

For outer conductor,

$$\overrightarrow{E_2} = \overrightarrow{v_2} \times \vec{B}_2$$
$$= \left[2.6179\vec{a}_\phi\right] \times [0.8\ \vec{a}_r]$$
$$= -2.09432\ \vec{a}_z$$
$$e.m.f2. = \int \overrightarrow{E}._1\, d\overrightarrow{L_2}$$
$$= \int_{z=0}^{0.5\ m} (-2.09432)\vec{a}_z).dz\vec{a}_z$$
$$= -2.09432[z]_0^{0.5}$$
$$= -1.04716\ V$$

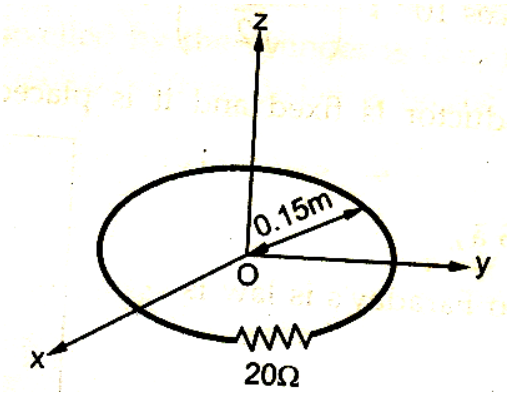
Hence current in the loop is given by

$$i = \frac{e.m.f.1 - e.m.f.2}{R} = \frac{-0.963 - (-1.04716)}{0.2}$$
$$i = \frac{0.85086}{0.2} = 4.2543\ A$$

14. The circular loop having a radius of 0.15 m is placed in X-Y plane. This loop consists of a resistance of 20 Ω as shown in the fig. if the magnetic flux density is $\vec{B} = 0.5 \sin 10^3 \vec{a}_z \, Tesla$. Find current flowing through this loop. (April/May 2017)

Sol:

The circular loop conductor is in X-Y plane. $\vec{B}$is in $\vec{a}_z$ direction which is perpendicular to X-Y plane.



Hence, we can write,

$$d\vec{S} = (r dr d\Phi)\vec{a}_z$$

$$total flux is given by$$

$$\Phi = \int_s \vec{B}.d\vec{S}$$

$$= \int_{\Phi=0}^{2\pi} \int_{r=0}^{0.15} [(0.5 \sin 10^3 t\,)\vec{a}_z] . [(r.dr d\Phi)\vec{a}_z]$$

$$= (0.5 \sin 10^3 t)\,[\Phi]_0^{2\pi} \left[\frac{r^2}{2}\right]_0^{0.15}$$

$$= (0.5 \sin 10^3 t)\,[2\pi] \left[\frac{(0.15)^2}{2}\right]$$

=35.3429 sin 10^2 tmWb

Now induced e.m.f .is given by,

$$e = -\frac{d\phi}{dt}$$

$$= -\frac{d}{dt}[35.3429 \times 10^{-3} \sin 10^3 t]$$

$$= -(35.3429 \times 10^{-3})(10^3) cos 10^3 t$$

$$= -35.3429\, cos 10^3 t V$$

$$hence current in the conductor is given by$$

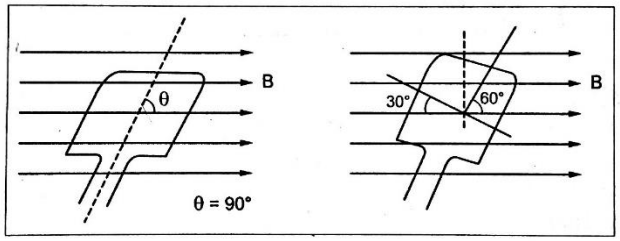
$$i = \frac{e}{R} = \frac{-35.3429\, cos 10^3 t}{20}$$

$$i = -1.7671\, cos 10^3 t A$$

15. A circular loop of wire is placed in a uniform magnetic field of flux density 0.5Wb/m^2. The wire has 200 turns and frequency of rotation of 1000 revolution /minute. If the radius of the coil is 0.2m, determine, (i) the induced emf, when the plane of the coil is perpendicular to the field, (ii) When the plane of the coil is 60^o to the flux lines. (Dec 2015)

Solution :

r = 0.2m
N = 200



F = 1000rpm= $\frac{1000}{60} rps$
ω = $\frac{2\pi\times1000}{60} rad/s$
B = 0.5 wb/m^2

(i) When the plane of the coil is perpendicular to the field ,the induced emf

e = NBA ωsinθ
= $200\times0.5\times\pi\times(0.2)^2\times\frac{2\pi\times1000}{60}\times in90^o$
= 1316 volts

(ii) When the plane of the coil is 60^o to the field, then the induced emf is

e = NBA ωsinθ
= $200\times0.5\times\pi\times(0.2)^2\times\frac{2\pi\times1000}{60}\times sin60^o$
= 1139.644 volts

16.In a material for which $\sigma = 5S/m$ and ε_r=1 the electric field intensity is E=250 sin 10^{10} t V/m. Find displacement current density and the frequency at which both have equal magnitudes. (Nov/Dec 2016)

Given data,
σ= 5S/m,ε_r=1
E=250 sin 10^{10} t V/m

$$J_D = \frac{\partial \vec{D}}{\partial t} = \frac{\partial}{\partial t}\left[\varepsilon\vec{E}\right] = (\varepsilon_0\varepsilon_r)\frac{\partial \vec{E}}{\partial t}$$
$$= 8.854\times10^{-12}\times1\times\frac{\partial}{\partial t}(250\sin10^{10}\mathrm{t})$$
$$= 8.854\times10^{-12}\times1\times(250\cos10^{10}\mathrm{t})$$

$$= 22.135\cos10^{10}t\ \ A/m^2$$
$$\frac{|J_c|}{|J_D|} = \frac{\sigma}{\varepsilon\omega} = 1$$
$$\omega = \frac{\sigma}{\varepsilon} = \frac{5}{8.854\times10^{-12}\times1} = 5.64\times10^{11}$$
$$2\pi f = 5.64\times10^{11}$$

$$f = \frac{5.64 \times 10^{11}}{2\pi} = 89.87\ GHz$$

17. An iron ring of cross sectional area 3cm2 and a mean circumference of 15cm is wound with 250 turns of wire carrying a current of 0.3A. The relative permeability of the ring is 1500. Calculate the flux established in the ring**(April/May 2018)**

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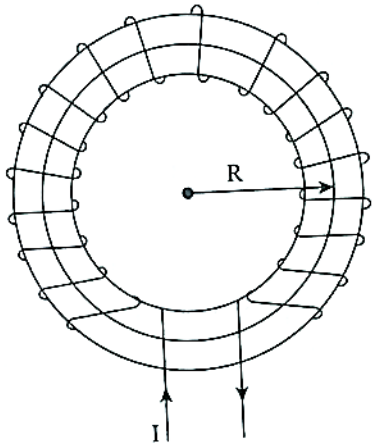

$A = 3\ cm^2 = 3 \times 10^{-4}\ m^2$

$2\pi R = 15\ cm = 15 \times 10^{-2}\ m$

$N = 250$

$I = 0.3\ A$

$\mu_r = 1500$

For a toroid, the flux established is given by $B = \mu_0 H = \left(\frac{\mu_0 \mu_r NI}{2\pi R}\right)$

$$B = \frac{\mu_0 (1500)(250)(0.3)}{15 \times 10^{-2}}$$

$$B = 0.942\ Wb/m^2$$

$$\phi = B \cdot A = 282.75 \times 10^{-6}\ Wb.$$

UNIVERSITY QUESTIONS
PART -A

1. State Faraday's law of electromagnetic induction. (May 2011, Dec 2010)
2. State Lenz's law (May 2010, June 2007, and Dec 2006)
3. What are displacement current density and its significance? (May 2011, May 2010, Dec 2012, Nov 2007, Dec 2012, June 2012)
4. What is motional emf? (Or) Write down the expression for the emf induced in the moving loop in static B field. (May 2008, Nov 2006)
5. What is transformer emf?
6. What is the equation for total emf due to motion (or) transformer action? Give one practical case.
7. Write about conduction current density.
8. Write down the differential form of Maxwell's equations (or) Write point from of Maxwell's equations.
9. Write down the integral form of Maxwell's equation. (Nov 2007)
10. State Maxwell's I and II equations (Dec 2010)
11. Compare field theory and circuit theory. (May 2010, June 2006, May 2006)
12. Mention the limitations of circuit theory. (May 2008)
13. Give the energy relation in an electromagnetic field.
14. A conductor of 1m length is moved with a velocity of 100 m/sec. perpendicularly to a field of 1 Tesla. What is the value of emf induced? (Dec 2012)
15. State continuity equations. (June 2012)
16. Define mutual and self inductance. (April/ May 2015)
17. Distinguish between transformer emf and motional emf. (April/May 2015)
18. Give the two important equations that provide a connection between field and circuit theory. (Nov/Dec 2014)

PART –B

1. State and explain Faraday's law of electromagnetic induction. Hence derive the expressions for statically and dynamically induced emf. (May 2006, May 2010)(May/June 2014)
2. Derive and explain Maxwell's equations both in integral and point forms.
(Dec 2012,May 2007, May 2006, Dec 2008 , Dec 2005,May 2003,June 2012,May 2010)
(May/June 2014)(Nov/Dec 2014)
Derive and explain Maxwell's equations in integral form from fundamental laws for a good conductor. (April/May 2015)
3. Derive general field relations for time – varying electric and magnetic field using Maxwell's equations. (Dec 2007, May 2005, May 2013)
4. Explain the relationship between the field theory and circuit theory using a simple RLC series circuit. (June 2007, Dec 2005, May 2010)
Explain how the circuit equation for series RLC circuit is derived from the field relation. (Nov/Dec 2014)
5. Compare the field theory and circuit theory (Dec 2012, Dec 2008, Dec2007, May 2005, May 2003, June 2012)(May/June 2014)(April/May 2015)
6. Drive the boundary condition for electromagnetic fields.
7. Compare and explain in detail conduction and displacement current. (April/May 2015)

8. Calculate the induced emf at $t = 10 \sec \ when$ the flux through each turns of a 200 turns coil is $t(t-1)m\ Wb$ (May 2007)

9. In a material for which $\sigma = 5.0 \ \frac{s}{m} and\ \varepsilon_r = 1\ and\ \vec{E} = 250 \sin 10^{10} t$ (V/m). Find the conduction and displacement current densities

10. A parallel plate capacitor with plate area of $5Cm^2$ and plate sepanatian of 3mm has a voltage $50 \sin 10^3 t\ V$ applied to its plates. Calculate the displacement current assuming$\varepsilon = 2\ \varepsilon_0$. (April/May 2014)

11. A Circular cross section conductor of radius 2mm carries a current$i_c = 2.5 \sin(5 \times 10^8 t)\mu A$. What is the amplitude of the displacement current density if $\sigma = 35 \frac{\mu s}{m}\ and\ \varepsilon_r = 1$.

12. The conduction current of passing through a wire with conductivity $\sigma = 3 \times 10^7 s/m$ and relative permeability $\varepsilon_r = 1$ is given by $I_c = 3 \sin \omega t\ mA\ if\ \omega = 10^8$ rad/s find the displacement current in wire.

13.A straight conductor of length 40 cm moves perpendicularly to its axis at a velocity of 50 meter per second is a uniform magnetic field of flux density 1.2 (tesla) . Evaluate the e.m.f induced in the conductor if the direction of motion is.

(a) Normal to the field
(b) Parallel to the field
(c) As an angle 60° to the orientation of the field

14. A rectangular conducting loop with a resistance of 0.2 Ω rotates at 500 r.p.m the vertical conductor at $r_1 = 0.03$ m is in the field $\vec{B}_1 = 0.25\ \vec{a}_r$ T and other conductor is at $r_2 = 0.05\ m$ and in the field $\vec{B}_2 = 0.8\ \vec{a}_r$ T .Find current flowing in the loop.

15. The circular loop having a radius of 0.15 m is placed in X-Y plane. This loop consists of a resistance of 20 Ω as shown in the fig. if the magnetic flux density is $\vec{B} = 0.5 \sin 10^3 t \vec{a}_z T$

Find current flowing through this loop.

UNIT- VELECTROMAGNETIC WAVES

Generation – Electro Magnetic Wave equations – Wave parameters; velocity, intrinsic impedance, propagation constant – Waves in free space, lossy and lossless dielectrics, conductors-skin depth, Poynting vector – Plane wave reflection and refraction – Transmission lines – Line equations – Input impedances – Standing wave ratio and power.

PART -A

1. Define poynting vector, what is the SI unit for this vector.(April 2008,May 2003,May 2010, 2011,2016)

The poynting vector is defined as the rate of flow of energy of a wave as it propagates .It is the vector product of electric field and magnetic field

$$\vec{p} = \vec{E} \times \vec{H}$$

Power=power density x area

Power density is measured in watt/m^2

2. What is a wave?(Nov 2007)

If a physical phenomenon that occurs at one place at a given time is reproduced at other place at later times the time delay being proportional to the space separation from the first location then the group of phenomenon constitute wave.

3. Brief about complex pointing vector?(June 2006)

The complex vector is given by

$$p = \frac{1}{2}\vec{E} \times \vec{H}$$

The product of E and H is a vector product mutually perpendicular components of E and H contribute to the power flaw. This power flow is directed along the normal to the plane containing E and H

4. Brief about the intrinsic impedance for a perfect dielectric medium?(June 2006)

Intrinsic impedance for a perfect dielectric medium is given by

$$\eta = \sqrt{\frac{j\omega\mu}{j\omega E}} = \sqrt{\frac{\mu}{E}}\,\boldsymbol{\Omega}$$

η Is a real positive phase angle is zero for a perfect dielectric both the field E and H are in phase

5.What is meant by linear Polarization?(June 2006)

The electric field E is the resultant of Ex and Ey, and the direction of it depend on the relative magnitude of Ex and Ey. The angle made by E with axis is given by $\theta = \tan^{-1}\frac{Ey}{Ex}$.If the resultant vector E is oriented in a direction which is constant with time the wave is said to be linearly polarized

6.What do you mean by skin depth or "Depth of penetration?(May 2010,2016, June 2006) (April/May 2017)(April/May 2018)(Nov/Dec 2017)

The distance δ through which the amplitude of a travelling plane wave decreases by a vector of e^{-1} (or) 0.368 is called skin depth (or) depth of penetration of the conductor $\delta = \frac{1}{\propto} = \sqrt{\frac{2}{j\omega\sigma}}$ for good conductor.

7.Define skin effect. (June 2007, May 2010)

The Electromagnetic energy (high frequency signal) is not transmitted in the inductor of a conductor it travels in the region surrounding the conductor .This is known as skin effect.

8.What is the velocity of electromagnetic wave in free apace and in lossless dielectric?(April 2008)

In free space $V = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}}$

$= 3 \times 10^8$m

In lossless dielectric

$V = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_\circ \epsilon_0 \mu_r \epsilon_r}} = \frac{3x10}{\sqrt{\mu_r \epsilon_r}}$

9.What is Brewster angle? (April 2008, May 2010)

Brewster angle is an incident angle at which there is no reflected wave for parallel polarized wave

$$\theta = \tan^{-1} \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

Where ϵ_1 is dielectric constant of medium 1

$\epsilon_2$is dielectric constant of medium 2

10. state one dimensional wave equation (April 2008)

$$\frac{\partial^2 E}{\partial x^2} = \mu\varepsilon \frac{\partial^2 E}{\partial t^2}$$

11. What is characteristic impedance? (Dec 2008,Dec 2012)

(Or)

What is intrinsic impedance?(Nov/Dec 2017)

It is the ratio of electric field to magnetic field (or) it is the ratio of square root of permeability to permittivity of the medium

$$\eta = \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}}\ ohms$$

12. What is uniform medium? (May 2010)

The uniform medium is a medium in which $\varepsilon, \mu\ and\ \sigma$ are constant throughout the medium.

13. Write the wave equation in conducting medium?

$$\nabla^2 E - \mu\varepsilon\frac{\partial^2 E}{\partial t^2} - \mu\sigma\frac{\partial E}{\partial t} = 0$$
$$\nabla^2 H - \mu\varepsilon\frac{\partial^2 H}{\partial t^2} - \mu\sigma\frac{\partial H}{\partial t} = 0$$

14. Write down the wave equation for E and H in a free space medium (Aril 2010,Dec2009)

$$\nabla^2 E - \mu_0\varepsilon_0\frac{\partial^2 E}{\partial t^2} = 0$$
$$\nabla^2 H - \mu_0\varepsilon_0\frac{\partial^2 H}{\partial t^2} = 0$$

15. A plane travelling wave in free space has an average pointing vector of 1-5 watts /m^2 calculate the average energy density. (Dec 2009)

Average pointing vector$= \frac{1}{2}E \times H$

$$= 1.5 \therefore H = \frac{E}{\eta}$$
$$E \times H = 3$$
$$\frac{E^2}{\eta} = 3$$
$$E^2 = 3\eta = 3 \times 120\pi$$

Average energy density $= \frac{1}{2}\varepsilon_0 E^2$

$$= \frac{1}{2} \times 8.854 \times 10^{-12} \times 3 \times 120$$
$$= 5 \times 10^{-9}\, w/m^2$$

16. Find the velocity of plane wave in a lossless medium having a relative permittivity of 5 and relative permeability of unity. (Dec2009)(Nov/Dec 2015)(April/May 2017)

Given

$$\varepsilon_r = 5 \qquad \mu_r = 1$$
$$v = \frac{v_0}{\sqrt{\mu_r\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{5}} = 1.34 \times 10^8\, m/s$$

17. Define standing wave ratio. (Dec 2009,June 2013)(Nov/Dec 2016)(April/May 2018)

The ratio of maximum to minimum magnitude of voltage or current on a line having standing wave is called the standing wave ratio (SWR)

$$SWR = \left|\frac{V_{max}}{V_{min}}\right| = \left|\frac{I_{max}}{I_{min}}\right|$$

18. State the properties of uniform plane wave? (Dec 2010)(Nov/Dec 2016)

The properties of uniform in plane wave are as follows.

I. At very point in space, the electric field E and magnetic field H are perpendicular to each and to the direction of the travel.

II. The fields vary harmonically with time and at the same frequency everywhere in space.

III. Each field has the same direction magnitude and phase at every point in any plane perpendicular to the dielectric of wave travel.

19. What is uniform wave? (June 2010)

If the phase of a wave is the same for all points on a plane surface, it is called uniform plane wave. If the amplitude is also constant in a plane wave, it is called uniform plane wave.

20. What is meant by circular polarization? (Dec 2010)

If x and y component of electric field E_x and E_y have equal amplitude and 90° phase difference ,the locus of the resultant electric field E is a circle and the wave said to be circularly polarized

21. Define Elliptical polarization

If x and y component of Electric field E_x and E_y hae different amplitude and 90° phase difference the locus of the resultant elutic field is a ellipse and the wave is said to be elliptically polarized.

22. Find the velocity of a plane wave in a lossless medium having a relative permittivity of 5 and relative permeability? (April 2005)

For lossless medium,

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{(\mu_0\mu_r)(\varepsilon_0\varepsilon_r)}} = \frac{1}{\sqrt{\sqrt{\dfrac{(4\pi \times 10^{-7} \times 2)}{(8.854 \times 10^{-2} \times 5)}}}}$$

$$V = 0.948 \times 10^8\ m/sec$$

23. Find the average power loss/ volume for a dielectric having $\varepsilon_r = 2\ and \tan\delta = 0.005, if\ E = \frac{1.0KV}{m}\ at\ 500\ mHz$ (Dec 2005)

Loss tangent $= \tan\delta = \frac{\sigma}{\omega\varepsilon} 0.005$

$$\sigma = (0.005)(2\pi f)(\varepsilon_0\varepsilon_r)$$

$$= 0.005 \times 2 \times \pi \times 500 \times 10^6 \times 8.854 \times 10^{-12} \times 2$$

$$= 2.7815 \times 10^{-5}\ Mho/m$$

The average power density is given by

Paws $= \frac{1}{2}\frac{|E|^2}{\eta}\omega/m^2$

Power loss per volume is given by

$$p = \frac{1}{2}|E|^2.\sigma$$

$$= \frac{1}{2}(1 \times 10^3)^2(2.7815 \times 10^{-5})$$

$$p = 13.91\ w/m^3$$

24. Calculate skin depth for medium with conductivity 100℧/m1 relative permeability 2, relative permittivity 3 at 50 Hz, 1MHz and 1 GHz (May 2013)

Skin depth is given by

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi\, f(\mu_0 \mu_r)\sigma}}$$

(i) 50Hz

$$\delta = \frac{1}{\sqrt{\pi \times 50 \times 4 \times \pi \times 10^{-7} \times 2 \times 100}} = 5.0329m$$

(ii) At 1 MHZ

$$\delta = \frac{1}{\sqrt{\pi \times 1 \times 10^{6} \times 4 \times \pi \times 10^{-7} \times 2 \times 100}} = 35.588m$$

(iii) At 1GHz

$$\delta = \frac{1}{\sqrt{\pi \times 1 \times 10^{9} \times 4 \times \pi \times 10^{-7} \times 2 \times 100}} = 1.1254\ mm$$

25. What is the significance of intrinsic impedance of free? What is its value?(Dec 2005, Dec 2011)

It is the ratio of electric field to magnetic field (or) It is the ratio of square root of permeability to permittivity of the medium.

$$\eta = \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}}\ ohm.$$

$For\ free\ space\ \eta_0$ Is purely resistive

$$\eta = \sqrt{\frac{4\pi * 10 - 7}{8.0854 * 10 - 12}}\ ohm.$$

= 377 Ω.

26. **Define skin effect and its significance at low frequency and at very high frequency applications to conductions**. (Dec 2003)

Skin effect is defined as the distance through which the amplitude of the travelling wave decreases to 37% of its original amplitude is called skin effect

$$\delta = \frac{1}{\alpha} = \frac{1}{\beta} = \frac{1}{\sqrt{\pi F\ \mu\sigma}} m$$

For the frequency in microwave range the Skin depth is very small for good conductors

27. Define loss tangent in an insulating material?

$$\theta = \tan^{-1} \frac{\sigma}{\omega\varepsilon}$$

θ is called loss angle, the term

$\frac{\sigma}{\omega\varepsilon}$is called loss tangent

If $\sigma \gg \omega\varepsilon$ the medium is good conductor

If $\sigma \gg \omega\varepsilon$ the medium is dielectric

29. How a dielectric medium can be identified as lossless and lossy for a given frequency.

For lossles dielectric $\sigma = 0$

For lossy dielectric $\sigma > 0$

However
Low loss and high loss dielectric can be identify by $\sigma/\omega\varepsilon$
$\frac{\sigma}{\omega\varepsilon} \ll 1$ for low loss dielectric
$\frac{\sigma}{\omega\varepsilon} \ll 1$ For high loss dielectric

When
σ =Is conductivity
ε =Is permittivity of the medium

$$\omega = 2\pi f$$

30. For a lossy dielectric material having$\mu_r = 1, \varepsilon_r = 48,\ = 20\ s/m$, calculate the propagation constant at a frequency of 16 Hz.(June 2006)

Propagation constant $\gamma = \alpha + j\beta$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right)$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right)$$

$$\frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{2\pi f\varepsilon_r\varepsilon_0} = \frac{20}{2\pi \times 16 \times 10^9 \times 48 \times 8.854 \times 10^{-12}}$$

$$= 0.46875$$

$$\alpha = 120.28$$

$$\beta = 388.59$$

31. Find the skin depth at a frequency of 2MHz is aluminum $where\ \sigma = \frac{38.2m}{m}$ and $\mu_r = 1$

$$\sigma = 38.2 \times 10^6 s/m, \mu_r = 1, \omega = 2\pi f$$

Skin depth $\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2}{2\pi\times2\times10^6\times1\times4\pi\times10^{-7}\times38.2\times10^6}}$

$$= 5.758 \times 10^{-5} m$$

32. Write Helmholtz's equation.

$$\nabla^2 E - \gamma^2 E = 0$$

When $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

33. Write down the expression for instantaneous mean power flow in electromagnetic field and instantaneous poynting vector.

Instantaneous power
$\omega = |V||I|\cos(\omega t + \theta_v)\ \ cos(\omega t + \theta_i)$

Instantaneous poynting vector

$$\vec{P} = \vec{E} \times \vec{H}$$

34. Write down the expression for average power flow in electromagnetic field and average pointing vector.

Average power $\omega_{av} = \frac{|V||I|}{2}\omega s\theta$

Average pointing vector $P_{av} = \frac{1}{2} real\ part[EXH]$

35. State slepian vector

Slepian vector is a vector which defined at every point such that its flux coming of any volume is zero.

$$(\nabla . S = 0)$$

It is given by

$$S = \nabla \times (VH)$$

V is electric potential

H is magnetic field intensity

36. State snell's law.

When a wave is travelling from one medium to another medium, the angle of in incidence is related to angle of reflection as follows.

$$\frac{Sin\ \theta_i}{Sin\ \theta_2} = \sqrt{\frac{\eta_1}{\eta_2}} (\mu_1 = \mu_2 = \mu_0)$$

When θ_i is angle incidence

$\theta_t$is angle of refraction

$\varepsilon_2$is dielectric constant of medium 2

ε_1 is dielectric constant of medium 1

37. Define surface impedance.

Surface impedance is defined as the ratio of tangential component of an electric field at the surface of a conductor to the linear current density.

$$Z_s = \frac{E_{tan}}{J_s} = \frac{\gamma}{\sigma}$$

When $\gamma$are propagation constant

$\sigma$is conductivity medium

39. What is the wave length and frequency of wave propagation in a free space? when β= 2? (April/May 2015)

Wave length, λ= $\frac{2\pi}{\beta}$

$= \frac{\mathbf{2\pi}}{\mathbf{2}}$

$= \pi$

Frequency,f = $\beta/(2\pi\sqrt{\mu_0\varepsilon_0})$

$$= \frac{2}{2\pi\sqrt{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}}$$

= 95.43 MHz

40. Compare equipotential plot for non-uniform and uniform fields. (April/May 2015)

In a uniform electric field the potential gradient is constant so that the equipotential lines are equally spaced as shown in the figure. However in the case of non – uniform field such as the field due to a positive point charge as shown in the figure. Field lines are widely separated as the distance from the charge increases.

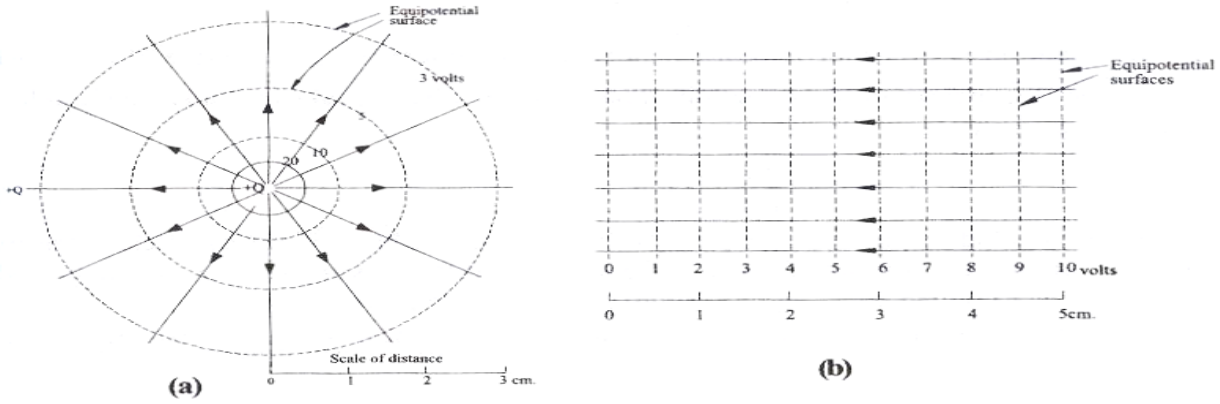

Fig. 2.26 : Equipotential surfaces in (a) Non-uniform field. (b) Uniform field.

41. If a plane is incident normally from medium 1 to medium 2, write the reflection and transmission coefficients. (Nov/Dec 2014)

(i) Reflection coefficient (or) reflectivity: It is defined as the fraction of incident energy flux reflected per unit area per second at the interface. It is denoted by the symbol Γ_R.

$$\Gamma_R = \frac{\text{Magnitude of reflected energy flux per unit area per second at the interface}}{\text{Magnitude of incident energy flux per unit per second at the interface}}$$

$$= \frac{|E_r|}{|E_i|}$$

(ii) Transmission coefficient (or) transmittivity: It is defined as the fraction of incident energy flux transmitted (refracted) per unit area per second at the interface. It is denoted by the symbol Γ_T.

$$\Gamma_T = \frac{\text{Magnitude of transmitted energy flux per unit area per second at the interface}}{\text{Magnitude of incident energy flux per unit area per second at the interface}}$$

$$\Gamma_T = \frac{|E_t|}{|E_i|}$$

42. The capacitance and inductance of an overhead transmission line are 0.0075 μF/km and 0.5 mH/km respectively. Determine the characteristics impedance of the line. (Nov/Dec 2014)

Characteristics impedance, $Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.5\times10^{-3}}{0.0075\times10^{-6}}}$

43.Find the characteristics impedance of the medium whose relative permittivity is 3 and relative permeability is 1. (Dec2009)(Nov/Dec 2015)

Solution:

$$\eta = \frac{E}{H} = \sqrt{\frac{\mu}{\varepsilon}}\ ohm.$$

$$\sqrt{\frac{4\pi * 10^{-7} * 1}{8.854 * 10^{-12} * 3}}\ ohm.$$

$= 377\Omega.$

44. What is the practical significance of skin depth?(Nov/Dec 2015)

The distance δ through which the amplitude of a travelling plane wave decreases by a vector of e^{-1} (or) 0.368 is called skin depth (or) depth of penetration of the conductor

$\delta = \frac{1}{\propto} = \sqrt{\frac{2}{j\omega\sigma}}$for good conductor

For low frequency, the skin depth δ is large.

For very high frequency, the skin depth δ is very small

Part B

1. From the Maxwell equation, derive the electromagnetic wave equation in conducting medium for $\overrightarrow{E}$ and $\overrightarrow{H}$ fields (APR 2011) (NOV 2010)(May/June 2016)(Nov/Dec 2016)(Nov/Dec 2017)

(OR)

Derive the Wave equations in general form.

Answer:

(i) <u>Electromagnetic wave equations in terms of electric field intensity</u>

Consider the Maxwell's equation obtained from faraday law

$\oint \vec{E}.d\vec{L} = -\iint \frac{\partial \vec{B}}{\partial t}\overrightarrow{ds}$ …….(1)

Appling stokes theorem to the LHS of the equation

$\int \vec{E}.d\vec{L} = \iint \nabla \times \vec{\mathrm{E}}.\overrightarrow{\mathrm{ds}} \int \vec{E}.d\vec{L} = \iint \nabla \times \vec{\mathrm{E}}.\overrightarrow{\mathrm{ds}}$……(2)

Sub equation (2) in equation(1)

$$\iint \nabla \times \vec{\mathrm{E}}.\overrightarrow{\mathrm{ds}} = -\iint \frac{\partial \vec{B}}{\partial t}.\overrightarrow{ds}$$

$\Rightarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\Rightarrow \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$---------- (3)

Taking curl on both sidesoftheequation (3)

$\nabla \times \nabla \times \vec{\mathrm{E}} = -\mu\nabla \times \frac{\partial \vec{H}}{\partial t}$--------- (4)

But Maxwell's equation obtained from amperes law

$\oint \vec{H}.d\vec{L} = \iint \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right).\overrightarrow{ds}$---------- (5)

Appling stokes theorem to L.H.S of the equation (5)

$\oint \vec{H}.d\vec{L} = \iint \nabla \times \vec{H}\,.\overrightarrow{ds}$---------- (6)

Sub, equation (6) in (5) $\iint \nabla \times \overrightarrow{H}.d\vec{S} = \iint (\vec{J} + \frac{\partial \vec{D}}{\partial t}).d\vec{s}$

$\nabla \times \vec{H} = (\vec{J} + \frac{\partial \vec{D}}{\partial t})$ ---------- (7)

But we know that $\vec{J}\ \alpha\ \vec{E}$ $\rightarrow \vec{J} = \sigma\ \vec{E}$---------- (8)

Where 'σ' → conductivity of the medium substitute.Equation(8)in Equation (7)

$\nabla \times \vec{H} = \sigma\, \vec{E} + \frac{\partial \vec{D}}{\partial t}$ ---------- (9)

Differentiating the above equation with respect to time

$$\frac{\partial}{\partial t}\left(\nabla \times \vec{H}\right) = \frac{\partial}{\partial t}\left(\sigma\, \vec{E} + \frac{\partial \vec{D}}{\partial t}\right)$$

$\nabla \times \frac{\partial \vec{H}}{\partial t} = \sigma \frac{\partial \vec{E}}{\partial t} + \frac{\partial^2 \vec{D}}{\partial t^2}$---------- (10)

Subequation, (10) in equation (4)

$$\nabla \times \nabla \times \vec{\mathrm{E}} = -\mu(\nabla \times \frac{\partial \vec{H}}{\partial t}) = -\mu(\sigma \frac{\partial \vec{E}}{\partial t} + \frac{\partial^2 D}{\partial t^2})$$

$\nabla \times \nabla \times \vec{\mathrm{E}} = -\mu\, \sigma \frac{\partial E}{\partial t} - \mu \frac{\partial^2 D}{\partial t^2}$---------- (11)

But $\vec{D}\ \alpha\ \vec{E} \Rightarrow \vec{D} = \varepsilon \vec{E}$

Where ε → permittivity of the medium

$\nabla \times \nabla \times \vec{\mathrm{E}} = -\mu\sigma \times \frac{\partial \vec{E}}{\partial t} - \mu\varepsilon \frac{\partial^2 \mathrm{E}}{\partial \mathrm{t}^2}$--- (12)

But as per vector Algebra

$\nabla \times \nabla \times \vec{\mathrm{E}} = \nabla(\nabla.\vec{\mathrm{E}}) - \nabla^2\, \vec{\mathrm{E}}$--------- (13)

Equating equation (12) and (13)

$\nabla(\nabla.\vec{\mathrm{E}}) - \nabla^2\, \vec{\mathrm{E}} = -\mu(\sigma \frac{\partial \vec{E}}{\partial t} + \varepsilon \frac{\partial^2 E}{\partial t^2})$---------- (14)

As per gauss law

$\nabla.\vec{D} = \rho_v$---------- (15)

But $\rho_v = 0$ outside the conductor $\Rightarrow \nabla.\vec{D} = 0$-------- (16)

$\Rightarrow \varepsilon(\nabla.\vec{E}) = 0 \Rightarrow \nabla.\vec{E} = 0$ ... ...(17)

Sub, equation (17) in (14)

$$\boxed{-\nabla^2 \mathbf{E} == -\mu\left(\sigma \frac{\partial \vec{E}}{\partial t} + \frac{\partial^2 \vec{E}}{\partial t^2}\right)}\text{--------} \quad (18)$$

(ii) Electromagnetic wave equation in terms of magnetic field intensity

Consider the Maxwell's equation obtained from ampere's law

$$\oint \vec{H}.\overrightarrow{dL} = \iint (\vec{J} + \frac{\partial \vec{D}}{\partial t}).\overrightarrow{ds} \text{---------- (19)}$$

Appling stokes theorem to LHS equation (19)

$$\iint \nabla \times \vec{H}.\overrightarrow{dS} = \iint (\vec{J} + \frac{\partial \vec{D}}{\partial t}).ds$$

$$\Rightarrow \nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \text{---------- (20)}$$

But $\vec{J} = \sigma \vec{E} and \ \vec{D} = \varepsilon \vec{E}$

Equation (20) becomes $\Rightarrow \nabla \times \vec{H} = \sigma \vec{E} + \frac{\varepsilon \partial \vec{E}}{\partial t}$ ----- (21)

Taking curl on both sides of equation (21)

$$\nabla \times \nabla \times \vec{H} = \sigma(\nabla \times \vec{E}) + \varepsilon[\nabla \times \frac{\partial \vec{E}}{\partial t}] \text{------- (22)}$$

Considering Maxwell's,equation from faradays' law

$$\oint \vec{E}.d\vec{L} = \iint (\nabla \times \vec{E}).\overrightarrow{ds} = -\iint \frac{\partial \vec{B}}{\partial t}.\overrightarrow{ds}$$

$$\Rightarrow \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \text{-------- (23)}$$

But $\vec{B} = \mu \vec{H}$ $\Rightarrow \nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$ ---------- (24)

Differentiatingequation (24) w.r.t 'time'

$$\frac{\partial}{\partial t}(\nabla \times \vec{E}) = \frac{\partial}{\partial t}(-\mu \frac{\partial \vec{H}}{\partial t})$$

$$\nabla \times \frac{\partial \vec{E}}{\partial t} = -\mu \frac{\partial^2 \vec{H}}{\partial t^2} \text{-------(25)}$$

Sub,equation(24) and (25) in equation(22)

$$\nabla \times \nabla \times \vec{H} = \sigma(\nabla \times \vec{E}) + \varepsilon \left(\nabla \times \frac{\partial \vec{E}}{\partial \mathrm{t}} \right)$$

$$\nabla \times \nabla \times \vec{H} = \sigma(-\mu \frac{\overrightarrow{\partial H}}{\partial t}) + \varepsilon \left(-\mu \frac{\partial^2 H}{\partial t^2} \right)$$

$\nabla \times \nabla \times \vec{H} = -\mu\,(\sigma \frac{\partial \vec{H}}{\partial t} + \varepsilon \frac{\partial^2 H}{\partial t^2})$---------(26)

As per vector Algebra

$\nabla \times \nabla \times \vec{H} = \nabla(\nabla \,.\, \vec{H}) - \nabla^2 \mathrm{H}$---------- (27)

Equating Equation (26) and (27)

$\nabla(\nabla \,.\, \vec{H}) - \nabla^2 \mathrm{H} = -\mu(\sigma \frac{\partial \vec{H}}{\partial t} + \varepsilon \frac{\partial^2 H}{\partial t^2})$ ---------- (28)

But as per magnetic gauss law $\nabla.\vec{B} = 0$

$$\mu(\nabla.\vec{H}) = 0$$

$\nabla.\vec{H} = 0$---------- (29)

Sub, equation (29) inequation (28)

$$\boxed{-\nabla^2 \mathbf{H} = -\mu(\sigma \frac{\partial \vec{H}}{\partial t} + \varepsilon \frac{\partial^2 \vec{H}}{\partial t^2})} \text{ ------ (30)}$$

---

2. Deduce the equation of the propagation of the plane electromagnetic waves in free space.(NOV 2012)(May/June 2016)(April/May 2017)(Nov/Dec 2017)

For free space $\sigma = 0$, since free space is a non- conducting medium.

Equation (18) and equation (30) becomes

$\nabla^2 \vec{\mathrm{E}} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$---------- (31)

$\nabla^2 \vec{\mathrm{H}} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}$---------- (32)

Equation (31) and (32) can be written as

$\nabla^2 \vec{\mathrm{E}} = \frac{1}{\mathrm{v}^2} \frac{\partial^2 \vec{E}}{\partial t^2}$--------- (33)

$\nabla^2 \vec{\mathrm{H}} = \frac{1}{\mathrm{v}^2} \frac{\partial^2 \vec{H}}{\partial t^2}$---------- (34)

Where $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ is called as wave velocity for free space

$$\Rightarrow v = 3 \times 10^8 m\backslash sec = velocity\ of\ light$$

Equation (33) and (34) is written as

$\frac{\partial^2 \vec{E}}{\partial t^2} = v^2 \nabla^2 \vec{E}$---------- (35)

$\frac{\partial^2 \vec{H}}{\partial t^2} = v^2 \nabla^2 \vec{H}$---------- (36)

Equation (35) and (36) are three dimensional wave equations

Assuming that the wave is propagating in 'Z' direction only the wave equation in one dimensional form are

$\boldsymbol{\frac{\partial^2 \vec{E}}{\partial t^2} = v^2 \frac{\partial^2 \vec{E}}{\partial z^2}}$**---------(37)**

$\boldsymbol{\frac{\partial^2 \vec{H}}{\partial t^2} = v^2 \frac{\partial^2 \vec{H}}{\partial z^2}}$**---------- (38)**

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3. Derive the relationship between electric field and magnetic field. Derive the wave equation for magnetic field in phasor form. (DEC 2010)

Consider Maxwell's,Equation derived from faraday's law

$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu \frac{\partial \vec{H}}{\partial t}$---------- (1)

Taking curl on both sides of the equation,

$$\nabla \times \nabla \times \vec{E} = -\mu \left[\nabla \times \frac{\partial \vec{H}}{\partial t}\right]$$

$\Rightarrow \nabla \times \nabla \times \vec{E} = -\mu \left[\frac{\partial}{\partial t}\left(\nabla \times \vec{H}\right)\right]$---------- (2)

From vector identity

$\nabla \times \nabla \times \vec{E} = \nabla\left(\nabla . \vec{E}\right) - \nabla^2 \vec{E}$---------- (3)

Equating equation(2) and (3)

$\nabla\left(\nabla . \vec{E}\right) - \nabla^2 \vec{E} = -\mu \left[\frac{\partial}{\partial t}\left(\vec{J} \times \frac{\partial \vec{D}}{\partial t}\right)\right]$----------(4)

Where $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ [Maxwell's eqn II]

Since most of the regions are source or charge free$\nabla . \vec{E} = 0$

⇒Equation, (4) becomes $-\nabla^2 \vec{E} = -\mu \left[\frac{\partial}{\partial t}\left(\vec{J} \times \frac{\partial \vec{D}}{\partial t}\right)\right]$

$\Rightarrow \nabla^2 \vec{E} = -\mu \left[\frac{\partial}{\partial t}\left(\vec{J} \times \frac{\partial \vec{D}}{\partial t}\right)\right]$---------- (5)

Consider a general electromagnetic wave both fields $\vec{E} and\ \vec{H}$ varying with time ,its partial derivative w.r.t 'time' can be replaced be jω

Equation (5) is written as $\nabla^2 \vec{E} = \mu\left[j\omega(\vec{J} + j\omega\vec{D})\right]$

$$\Rightarrow \nabla^2 \vec{E} = j\omega\mu\left[(\sigma\vec{E}) + j\omega(\varepsilon\vec{E})\right]$$

$$\Rightarrow \nabla^2 \vec{E} = \left[j\omega\sigma\mu\vec{E} + (j\omega)\varepsilon\mu\vec{E})\right]$$

$\Rightarrow \boldsymbol{\nabla^2 \vec{E} = [j\omega\mu(\sigma + j\omega\varepsilon)]\vec{E}}$---------- (6)

In similar way, we can write another phasor equation

$\Rightarrow \boldsymbol{\nabla^2 \vec{H} = [j\omega\mu(\sigma + j\omega\varepsilon)]\vec{H}}$---------- (7)

The terms inside the bracket in equations (6) and (7) are exactly similar and represent the properties of the medium in which wave in propagating.

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4. Derive the EM wave equation in frequency domain and obtain the expressions for intrinsic impedance and propagation constant for free space, conductor and dielectric medium. (APR 2010)(Include all the below cases for question no.4)(May/June 2014,2016)

Define wave and derive the wave equations for non-dissipative medium.(June 2013)
(Includecase (i) alone for question no.5)

Discuss the wave motion in good conductors (APR 2010)
(Includecase (iv) alone for question no.6)

Describe the concept of wave propagation in a linear, isotropic, homogeneous, and lossy dielectric medium. (Nov/Dec 2014)

Obtain an expression for wave propagationinlossy dielectrics. (Only case (ii)) (April/May 2015)

Obtain the electromagnetic wave equation for free space in terms of electric field and explain the wave propagation with necessary parameters. (Nov/Dec 2015)(April/May 2018)

Answer:

Wave equation in phasor form

$\Rightarrow \nabla^2\vec{E} = [j\omega\mu(\sigma + j\omega\varepsilon)]\vec{E} = r^2\vec{E}$---------- (8)

$\Rightarrow \nabla^2\vec{H} = [j\omega\mu(\sigma + j\omega\varepsilon)]\vec{H} = r^2\vec{H}$ ---------- (9)

Where $r^2 = j\omega\mu(\sigma + j\omega\varepsilon)$---------- (10)

r- Propagation constant of the medium

$\nabla^2\vec{E} = \gamma^2\vec{E}$ *and* $\nabla^2\vec{H} = \gamma^2\vec{H}$Are called Helmholtz'sequation (or) vector wave equations
since r is a complex quantity, let $r = \alpha + j\beta$

$r^2 = (\alpha + j\beta)^2 = \alpha^2 - \beta^2 + j2\alpha\beta$---------- (11)

Equating (10) and (11)$\alpha^2 - \beta^2 + j2\alpha\beta = j\omega\mu\sigma - \omega^2\mu\varepsilon$---------- (12)

Equating real and imaginary parts ofequation (12)

$$\alpha^2 - \beta^2 = -\omega^2\mu\varepsilon$$

$$\beta^2 - \alpha^2 = -\omega^2\mu\varepsilon$$

$$2\alpha\beta = \omega\mu_0$$

From equation (11) $|r^2| = \sqrt{(\alpha^2 - \beta^2) + (2\alpha\beta)^2}$

$|r^2| = \alpha^2 + \beta^2$---------- (13)

From equation (10)

$|r^2| = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2}$---------- (14)

Equating (13) and (14)

$$\alpha^2 + \beta^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2}$$----------(15)

$$\beta^2 - \alpha^2 = -\omega^2\mu\varepsilon$$---------- (16)

Adding (15) and (16)

$$2\beta^2 = \omega^2\mu\varepsilon + \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2}$$

$2\beta^2 = \omega^2\mu\varepsilon[1 + \frac{1}{\omega\varepsilon}\sqrt{\sigma^2 + \omega^2\varepsilon^2}]$

$$\beta^2 = \frac{\omega^2\mu\varepsilon}{2}\left[1 + \frac{\omega\varepsilon}{\omega\varepsilon}\sqrt{\frac{\sigma^2}{\omega^2\varepsilon^2} + 1}\right]$$

$\beta^2 = \frac{\omega^2\mu\varepsilon}{2}[1 + \sqrt{\frac{\sigma^2}{\omega^2\varepsilon^2} + 1}]$---------- (17)

Subtracting (15) and (16)

$$2\alpha^2 = \omega\mu\sqrt{\sigma^2 + \omega^2\varepsilon^2} - \omega^2\mu\varepsilon$$

$\alpha^2 = \frac{\omega^2\mu\varepsilon}{2}[\frac{1}{\omega\varepsilon}\sqrt{\sigma^2 + \omega^2\varepsilon^2}$-1]

$$\alpha^2 = \frac{\omega^2\mu\varepsilon}{2}\left[\frac{\omega\varepsilon}{\omega\varepsilon}\sqrt{\frac{\sigma^2}{\omega^2\varepsilon^2} + 1} - 1\right]$$

$\alpha^2 = \frac{\omega^2\mu\varepsilon}{2}\sqrt{\frac{\sigma^2}{\omega^2\varepsilon^2} + 1} - 1$---------- (18)

Equation (18) and (17) becomes

$$\boldsymbol{\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{\frac{\sigma^2}{\omega^2\varepsilon^2} + 1} - 1\right)}}$$

$$\boldsymbol{\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}(1 + \sqrt{\frac{\sigma^2}{\omega^2\varepsilon^2} + 1})}}$$

An EM wave is completely defined in mediumif intrinsic impedance (η) and propation constant(r) of medium is known.

Thus it is necessary to derive the expressions for η and in terms of the properties of a medium such aspermeability(μ), permittivity(ε), conductivity (μ)etc.

Parameter	Expression	Unit
Propagation constant	$\gamma = \alpha + j\beta = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$	m^{-1}
Attenuation constant	$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}(\sqrt{1 + (\frac{\sigma}{\omega\varepsilon})^2} - 1)}$	Np/m
Phase constant	$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}(\sqrt{1 + (\frac{\sigma}{\omega\varepsilon})^2} + 1)}$	Rad/m

Intrinsic impedance	$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$	Ω
Wave velocity	$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{\omega}{\beta} = f$	m/s

Case (i) EM wave in perfect dielectric or lossless dielectric:

For perfect dielectric, conductivity $\sigma = 0,\ \ \varepsilon = \varepsilon_0\varepsilon_r,\ \ and\ \mu = \mu_0\mu_r$

I) $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)} = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

$$\Rightarrow\ \gamma = \pm j\omega\sqrt{\mu\varepsilon}$$

II) $\gamma = \alpha + j\beta \Rightarrow\ \ \alpha + j\beta = \pm j\omega\sqrt{\mu\varepsilon}$

$$\Rightarrow\ \alpha = 0$$

III) $\beta = \omega\sqrt{\mu\varepsilon}$

$$IV)\ \eta = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\varepsilon)}} = \sqrt{\frac{\mu}{\varepsilon}} \qquad \Omega$$

$\eta = \sqrt{\frac{\mu}{\varepsilon}}$Is real resistivei.e.,phase angle of $\eta = zero \Rightarrow\ \vec{E}\ and\ \vec{H}\ are\ in\ phase$

In perfect dielectric, $\sigma = 0, \alpha = 0$ means, as the wave propagates only phase (β) changes

The no attenuation..i.e.$\alpha = 0$ means no loss

$$\boldsymbol{v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{\omega}{\beta} = \lambda f \ \ m/s}$$

This lossless dielectric

Case (ii)EM wave inloss dielectric $\sigma \neq 0$

$$\gamma = \pm\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

Rearranging the terms

$$\gamma = \sqrt{j\omega\mu(\frac{\sigma}{j\omega\varepsilon} + 1)j\omega\varepsilon}$$

$$\gamma = j\varepsilon\sqrt{\mu\varepsilon}\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}$$

$$\alpha \neq 0$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}(\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1)}$$

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}(\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1)}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon(1 + \frac{\sigma}{j\omega\varepsilon})}}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\sqrt{1 - j\frac{\sigma}{\omega\varepsilon}}} \quad \Omega$$

$$\eta = |\eta| < \theta n \ \ where \ \boldsymbol{\eta} = \sqrt{\frac{\boldsymbol{\mu}}{\boldsymbol{\varepsilon}}}.\frac{\mathbf{1}}{\sqrt{\mathbf{1} - \boldsymbol{j}\frac{\boldsymbol{\sigma}}{\boldsymbol{\omega\varepsilon}}}}$$

And $\boldsymbol{\theta_n} = \frac{\mathbf{1}}{\mathbf{2}}[\frac{\boldsymbol{\pi}}{\mathbf{2}} - \mathbf{tan}^{-\mathbf{1}}\frac{\boldsymbol{\omega\varepsilon}}{\boldsymbol{\sigma}}]$

Thus in lossdielectric, the electric and magnetic fields are not in phase

<u>Case (iii) EM wave in practical dielectric</u>

The propagation constant

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$$

$$\gamma = j\omega\sqrt{\mu\varepsilon}\sqrt{1 + \frac{\sigma}{j\omega\varepsilon}}$$

$$\gamma = \alpha + j\beta = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} + j\omega\sqrt{\mu\varepsilon}$$

$$\gamma = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} + j\omega\sqrt{\mu\varepsilon}$$

Comparing real terms and imaginary terms

Atténuationconstant$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$Np/m

Phase constant $\beta = \omega\sqrt{\mu\varepsilon}$ rad/m

In general the intrinsic impedance is given by,

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{j\omega\mu}{j\omega\varepsilon(1 + \frac{\sigma}{j\omega\varepsilon})}}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\sqrt{1 + \frac{\sigma}{j\omega\varepsilon}}}$$

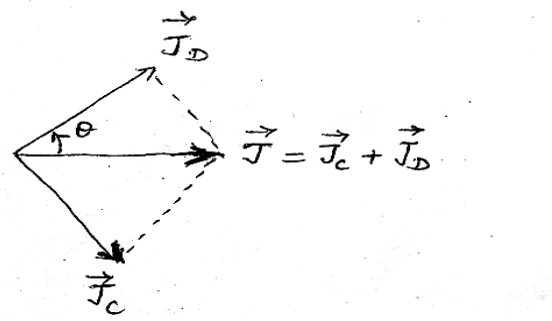
$$\vec{J}_c = \sigma E$$

$$\vec{J}_c = \frac{\partial D}{\partial t} = j\omega\varepsilon E$$

$$\Rightarrow \frac{\vec{J}_c}{\vec{J}_D} = \frac{\sigma}{j\omega\varepsilon} \Rightarrow$$

$$\boxed{\boldsymbol{loss\ angle\ \theta = \tan^{-1}\frac{\sigma}{\omega\varepsilon}}}$$

$$\boxed{\boldsymbol{loss\ tangent\ tan\theta = \frac{\sigma}{\omega\varepsilon}}}$$



Case (iv) EM wave in good conductor

For good conductors $\frac{\sigma}{\omega\varepsilon} \gg 1$

The propagation constant $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

Ad $\sigma \gg \omega\varepsilon$, we neglect imaginary part

$$\Rightarrow \gamma = \sqrt{j\omega\mu\sigma} = \sqrt{\omega\mu\sigma}\sqrt{j}$$

$$j = 1 < 90°$$

$$\Rightarrow \gamma = \sqrt{\omega\mu\sigma}\sqrt{1\ angle\ 90°}$$

$$= \sqrt{\omega\mu\sigma}\ 45°$$

$$= \sqrt{\omega\mu\sigma}\ [\cos 45^{\circ} + j \sin 45°]$$

$$= \sqrt{\omega\mu\sigma}\ [\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}]$$

$$= \sqrt{2\pi f\mu\sigma}[\frac{1}{\sqrt{2}}(1 + j1)]$$

$$\boldsymbol{\gamma = \sqrt{\pi f\mu\sigma} + \sqrt{\pi f\mu\sigma}}$$

For good conductors,

$$\boldsymbol{\alpha = \sqrt{\pi f\mu\sigma}}\ \textbf{NP/m}$$

$$\boldsymbol{\beta = \sqrt{\pi f\mu\sigma}}\textbf{Rad /m}$$

Intrinsic impedance $\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$

$$\sigma \gg j\omega\varepsilon \Rightarrow \eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} = \sqrt{\frac{\omega\mu}{\sigma}}\sqrt{j}$$

$$\boldsymbol{\eta = \sqrt{\frac{\omega\mu}{\sigma}}\left[\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right] = \sqrt{\frac{\pi f\mu}{\sigma}}(1 + j1)}$$

5. State and prove poynting theorem.(APR 2011)(NOV 2012))(April/May 2017)(April/May 2018)

Explain how the rate of energy transportation by means of EM waves from one point to another point can be obtained from Maxwell's equation (June 2013)

State poynting theorem and thus obtain an expression for instantaneouspower density vector associated with the electromagnetic field. (April/May 2015) (Or)

Derive poynting theorem from Maxwell"s equation and explain. (Nov/Dec 2015)(Nov/Dec 2017)

The rate of energy transported from one point [where a transmitter is located] to another point [where a receiver is located] by means of electromagnetic waves is obtained by using the Maxwell's equations.

$\nabla \times \vec{E} = -\mu \frac{\partial \vec{H}}{\partial t}$---------- (1)

$$\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t}$$---------- (2)

Dotting both sides of equation 2 with $\vec{E}$

$\vec{E}.(\nabla \times \vec{H}) = \sigma \vec{E}.\vec{E} + \varepsilon \vec{E}.\frac{\partial \vec{E}}{\partial t}$----------(3)

$\vec{E}.(\nabla \times \vec{H}) = \sigma \vec{E}^2 + \varepsilon \vec{E}.\frac{\partial \vec{E}}{\partial t}$----------(4)

From vector identity

$$\nabla.\left(\vec{A} \times \vec{B}\right) = \vec{B}.\left(\nabla \times \vec{A}\right) - \vec{A}.(\nabla \times \vec{B})$$

Let$\vec{A} = \vec{H} and\ \vec{B} = \vec{E}$

$$\Rightarrow \nabla.\left(\vec{H} \times \vec{E}\right) = \vec{E}.\left(\nabla \times \vec{H}\right) - \vec{H}.(\nabla \times \vec{E})$$---------- (5)

$\vec{E}.\left(\nabla \times \vec{H}\right) = \nabla.\left(\vec{H} \times \vec{E}\right) + \vec{H}.(\nabla \times \vec{E})$---------- (6)

Equating (4) and (6)

$\sigma E^2 + \varepsilon \vec{E} \frac{\partial \vec{E}}{\partial t} = \nabla.\left(\vec{H} \times \vec{E}\right) + \vec{H}.(\nabla.\vec{E})$---------- (7)

Sub (2) in (7)

$$\sigma E^2 + \varepsilon \vec{E} \frac{\partial \vec{E}}{\partial t} = \nabla.\left(\vec{H} \times \vec{E}\right) + \vec{H}.(-\mu \frac{\partial \vec{H}}{\partial t})$$

$$\Rightarrow \nabla.(\vec{H} \times \vec{E}) = \mu\vec{H}.\left(\frac{\partial \vec{H}}{\partial t}\right) + \varepsilon\vec{E}.\left(\frac{\partial \vec{E}}{\partial t}\right) + \sigma E^2$$

Interchanging the cross product

$$\nabla.\left(\vec{E} \times \vec{H}\right) = \mu\vec{H}.\left(\frac{\partial \vec{H}}{\partial t}\right) - \varepsilon\vec{E}.\left(\frac{\partial \vec{E}}{\partial t}\right) - \sigma E^2$$

Only the vector identities

$$-\mu\vec{H}.\left(\frac{\partial \vec{H}}{\partial t}\right) = -\frac{1}{2}\,\mu\frac{\partial H^2}{\partial t}$$

$$-\varepsilon\vec{E}.\left(\frac{\partial \vec{H}}{\partial t}\right) = -\frac{1}{2}\,\varepsilon\frac{\partial E^2}{\partial t}$$

$$\Rightarrow \nabla.\left(\vec{E} \times \vec{H}\right) = -\frac{1}{2}\,\mu\frac{\partial H^2}{\partial t} - \frac{1}{2}\,\varepsilon\frac{\partial E^2}{\partial t} - \sigma E^2 \text{--------(8)}$$

Taking volume integral on both sides and interchanging the order differentiation and integration.

$$\int_v \nabla.\left(\vec{E} \times \vec{H}\right)dv = -\frac{\partial}{\partial t}\int_v \left(\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right)dv - \int_v \sigma E^2 dv \text{---------- (9)}$$

Applying divergence theorem to L.H.S ofequation(9)

$$\int_s \left(\vec{E} \times \vec{H}\right).ds = -\frac{\partial}{\partial t}\int_v \left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right)dv - \int_v \sigma E^2 dv \text{----------(10)}$$

The above equation is known as poyntingtheorem

$\oint_s \left(\vec{E} \times \vec{H}\right).ds$Represents the total power leaving the volume.

- The first term in the R.H.S represents rate of decrease of energy stored in electric and magnetic field.
- The second term represents the ohmic power dissipated
- Pointing theorem states that the net power flowing out of a given volume v is equal to the time rate of decrease in the energy stored within the volume 'v' minus the conduction losses.
- The quantity $\vec{E} \times \vec{H}$ on L.H.S is known as poyntingvector$\vec{P}$ in watts per square meter. i.e., $\vec{P} = \vec{E} \times \vec{H}$
- It represents the instantaneous power density vector associated with the electromagnetic field at a given point.

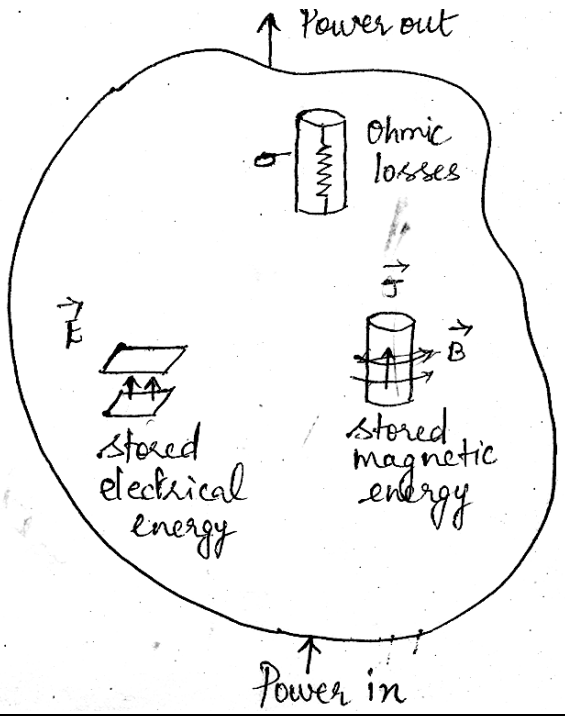
- The integration of the pointing vector over any closed surface gives the net power flowing out of that surface.
- $\vec{P}$is always normal to both $\vec{E}$ and $\vec{F}$ and is therefore directed along the direction of wave propagation.
- Equation(10) can be written as

$$\oint \vec{P}.\overrightarrow{ds} = \int_v \sigma E^2 dv - \frac{\partial}{\partial t}\int \frac{1}{2}[\mu H^2 + \varepsilon E^2]dv$$

$$\vec{\boldsymbol{P}}_{\boldsymbol{avg}} = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{Re}\left[\vec{\boldsymbol{E}} \times \vec{\boldsymbol{H}}^*\right] = \frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{Re}\left[\vec{\boldsymbol{E}}^* \times \vec{\boldsymbol{H}}\right]$$

Where $\vec{H}^*$is the complex conjugate of$\vec{H}$

$\vec{H}^*$Is the complex conjugate of$\vec{E}$



~~Fig~~

Power balance representation in EM fields.

6. Derive the transmission and reflection co-efficient for the electromagnetic waves

(APR 2008) (NOV 2012) (Nov/Dec 2017)

- Practically, very often, waves propagate in bound regions, consisting several media of different constitute parameters such as $\varepsilon, \mu, \sigma, \eta$ etc
- Depending upon the manner in which the uniform plane is incident on the boundary, there are two cases of incident

  i. Normal incidence ii. Oblique incidence

- Depending upon interface, we have

  1. Two dielectric medium

     A). Lossy dielectric –dielectric

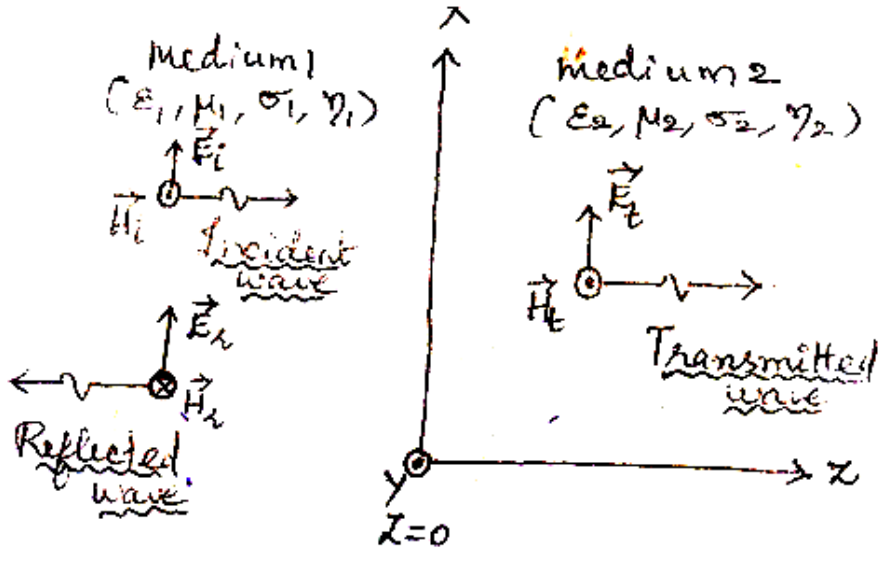
     B). Perfect dielectric –dielectric

  2. Dielectric and conductive media.

I). General case: lossy dielectric Di-electric boundary

Assume that a plane wave propagating along the +z direction is incident normally on the boundary z=0 between medium characterized by $\sigma_1, \varepsilon_1, \mu_1$ and medium 2 (z> 0 charactriser by $\sigma_2, \varepsilon_2, \mu_2$ as in fig

Subscripts I,r,tdenotes incident ,reflected transmitter wave respectively



Let $\vec{E}_i, \vec{H}_i \rightarrow$ field strength of incident wave in medium 1

$\vec{E}_r, \vec{H}_r$ →Field strength of reflected wave in medium 1

$\vec{E}_t, \vec{H}_t$ →fieldstrength of transmitted wave in medium 2

$\vec{E}_1 = \vec{E}_i + \vec{E}_r \; and \; \vec{H}_1 = \vec{H}_i + \vec{H}_r$---------- (1)

$\vec{E}_2 = \vec{E}_t \; and \; \vec{H}_2 = \vec{H}_t$---------- (2)

According to boundary conditions,

$$\vec{E}_{t1} = \vec{E}_{t2}$$

$\vec{H}_{t1} = \vec{H}_{t2}$Tangential component are continuous EM boundary

$$\vec{E}_i + \vec{E}_r = \vec{E}_t$$

$$\vec{H}_i + \vec{H}_r = \vec{H}_t$$

The relationship between the magnitudes of $\vec{E} and \; \vec{H} \; at \; z = 0$ are given by following expressions

$$E_i = \eta_1 H_i$$

$E_r = -\eta_1 H_r$as direction of reflected wave is opposite of that of incident wave

$$E_t = \eta_2 H_r$$

In terms of magnitude of fields,

$$E_i + E_r = E_t \text{---------- (3)}$$

$$H_i + H_r = H_t \text{---------- (4)}$$

In equation (4) putting of $H_i, H_r \; and \; H_t$ in terms of $E_i, E_r \; and \; E_t$ we get

$$\frac{E_i}{\eta_1} - \frac{E_r}{\eta_1} = \frac{E_t}{\eta_2}$$

$E_i - E_r = \frac{\eta_i}{\eta_r} E_t$----------(5)

Adding (3) and (5) $2E_i = \left(1 + \frac{\eta_1}{\eta_2}\right) E_t \Rightarrow 2E_i = \frac{\eta_1 + \eta_2}{\eta_2} E_t$

$$\Rightarrow E_i = \frac{2\eta_2}{\eta_1 + \eta_2} . E_i \text{----------(6)}$$

The transmission coefficient is denoted by τ and it given by

$$\boldsymbol{\tau = \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1+\eta_2}}\text{----------(7)}$$

Eliminating E_t from equation (3) and (5)

$$E_i + E_r = \frac{\eta_2}{\eta_1}(E_i - E_r)$$

$$\eta_1(E_1 + E_r) = \eta_2(E_i - E_r)$$

$$E_i(\eta_1 - \eta_2) = -E_r(\eta_1 + \eta_2)$$

$$E_r = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}.E_i\text{----------(8)}$$

The reflection co-efficient is denoted by Γ and it is giving

$$\boldsymbol{\Gamma = \frac{E_r}{E_i} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}}\text{----------(9)}$$

From (7) and (9) we can write as

$1+ \Gamma = \tau$

$$0 \leq |\Gamma| \leq 1$$

Both Γ and τ are dimensionless and complex.

---

7. Define Brewster angle and derive its expression (NOV 2010)

Incident wave is oblique to the boundary:

An incident beam meets the boundary over the region AB and gets partly reflected and transmitted.however, in this case the transmitted ray will be refracted. That is direction of propagation will be altered.

(a) Horizontal polarization:

In this case electric vector $\vec{E}$ is perpendicular to the plane of incidence and parallel to the reflecting surface. While the plane of incidence is the plane containing incidence ray normal to the surface. More significant designation for horizontal polarization is perpendicular polarization because electric vector is perpendicular to the pane of incidence.

$$\Gamma_R = \frac{\cos\theta_i - \sqrt{\left(\varepsilon_2/\varepsilon_1\right) - sin^2\theta_i}}{\cos\theta_i + \sqrt{\left(\varepsilon_2/\varepsilon_1\right) - sin^2\theta_i}}$$

It is angle of incidence.

(b) **Vertical polarization**:

In this case $\vec{E}$ is parallel to the plane of incidence, and magnetic vector$\vec{H}$ must be perpendicular to the plane of incidence. Hence this refereed as parallel polarization.

$$\Gamma_R = \frac{\left(\varepsilon_2/\varepsilon_1\right)\cos\theta_i - \sqrt{\left(\varepsilon_2/\varepsilon_1\right) - sin^2\theta_i}}{\left(\varepsilon_2/\varepsilon_1\right)\cos\theta_i + \sqrt{\left(\varepsilon_2/\varepsilon_1\right) - sin^2\theta_i}}$$

Brewster Angle:

Brewster angle is the angle for which there is no reflection when the wave is vertically polarized.

This condition can be obtained from equating the reflection coefficient of vertically polarized wave to zero.

$$\Gamma_R = \frac{\left(\varepsilon_2/\varepsilon_1\right)\cos\theta_i - \sqrt{\left(\varepsilon_2/\varepsilon_1\right) - sin^2\theta_i}}{\left(\varepsilon_2/\varepsilon_1\right)\cos\theta_i + \sqrt{\left(\varepsilon_2/\varepsilon_1\right) - sin^2\theta_i}} = 0$$

This occurs when the numerator is zero.

$$\sqrt{\left(\varepsilon_2/\varepsilon_1\right) - sin^2\theta_i} = \frac{\varepsilon_2}{\varepsilon_1}\cos\theta_i$$

$$\frac{\varepsilon_2}{\varepsilon_1} - sin^2\theta_i = \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 cos^2\theta_i$$

$$=\left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 (1 - sin^2\theta_i)$$

$$\frac{\varepsilon_2}{\varepsilon_1} - sin^2\theta_i = \left(\frac{\varepsilon_2}{\varepsilon_1}\right)^2 - \frac{\varepsilon_2^2}{\varepsilon_1^2} sin^2\theta_i$$

$$(\varepsilon_1^2 - \varepsilon_2^2) sin^2\theta_i = \varepsilon_2(\varepsilon_1 - \varepsilon_2)$$

$$sin^2\theta_i = \frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}$$

$$COS^2\theta_I = \frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}$$

$$Tan\theta_i = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

If the incident wave is not entirely parallel polarized, there will be some reflection, but the reflected wave will be entirely of perpendicular polarization.

The Brewster angle is often referred as polarsing angle since a wave composed of both perpendicular and parallel components and incident at the Brewster angle, produces a reflected wave only a perpendicular component.

Brewster angle and degree of polarization:

In 1982, Sir David Brewster found that maximum polarization of the reflected ray occurs when the reflected ray is at right angles to the refracted ray.

$$\theta_i + \theta_r = 90^{\circ} = {}^{\pi}/_{2}$$

AtBrewster angle reflection coefficient will be zero

From Snell's law

$$\frac{sin\theta_i}{sin\theta_r} = \frac{\eta_2}{\eta_1}$$

For Brewster angle

$$\theta_r = {}^{\pi}/_{2} - \theta$$

$$\theta_i = \theta_b$$

$$\frac{sin\theta_b}{cos\theta_b} = \frac{\eta_2}{\eta_1}$$

$$tan\theta_b = \frac{\eta_2}{\eta_1}$$

$$\theta_b = tan^{-}\left(\frac{\eta_2}{\eta_1}\right)$$

This called Brewster angle.

If the unpolarised wave is reflected at other than Brewster angle, the degree of polarization is given by

$$p = \frac{\Gamma_{R_1} - \Gamma_{R_2}}{\Gamma_{R_1} + \Gamma_{R_2}}$$

Where$\Gamma_{R_1}$and Γ_{R_2} are respectively reflection co-efficient for perpendicular and parallel components of reflected light and are given by

$$\boldsymbol{\Gamma_{R_1} = \frac{sin^2(\theta_i - \theta_r)}{sin^2(\theta_i + \theta_r)}}$$

$$\boldsymbol{\Gamma_{R_2} = \frac{tan^2(\theta_i - \theta_r)}{tan^2(\theta_i + \theta_r)}}$$

8.A plane wave travelling in air normally incident on a block of paraffis with $\varepsilon_r = 2.2$, find the reflection coefficient. (April / May 2015)

$$\eta_1 = 120\,\pi$$

$$\eta_2 = \sqrt{\frac{\mu_r \mu_0}{\varepsilon_0 \varepsilon_T}}$$

$$\mu_r = 1 \; and \; \varepsilon_T = 2.2$$

$$\eta_2 = \sqrt{\frac{1}{2.3}}\; 120\,\pi$$

$$= 254 \; ohms.$$

$$reflection\ coefficient = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} = \frac{254 - 377}{254 + 377}$$

= -0.195

---

9. A plane wave travelling in air normally incident on a block of paraffins with $\varepsilon_r = 2.3$, find the reflection coefficient. (Nov/Dec 2015)

$$\eta_1 = 120\,\pi$$

$$\eta_2 = \sqrt{\frac{\mu_r \mu_0}{\varepsilon_0 \varepsilon_T}}$$

$$\mu_r = 1 \; and \; \varepsilon_T = 2.3$$

$$\eta_2 = \sqrt{\frac{1}{2.3}}\; 120\,\pi$$

$$= 248.5 \; ohms.$$

$$reflection\ coefficient = \frac{\eta_2 - \eta_1}{\eta_1 + \eta_2} = \frac{248.5 - 377}{248.5 + 377}$$

= -0.20

10.Find the velocity of a plane wave in a lossless medium having a relative permittivity of 4 and a relative permeability of 1.2. (Nov / Dec 2014)

$$\varepsilon_r = 4$$

$$\mu_r = 1.2$$

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}}$$

$$v_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 m/sec$$

$$rv = \frac{v_0}{\sqrt{\mu_r \varepsilon_r}}$$

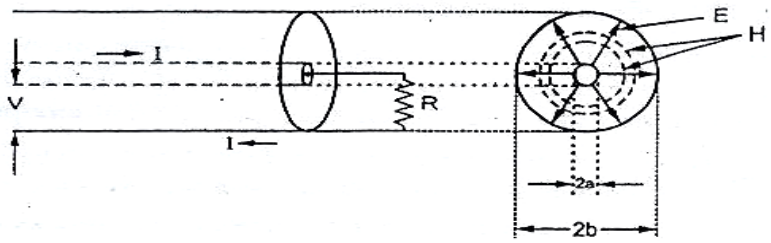
$$= \frac{3 \times 10^8}{\sqrt{1.2 \times 4}}$$

$$= 1.37 \times 10^8 m/sec$$

11. Show that the total power flow along a coaxial cable will be given surface integration of the pointing vector over any closed surface. (Nov/Dec 2014)

Consider a co – axial cable in which the power is transferred to the load resistance R along a cable. There are two conductors namely inner conductor and outer conductor concentric to each other. Let the radius of the inner conductor be 'a' units band the inner radius of the outer conductor be 'be' units as shown in the figure. 15.1

In the cable, there exists a D.C. voltage V between the two conductors, while the steady current I flows in the inner and outer conductors as shown in the figure. 15.1



15.1 Voltage and current in a co-axial cable

Magnetic field strength H will be directed in the circular path about the axis as shownin the fig. In the region between the two conductors, the current enclosed is equal to the magneto motive force around any of the circles of H.

$$\oint \overline{H} \cdot d\overline{S} = I$$

The magnetic field H is constant along any of the circular path. Let r be the radius of any circle considered then the magneto motive force is given by,

$$\oint \overline{H} \cdot d\overline{S} = 2\pi r H = I$$

$$H = \frac{I}{2\pi r}$$

Let q e the charge per unit length, then the potential difference between inner and outer conductor of a co-axial cable is given by,

$$V = \frac{q}{2\pi\varepsilon} \log\left(\frac{b}{a}\right)$$

Similarly the magnitude of electric field intensity $\vec{E}$ for a co-axial cable is given by,

$$E = \frac{q}{2\pi\varepsilon r}$$

Fromequations (3) and (4),we can write,

$$E = \frac{V}{r\left[\log\left(\frac{b}{a}\right)\right]}$$

According to the pointing theorem, thepoynting vector is given by,

$$\vec{P} = \overline{E} \times \overline{H}$$

But the power flows in the direction parallel to the axis of the cable. As $\vec{E}$ and $\vec{H}$ are mutually perpendicular to each other everywhere, the magnitude of the Poynting vector is given by,

$$P = E \bullet H$$

The total power flow along the cable can be obtained by integrating the Poynting vector over any cross-sectional surface with area $2\pi rdr$

$$W = \text{Total power flow} = \int_{\text{surface}} \bar{P} \bullet d\bar{a}$$

$$\therefore \quad W = \int (\bar{E} \times \bar{H}) \bullet (d\bar{a})$$

$$\therefore \quad W = \int_a^b \frac{V}{r \log\left(\frac{b}{a}\right)} \bullet \left(\frac{I}{2\pi r}\right) (2\pi r\, dr$$

$$\therefore \quad W = \frac{V I}{\log\left(\frac{b}{a}\right)} \int_a^b \frac{dr}{r}$$

$$\therefore \quad W = \frac{V \bullet I}{\log\left(\frac{b}{a}\right)} [\log r]_a^b$$

$$\therefore \quad W = \frac{V \bullet I}{\log\left(\frac{b}{a}\right)} [\log b - \log a]$$

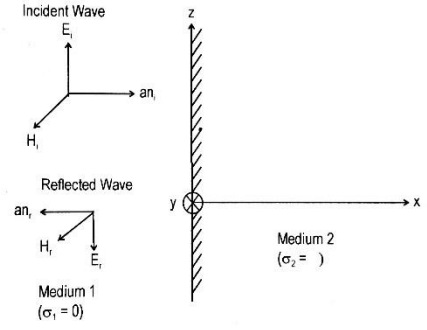
$$\therefore \quad W = \frac{V \bullet I}{\log\left(\frac{b}{a}\right)} \log\left(\frac{b}{a}\right)$$

$$\therefore \quad \boxed{W = V \bullet I}$$

Above result is certainly a universal result the power flow is product of voltage and current. The important point to be noted here that the area over which the Poynting vector is integrated did not included any of the conductor. Hence we can conclude that in case of a perfect conductor, the power is entirely external to the conductors.

12. Write a short notes on standing waves (May/June 2016)(Nov/Dec 2017)

When a plane electromagnetic wave in air is incident normally upon the surface of a perfect conductor, the electromagnetic wave is entirely reflected. In time-varying the fields, neither $\vec{E}$ or $\vec{H}$ can exist within a perfect conductor so that none of the energy of the incident wave con be transmitted. Since energy is absorbed by perfect conductor, the amplitude of $\vec{E}$ and $\vec{H}$ in the reflected wave are the same as the incident wave and only difference is in the direction of power flow. If the electric field of the incident electromagnetic wave is



$$\overrightarrow{E_i} = E_i\, e^{j(\omega t - \beta x)}$$

The surface of the conductor is taken to the x=0 plane. The reflected electromagnetic wave is given by

$$\overrightarrow{E_r} = E_{ro}\, e^{j(\omega t + \beta x)}$$

Electric field E_r is determined by the boundary condition. The tangential component of electric field $\vec{E}$ is continuous across the boundary and $\vec{E}$ is zero inside the conductor. The tangential component of $\vec{E}$ just outside the conductor must also be zero. It means the sum of incident and reflected electric field strengths must be zero in the plane x=0. Therefore

$$\overrightarrow{E_{io}} + \overrightarrow{E_{ro}} = 0$$

$$\overrightarrow{E_{io}} = -\overrightarrow{E_{ro}} \text{ at } x = 0.$$

Both have level equal magnitude and reverse phase.

Resultant electric field strength= Electric field strength of incident wave + Electric field strength of reflected wave,

$$\overrightarrow{E_T} = \overrightarrow{E_i} + \overrightarrow{E_r}$$

$$= E_{io}\, e^{j(\omega t - \beta x)} + E_{ro}\, e^{j(\omega t + \beta x)}$$

$$= E_{io}\, e^{j(\omega t - \beta x)} - E_{io}\, e^{j(\omega t + \beta x)}$$

$$= E_{io}[e^{j(\omega t - \beta x)} - e^{j(\omega t + \beta x)}]$$

$$= E_{io}[e^{j\omega t}\, e^{-j\beta x} - e^{j\omega t}\, e^{j\beta x}]$$

Since $\boldsymbol{E_{ro} = -E_{io}}$

$$= -2j\, E_{io}\, e^{j\omega t} \left[\frac{e^{j\beta x} - e^{-j\beta x}}{2j}\right]$$

$$= -2j\, E_{io}\, e^{j\omega t} \sin \beta x$$

$$= -2j\, E_{io}[(\cos \omega t + j \sin \omega t)] \sin \beta x$$

$$= 2E_{io}[-j \cos \omega t - j^2 \sin \omega t] \sin \beta x$$

$$= 2E_{io}[\sin \omega t - j \cos \omega t] \sin \beta x$$

$$= 2E_{io} \sin \omega t \sin \beta x - j2E_{io} \cos \omega t \sin \beta x$$

$$\overrightarrow{E_T} = \text{Real } \{E_T\} + j\, Im\, (E_T)$$

$$\text{Real } \{E_I\} = 2E_{io} \sin \omega t \sin \beta x.$$

The above equation shows that the combination of incident and reflected waves gives a standing wave which does not progress. The magnitude of electric field changes sinusoidal with distance from the reflected plane. The magnitude of electric field is zero at the surface and at multiples of half wavelength(λ/2) from the surface.

Magnetic field strength is reflected at the surface without phase reversal. If $\vec{E}$ and $\vec{H}$ both are reversed, the power direction does not change which is required here. So the phase of $\overrightarrow{H_r}$ is same as phase of $\overrightarrow{H_l}$ at the surface of reflection x=0.

The resultant magnetic field at any point that is at a distance-x from x=0, is given by

$$\overrightarrow{H_T} = H_{io}\, e^{j(\omega t - \beta x)} + H_{ro}\, e^{j(\omega t + \beta x)}$$

$$= H_{io}\, e^{j(\omega t - \beta x)} + H_{io}\, e^{j(\omega t + \beta x)}$$

$$= H_{io}\, e^{j\omega t}\, [e^{-j\beta x} + e^{j\beta x}]$$

$$= 2H_{io}\, e^{j\omega t} \left[\frac{e^{j\beta x} + e^{-j\beta x}}{2}\right]$$

$$= 2H_{io}\, e^{j\omega t} \cos \beta x$$

$$= 2H_{io} [\cos \omega t + j \sin \omega t] \cos \beta x$$

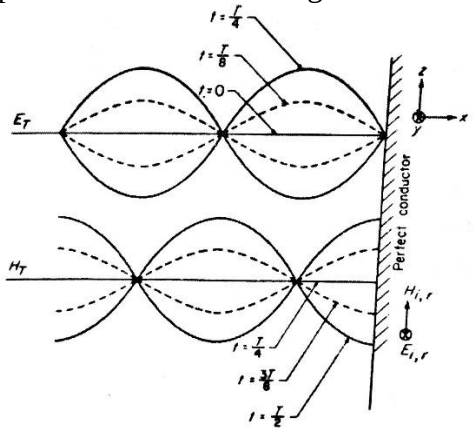
$$= 2H_{io} \cos \omega t \cos \beta x + j2H_{io} \sin \omega t \cos \beta x$$

$$= \text{Real } \left(\overrightarrow{H_T}\right) + j\, I_m \left(\overrightarrow{H_T}\right)$$

$$\text{Real } \left\{\overrightarrow{H_T}\right\} = 2H_{io} \cos \omega t \cos \beta x.$$

Magnetic field $\overrightarrow{H_r}$ has maximum value at the surface of the conductor and at multiples of half wave length (λ/2) from the surface.

Electric field $\overrightarrow{E_r}$ and magnetic field $\overrightarrow{H_r}$ are 90º out of time phase because of factor *j*. Standing waves of $\vec{E}$ and $\vec{H}$ are represented as shown in figure 16.2



16.2Illustration of standing waves of E and H

13.Given the conduction current density in a lossy dielectric as$J_c = 0.02Sin\ 10^9 t\left(\frac{A}{m^2}\right)$. Find the displacement current density if $\sigma = 10^3 \text{℧}/m$ and $\varepsilon_{r=6.5}$

Sol

$$J_c = \sigma E = 0.02Sin10^9 t = 10^3 E$$

When $E = 2 \times 10^{-5} sin10^9 t$

$$J_d = \frac{\partial D}{\partial t}$$

$$= \frac{d}{dt}(\varepsilon_0 \varepsilon_r 2 \times 10^{-5} \sin 10^9\ t)$$

$$= 8.854 \times 10^{-12} \times (6.5) \times 2 \times 10^{-5} x 10^9 \times cos10^9 t$$

$$= 115 \times 10^{-8} \cos 10^9 t$$

$$Jd = 1.15 \times 10^{-6} \cos 10^9 t\ (A/m)$$

14. Fine the frequency at which conduction current density and displacement current density are equal in

(a) Distilled water for which $\varepsilon_r = 81$ and $\sigma = 2.0 \times 10^{-4}$ ℧ \m

(b) Sea water, for which ε_r =1 and $\sigma = 4.027$ ℧ \m .

(a) For equal values of conduction and displacement current densities

$$\sigma = \omega\varepsilon$$

$$f = \frac{\sigma}{2\pi\varepsilon} = \frac{2.0 \times 10^{-4}}{(2\pi \times 8.854 \times 10^{-10} \times 81)}$$

$$f = 4.44 \times 10^4\ Hz$$

$$= 4.44\ KHz$$

(b) In case $\varepsilon_r = r, \sigma$ is much higher than for distilled water

$$f = \frac{\sigma}{2\pi\varepsilon} = \frac{4.0}{2\pi \times (8.854 \times 10^{-12})}$$

$$= 7.19 \times 10^{10} Hz$$

$$= 71.9 GHz$$

15. The wave length of a 600 MHz wave propagating through a non magnetic dialectic is 20 cm, what is the relative permittivity by ε_r of the material?

Sol

Velocity of propagation $V = F\lambda$

F=frequency $= 600 mHz = 6 \times 10^8 Hz$

$$\lambda = 20cm = 0.20m$$

$$V = 6 \times 10^8 \times 0.20 = 1.2 \times 10^8\ m/sec$$

$$1.2 \times 10^8 = \frac{3.0 \times 10^8}{\sqrt{\varepsilon_r}}, \varepsilon_r = 6.25$$

16. The current density at the surface of a thick metal plate is 100 A\$m^2$.what is the skin depth if current density at a depth of 0.01 cm is 28 A\$m^2$?

Sol

Current density at a depth x is given by

$$J_x = J_0 e^{-x/\delta}$$

Where, δ is the skin depth

$$e^{x\backslash\delta} = \frac{J_0}{J_x} = \frac{100}{28} = 3.57$$

$$\frac{x}{\delta} = l_n 3.57 = 1.27$$

$$\delta = \frac{x}{1.27} = \frac{0.01}{1.27} = 0.0079cm$$

17. Calculate the intrinsic impedance and wave velocity for a conducting medium,$\sigma = 58\ \mho\backslash m$ and $\mu_r = 1$ at a freguncy of 100 mHz.

Sol

$$f = 100 \times 10^6 = 10^8 Hz$$

$$\omega = 2\pi \times 10^8$$

Intrinsic Impedance $\eta_m = \sqrt{\frac{\omega\mu m}{\sigma m}} < 45° ohm$

$$= \sqrt{\frac{2\pi \times 10^8 \times 4 \times 10^{-7}}{58 \times 10^6}} < 45°$$

$$= 3.69 \times 10^{-3} < 45° \boldsymbol{\Omega}$$

$$\lambda m = \sqrt{\omega\mu m\sigma m.} < 45°$$

$$= \sqrt{2 \times \pi \times 10^8 \times 4\pi \times 10^{-7} \times 58} \times 10^6$$

$$= 214 \times 10^3 < 45$$

$$\beta_m = \frac{214 \times 10^3}{\sqrt{2}} = 151 \times 10^3 red/m$$

Wave velocity $= \frac{w}{\beta_m} = \frac{2\pi \times 10^8}{151 \times 10^3} = 4.161 \times 10^3\ m/sec$

18. At what frequencies may earth be considered as a perfect dielectric?
$\sigma = 5 \times 10^{-3}\ \mho/m$ and $\mu_r = 1, \varepsilon_r = 8$) Can $\propto$ be asemed zero as these frequency?

Sol

For $\frac{\sigma}{\omega\varepsilon} \ll$,earth can be considered as perfect dielectric

$$\frac{\sigma}{\omega\varepsilon} = 0.1$$

$$\omega_c = \frac{\sigma}{0.1\varepsilon} = \frac{5 \times 10^{-3} \times 36\pi \times 10^9}{0.1 \times 8.0}$$

$$\varepsilon_0 = \frac{1}{36\pi \times 10^9}$$

$$= 706.85\ mega - red\ sec$$

$$f_c = 112.5\ mHz$$
$$\alpha\ can\ \ not\ be\ zero$$
$$\propto = \beta/2 \text{ Tan } \delta$$
$$= \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$$
$$= \frac{5 \times 10^{-3}}{2}\sqrt{\frac{4\pi \times 10^{-7} \times 8.854 \times 10^{-12}}{8}}$$
$$= 0.332\ Neper/m$$

19. A 2000 MHz standing wave pattern exists in a non-magnetic dielectric. What is the value of ε_r if the distance between a maximum and its adjacement minimum is 1.5 cm?

Sol

This distance between a max and its adjacent minimum is $\frac{\lambda}{4} = 1.5\ cm$

Wave length $\lambda = 6.0\ cm$

Velocity $v = \lambda f$

$$= (6 \times 10^{-2})(2000 \times 10^{6}) = 1.2 \times 10^{8}\ \frac{m}{sec}$$
$$1.2 \times 10^{8} = \frac{3.0 \times 10^{8}}{\sqrt{\varepsilon_r}}$$
$$\varepsilon_r = 6.25$$

20. A 500 MHz incident wave propagates from a lossless insulator to free space .Determine $\mu_r, and\ \ \varepsilon_{r1}\ if\ SWR$ in reign 1 is 1.6 and the wave length is 35 cm assume $\varepsilon_{r1} > \varepsilon_{r1}$.

Sol

Wave length $\lambda = 0.35m$

$$f = 5 \times 10^{8}\ Hz$$

Velocity of propagation $v = \lambda f$

$$= 5 * 10^{8} \times 0.35$$
$$= 1.75 \times 10^{8}\ m/sec$$

$\sqrt{\mu_r \varepsilon_r} = \frac{3}{1.75}$----1

$$\rho = \frac{S-1}{S+1} = \frac{1.6-1}{1.6+1} = \frac{-\sqrt{\frac{\mu_r}{\varepsilon_r}+1}}{\frac{\mu_r}{\varepsilon_r}+1}$$

$\therefore$ $\sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{1}{1.6}$----------2

From (1) & (2)

$$\varepsilon_r = \frac{1.6 \times 3}{1.75} = 2.743$$
$$\mu_r = \frac{3}{1.75} \times 1.6 = 1.071$$

21. Calculate the attenuation constant and phase constant for a uniform plane with frequency 10 GHz in a medium for which $\mu = \mu_0$, $\varepsilon_r = 2.3$ and $\sigma = 2.56 \times 10^{-8}$ ℧/m.

Sol

$$\text{Let } \gamma = \alpha + jP$$

$$= \sqrt{j\omega\mu(\sigma + j\omega e)}$$

$$\gamma = \sqrt{j(2\pi f)(\mu_0\mu_r)[\sigma + j + (2\pi f)(\varepsilon_0\varepsilon_r)]}$$

$$= \sqrt{2 \times \pi \times 10 \times 10^9 \times 4 \times \pi \times 10^{-7}[2.56 \times 10^{-4} + j(2\pi \times 10^9) \times 8.854 \times 10^{12} \times 2.3]}$$

$$\gamma = \sqrt{j(78.96 \times 10^3)(2.56 \times 10^{-4} + j1.28)}$$

$$= \sqrt{(78.96 \times 10^3 < 90°)(1.28 \times < 89.98°)}$$

$$= \sqrt{101.25 \times 10^3 < 179.98°}$$

$$= 317.84 < 89.99°$$

$$\gamma = 0.0554 + j317.84$$

$$\lambda = 0.0554 N_p l_m$$

$$\beta = 317.84 \frac{rad}{m}$$

22. A uniform plane wave is travelling at a velocity of 2.5$\times 10^5$ m/s having Wavelength $\lambda = 0.25$ $mmin$ good conductor .calculate the frequency of wave and the conductivity of a medium.

Sol

$$v = 2.5 \times 10^5 m\backslash s, \quad \lambda\, 0.25 \times 10^3 m$$

$$f = \frac{V}{\lambda}$$

$$= \frac{2.5 \times 10^5}{0.25 \times 10^{-3}}$$

$$= 10 \times 10^8 Hz = 1\ GHz$$

$$velocity\ of\ the\ wave\ in\ a\ good\ canductor$$

$$V = \sqrt{\frac{2\omega}{\mu\sigma}}$$

$$V^2 = \frac{2w}{\mu\sigma}$$

$$\sigma = \frac{2w}{\mu v^2}$$

$$= \frac{2 \times 2\pi f}{\mu_0\mu_r V^2} [\because\ \mu_r = 1]$$

$$= \frac{4\pi \times 10^9}{4\pi \times 10^{-7} \times (2.5 \times 10^5)^2} = 0.16 \times 10^6\ \text{℧}/m$$

23. A uniform plane wave propagating in a medium has E=$2e^{-\alpha z \sin(10^8 t - \beta z)}\overrightarrow{a_y} V/m$ .If the medium is characterized by $\varepsilon_r = 1, \mu_r = 20$ and $\sigma = 3$ mhos/m, find loss tangents α, β and H (Nov/Dec 2015)

Solution:
Given:
E=$2e^{-\alpha z \sin(10^8 t - \beta z)}\overrightarrow{a_y} V/m$
$\omega = 10^8$
$\varepsilon_r = 1\eta\eta$
$\mu_r = 20$
$\sigma = 3$mhos/m,

$$\frac{\sigma}{\omega\varepsilon} = \frac{3}{10^8 \times 80854 \times 10^{-12}}$$

$\frac{\sigma}{\omega\varepsilon} \gg 1$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{10^8 \times 4\pi \times 10^{-7} \times 20 \times 3}{2}}$$

α = 61.40

H=$\frac{E}{\eta}$

$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_o \mu_r}{\varepsilon_o \varepsilon_r}}$

$=120\,\pi\sqrt{\frac{\mu_r}{\varepsilon_r}}$

$=120\,\pi\sqrt{\frac{120}{1}}$

$= 120\pi\sqrt{20}$

$=536.65\,\pi$

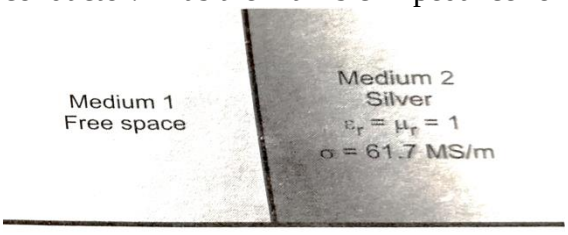
=1685.08

H=$\frac{2e^{-\alpha z \sin(10^8 t - \beta z)}\overrightarrow{a_y}}{1685.08}$

=$1.1868e^{-\alpha z}e^{-\alpha z \sin(10^8 t - \beta z)}\overrightarrow{a_y}$mA/m

24. A free space – silver interface has E(incident) = 100 V/m on the free space side. The frequency is 15 Mhz and the silver constants are $\varepsilon_r = \mu_r = 1, \sigma = 61.7\,MS/m$. Determine E (reflected) and E (transmitted) at the interface. (April/May 2017)

The conductivity of silver is given as $\sigma = 61.7\,MS/m \gg 1$ it can be considered as good conductor. Thus the intrinsic impedance for good conductor is given by '



$$\eta_2 = \sqrt{\frac{\mu\omega}{\sigma}} \angle 45°$$

$$= \sqrt{\frac{(\mu_0\mu_r)(2\pi f)}{\sigma}} \angle 45° = \sqrt{\frac{\left(4\times\pi\times10^{-7}\times1\right)\left(2\times\pi\times15\times10^{6}\right)}{\left(61.7\times10^{6}\right)}} \angle 45°$$

$$= 1.3854\times10^{-3} \angle 45° \ \Omega = \left(9.7962\times10^{-4} + j\,9.7962\times10^{-4}\right)\Omega$$

As medium 1 is free space, the intrinsic impedance is given by

$$\eta_1 = \eta_0 = 377 \angle 0° \ \Omega = (377 + j\,0)\ \Omega$$

Also in free space, $E_i = 100$ V/m

The transmission coefficient is given by

$$\tau = \frac{E_t}{E_i} = \frac{2\eta_2}{\eta_1 + \eta_2}$$

$$\therefore \quad \frac{E_t}{100} = \frac{2\left(1.3854\times10^{-3} \angle 45°\right)}{(377 + j0) + \left(9.7962\times10^{-4} + j\,9.7962\times10^{-4}\right)} = \frac{0.2771 \angle 45°}{377 + j\,9.7962\times10^{-4}}$$

$$\therefore \quad E_t = \frac{0.2771 \angle 45°}{377 \angle 1.48\times10^{-4}} = \mathbf{7.35\times10^{-4} \angle 45° \ V/m}$$

The reflection coefficient is given by,

$$\Gamma = \frac{E_r}{E_i} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

$$\therefore \quad \frac{E_r}{100} = \frac{\left(9.7962\times10^{-4} + j\,9.7962\times10^{-4}\right) - 377}{\left(9.7962\times10^{-4} + j\,9.7962\times10^{-4}\right) + 377}$$

$$\therefore E_r = 100\left[\frac{-377 + j\,9.7962\times10^{-4}}{377 + j\,9.7962\times10^{-4}}\right] = 100\left[\frac{377 \angle +180°}{377 \angle 1.48\times10^{-4}}\right] = 100 \angle 180° = \mathbf{-100\ V/m}$$

25. ***A 9375 MHz uniform plane wave is propagating in polystyrene. If the amplitude of the electric field intensity is 20 V/m and the material is assumed to be lossless, find :***

i) Attenuation constant ii) Phase constant iii) Wavelength in polystyrene

iv) Velocity of propagation v) Intrinsic impedance vi) Propagation constant vii) Amplitude of the magnetic field intensity.

Solution : For a lossless medium, $\sigma = 0$. Then

i) Attenuation constant, $\alpha = 0$

ii) Phase constant,

$$\beta = \omega\sqrt{\mu\varepsilon} = \omega\sqrt{(\mu_0\mu_r)(\varepsilon_0\varepsilon_r)}$$

$$= \left(2\times\pi\times 9375\times 10^6\right)\sqrt{\left(4\times\pi\times 10^{-7}\right)(1)\left(8.854\times 10^{-12}\right)(2.56)} = \mathbf{314.37 \ \ rad/m}$$

iii) Wavelength (in polystyrene) is given by,

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{314.37} = \mathbf{0.01998 \ \ m}$$

iv) The velocity of propagation is given by,

$$v = f\lambda = \left(9375\times 10^6\right)(0.01998) = \mathbf{1.873\times 10^8 \ \ m/s}$$

v) The intrinsic impedance is given by,

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \sqrt{\frac{4\times\pi\times 10^{-7}\times 1}{8.854\times 10^{-12}\times 2.56}} = \mathbf{235.45 \ \ \Omega}$$

vi) The propagation constant γ is given by

$$\gamma = \alpha + j\beta = 0 + j\,314.37 \ \ m^{-1} = \mathbf{j\,314.37 \ \ m^{-1}}$$

vii) The amplitude of electric field intensity is given as 20 V/m. $\therefore$ $E_x = 20$ V/m

But $\eta = \frac{E_x}{H_y}$ where H_y is amplitude of magnetic field intensity in y-direction.

$$\therefore \quad \eta = \frac{E_x}{H_y} = \frac{20}{H_y} = 235.45$$

The amplitude of magnetic field intensity is given by

$$H_y = \frac{E_x}{\eta} = \frac{20}{235.45} = \mathbf{0.08494 \ \ A/m}$$

26. A plane wave travelling in +z direction in free space (z>0) is normally incident at z=0 on a conductor (z>0) for which $\sigma = 61.7\frac{MS}{m}, \mu_r = 1$. The free-space E wave has a frequency f= 1.5 MHz and an amplitude of 1.0 V/m; at the interface it is given by E(0,t) =1.0 sin 2лfta_y. analyse the wave and predict the magnetic wave H(z,t) at z>0 (Nov/Dec 2016)(Part C)

For z> 0, and in complex form,

$$E(z,t) = 1.0\, e^{-\alpha z} e^{j(2\pi f t - \beta z)} a_y \ (V/m)$$

Where the imaginary part will ultimately be taken.
In the conductor,

$$\alpha = \beta = \sqrt{\pi(1.5 \times 10^6)(4\pi \times 10^6)} = 1.91 \times 10^4$$

$$\eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45° = 4.38 \times 10^{-4} e^{j\pi/4}$$

Then, Since $E_y / (-H_X) = \eta$

$$H(z,t) = -2.28 \times 10^3 e^{-\alpha z} e^{j(2\pi f t - \beta z - \pi/4)} a_x \ (A/m)$$

Taking the imaginary part

$$H(z,t) = -2.28 \times 10^3 e^{-\alpha z} sin\,(2\pi f t - \beta z - \pi/4) a_x \ (A/m)$$

Where f, α, and β are as given above

27. The electric field intensity associated with a plane wave travelling in a perfect dielectric medium is given by $E_x(z,t) = 10cos\,(2\pi\ 10^7 t - 01.\pi z)$v/m. What is the velocity of propogation(April/May 2018)

$$E_x(z,t) = C \cdot \cos(\omega t - \beta z)$$

$$\omega = 2\pi f = 2\pi \times 10^7 \cdot$$

$$\Rightarrow f = 10^7 \text{ Hz.}$$

$$\beta = \left(\frac{\omega}{v}\right) = \frac{2\pi f}{\lambda f} = \frac{2\pi}{\lambda} = 0.1\pi \quad \Rightarrow \lambda = 20 \text{ m}$$

a)

$$v = f \cdot \lambda = 10^7 \times 20 = 2 \times 10^8 \text{ m/sec.}$$

$$v_0 \text{ (in free space)} = 3 \times 10^8 \text{ m/sec.}$$

$$\sqrt{\varepsilon_r \mu_r} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$$

$$\mu_r = 1 \quad \Rightarrow \varepsilon_r = (1.5)^2 = 2.25$$

b)

$$\eta = \sqrt{\left(\frac{\mu}{\varepsilon}\right)} = \frac{120\pi}{1.5} = 80\pi.$$

$$H_y = \left(\frac{10}{80\pi}\right) \cdot \cos(2\pi \times 10^7 t - 0.1\pi z) \quad \text{A/m}$$

28. A lossless transmission line has characteristics impeadance of 70Ω and phase constant of 3rad/m at 100MHz, calculate the inductance per meter and the capacitanceper meter length of the line(Nov/Dec 2017)

$$Z_0 = 70\Omega \ ; \ \beta = 3\,\text{rad/m}; \ f = 100\times10^6\ \text{Hz}$$

But for a loss less line, $z_0 = R_0 = 70\Omega$

$$R_0 = \sqrt{\left(\frac{L}{C}\right)} \ ; \ \omega = \frac{\beta}{\sqrt{LC}}$$

(ie) $70 = \sqrt{\frac{L}{C}} \ ; \ \Rightarrow \ 70^2 = \left(\frac{L}{C}\right); \ L = 70^2 C$

$$2\pi(100\times10^6) = \frac{3}{\sqrt{LC}}$$

$$\Rightarrow \ \sqrt{LC} \ = \frac{3}{2\pi(100\times10^6)} = 4.77\times10^{-9}$$

$$LC = (4.77\times10^{-9})^2$$

$$(70)^2 C\cdot C = (4.77\times10^{-9})^2$$

$$C = 68.21\times10^{-12}\ F$$

$$L = (70)^2 C = 334.22\times10^{-9}\ H.$$

UNIVERSITY QUESTIONS

PART A

1) Define poynting vector, what is the SI unit for this vector.(April 2008,May 2003,May 2010, 2011)
2) What is a wave?(Nov 2007)
3) Brief about complex pointing vector?(June 2006)
4) Brief about the intrinsic impedance for a perfect dielectric medium?(June 2006)
5) What is meant by linear Polarization?(June 2006)
6) What do you mean by "Depth of penetration?(May 2010, June 2006)
7) Define skin effect. (June 2007, May 2010)
8) What is the velocity of electromagnetic wave in free apace and in lossless dielectric?(April 2008)
9) What is Brewster angle? (April 2008, May 2010)
10) State one dimensional wave equation (April 2008)
11) What is characteristic impedance? (Dec 2008,Dec 2012) or What is intrinsic impedance?
12) What is uniform medium? (May 2010)
13) Write the wave equation in conducting medium?
14) Write down the wave equation for E and H in a free space medium (Aril 2010,Dec2009)

15) A plane travelling wave in free space has an average pointing vector of 1-5 watts /m^2 calculate the average energy density. (Dec 2009)
16) Find the velocity of plane wave in a lossless medium having a relative permittivity of 5 and relative permeability of unity. (Dec2009)
17) Define standing wave ratio. (Dec 2009,June 2013)
18) Mention any two properties of uniform plane wave? (Dec 2010)
19) What is uniform wave? (June 2010)
20) What is meant by circular polarization? (Dec 2010)
21) Define Elliptical polarization
22) Find the velocity of a plane wave in a lossless medium having a relative permittivity of 5 and relative permeability? (April 2005)
23) Find the average power loss/ volume for a dielectric having $\varepsilon_r = 2\ and \tan\delta = 0.005, if\ E = \frac{1.0KV}{m} at\ 500\ mHz$ (Dec 2005)
24) Calculate skin depth for medium with conductivity 100℧/m1 relative permeability 2, relative permittivity 3 at 50 Hz, 1MHz and 1 GHz (May 2013)
25) What is the significance of intrinsic impedance of free? What is its value?(Dec 2005, Dec 2011)
26) Define skin effect and its significance at low frequency and at very high frequency applications to conductions. (Dec 2003)
27) Define loss tangent in an insulating material?
28) How a dielectric medium can be identified as lossless and lossy for a given frequency.
29) For a lossy dielectric material having$\mu_r = 1, \varepsilon_r = 48,\ = 20\ s/m$, calculate the propagation constant at a frequency of 16 Hz.(June 2006)

30) Find the skin depth at a frequency of 2MHz is aluminum $where\ \sigma = \frac{38.2m}{m}$ and $\mu_r = 1$

31) Write Helmholtz's equation.
32) Write down the expression for instantaneous mean power flow in electromagnetic field and instantaneous poynting vector.
33) Write down the expression for average power flow in electromagnetic field and average pointing vector.
34) State slepian vector
35) State snell's law.
36) Define surface impedance.
37) What is the wave length and frequency of wave propagation in a free space? when β= 2? (April/May 2015)
38) Compare equipotential plot for non-uniform and uniform fields. (April/May 2015)
39) If a plane is incident normally from medium 1 to medium 2, write the reflection and transmission coefficients. (Nov/Dec 2014)
40) The capacitance and inductance of an overhead transmission line are 0.0075 μF/km and 0.5 mH/km respectively. Determine the characteristics impedance of the line. (Nov/Dec 2014)

PART B

1) From the Maxwell equation, derive the electromagnetic wave equation in conducting medium for $\overrightarrow{E}$ and $\vec{H}$ fields (APR 2011) (NOV 2010)
2) Derive the Wave equations in general form.

3) Deduce the equation of the propagation of the plane electromagnetic waves in free space. (NOV 2012)

4) Derive the relationship between electric field and magnetic field. Derive the wave equation for magnetic field in phasor form. (DEC 2010)

5) Derive the EM wave equation in frequency domain and obtain the expressions for intrinsic impedance and propagation constant for free space, conductor and dielectric medium. (APR 2010) (Include all the below cases for question no.4)(May/June 2014)

6) Define wave and derive the wave equations for non-dissipative medium.(June 2013)
   i. (Include case (i) alone for question no.5)

7) Discuss the wave motion in good conductors (APR 2010)
   i. (Include case (iv) alone for question no.6)
8) Describe the concept of wave propagation in a linear, isotropic, homogeneous, and lossy dielectric medium. (Nov/Dec 2014)
9) Obtain an expression for wave propagation in lossy dielectrics. (Only case (ii))
   i. (April/May 2015)

10) State and prove poynting theorem. (APR 2011)(NOV 2012))

11) Explain how the rate of energy transportation by means of EM waves from one point to another point can be obtained from Maxwell's equation (June 2013)(OR)

12) State poynting theorem and thus obtain an expression for instantaneous power density vector associated with the electromagnetic field. (April/May 2015)

13) Reflection of plane waves:(OR)

14) Derive the transmission and reflection co-efficient for the electromagnetic waves

15) (APR 2008) (NOV 2012)

16) Define Brewster angle and derive its expression (NOV 2010)

17) Show that the total power flow along a coaxial cable will be given surface integration of the pointing vector over any closed surface. (Nov/Dec 2014)

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 50474

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

Third Semester

Electrical and Electronics Engineering

EE 6302 – ELECTROMAGNETIC THEORY

(Regulations 2013)

: Three Hours | Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

State Coulomb's law.

Define gradient. What does it indicate ?

State Poisson's and Laplace's equations.

Define dielectric strength.

State Ampere's circuital law.

What is inductance ? Give its formula in electrical parameters.

What is displacement current ?

State any two major differences between circuit theory and field theory

Define skin depth.

What is intrinsic impedance ?

4

PART – B

(5×13=65 Marks)

1. a) i) A charge 20 μ C is located at A(– 6,4,7) and another charge 50 μ C is at B(5, 8, –2) in free space. If distances are given in metres, determine the vector force exerted by the first charge on the second one. (4

ii) A 50 cm length coaxial cable having an inner radius 1 mm and an outer radius 4 mm, has its inner space between the conductors filled with air. The total charge on the inner conductor is 30 nC. Find the charge density on each conductor. (4

iii) State the divergence theorem and explain its significance. (5

(OR)

b) i) A charge of 0.3 μC is located at A(25, – 30, 15) (in cm) and a second charge of 0.5 μ C is at B(–10, 8, 12) cm. Find the Electric field intensity at the origin. (4

ii) Find the total charge enclosed in an incremental volume of 10^{-9} m^3 located at the origin, if $D = e^{-x} \sin ya_x + 2za_z$ C/m^2. (4

iii) State Gauss's law and give any two of its applications (5

12. a) i) State and prove electrostatic boundary conditions. (7

ii) Describe the concept of dielectric polarization. (6

(OR)

b) i) Derive the energy density of capacitance. (7

ii) Discuss in detail, the electric field in multiple dielectrics (6

13. a) i) Derive the H due to current 'I' flowing in a circular loop. (6

ii) State and prove magnetostatic boundary conditions. (7

(OR)

b) i) Describe magnetic materials. (

ii) Compare and contrast scalar and vector potentials. (

iii) Define Torque and Magnetic force. (

14. a) Derive all the Maxwell's equations, in both differential and integral form. (1

(OR)

b) i) Describe the function of a transformer starting from fundamental principles.

ii) Describe the applications where circuit theory is used and applications, wher field theory is used.

5. a) i) Derive wave equation from Maxwell's equations. (7)
ii) Describe the wave propagation in free space. (6)

(OR)

b) i) Describe plane wave reflection. (7)
ii) Derive Poynting vector and state its significance. (6)

PART – C (1×15=15 Marks)

16. a) Consider an Antenna radiating signals from a Transmitter section. Also, there exists an Amplifier which is actually used in a Receiver section to boost the very weak signals. Compare the role played/not played by Field theory and/or Circuit theory in both these electronic equipments. Clearly bring out a deeper analysis of both the modules with the relevant theory. Hence validate "Both field theory and circuit theory are equally important".

(OR)

b) Clearly bring out the distinction between a 'Standing wave' and a 'propagating wave'. What difference does it mean, in terms of power flow given by Poynting vector in both these kinds of waves ? In detail, bringout the concepts behind the two waves. Is standing wave finding an application anywhere ? Why ?

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 40992

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018
Third Semester
Electrical and Electronics Engineering
EE 6302 – ELECTROMAGNETIC THEORY
(Regulations 2013)

Time : Three Hours | Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. Find the unit vector extending from the origin toward the point P(3, – 1, – 2).
2. Determine the electric field intensity in free space if $\vec{D} = 30\vec{a}_x \ C/m^2$.
3. Mention the properties of electric flux lines.
4. State the electrostatic boundary conditions at the interface between two dielectrics.
5. What is the total force acting on a moving charge, Q in the presence of both electric and magnetic fields.
6. Compare magnetic scalar potential and magnetic vector potential.
7. Define Reluctance and Permeability.
8. Distinguish between conduction and displacement currents.
9. Mention the practical importance of 'Skin depth'.
10. What is 'Standing Wave Ratio' ?

PART – B (5×13=65 Marks)

11. a) i) With neat diagrams, explain the spherical system with co-ordinates (R, θ, ϕ). (6)

ii) Apply Coulomb's law to find the electric field intensity at any point P due to a straight, uniformly charged wire of linear charge density + λ C/m. The point P is at a distance of 'h' m above the wire. (7)

(OR)

b) i) Explain the divergence of a vector field and divergence theorem. (6)

ii) By mean of Gauss's law, determine the electric field intensity inside and outside a spherical shell of radius R. The shell contains a total charge Q uniformly distributed over the surface. (7)

12. a) i) Two point charges – 4 μC and 5 μC are located at (2, – 1, 3) and (0, 4, – 2) respectively. Find the potential at (1, 0, 1) assuming zero potential at infinity. (6)

ii) A parallel plate capacitor has a plate separation t. The capacitance with air only between the plates is C. When a slab of thickness t′ and relative permitivity ε' is placed on one of the plates, the capacitance is C′ Show that $\frac{C'}{C} = \frac{\varepsilon' t}{(t' + \varepsilon'(t - t'))}$. (7)

(OR)

b) i) Explain briefly the polarization in dielectrics. (6)

ii) Derive Laplace's and Poisson's equations from Gauss's law for a linear material medium. State the importance of these equations. (7)

13. a) i) By means of Biot-Savart's law, derive an expression for the magnetic field intensity at any point on the line through the centre at a distance 'h' from the centre and perpendicular to the plane of a circular loop of radius 'ρ' and carrying current 'I.' (6)

ii) An iron ring, 0.2 m in diameter and 10 cm^2 sectional area of the core, is uniformly wound with 250 turns of wire. The wire carries a current of 4 A. The relative permeability of iron is 500. Determine the value of self-inductance and the stored energy. (7)

(OR)

b) i) What is 'Magnetization' ? Explain the classification of magnetic materials. (6)

ii) What is the maximum torque on a square loop of 1000 turns in a field of uniform flux density of 1 Tesla ? The loop has 10 cm sides and carries a current of 3 A. What is the magnetic moment of the loop ? (7)

14. a) An iron ring with a cross-sectional area of $3cm^2$ and a mean circumference of 15 cm is wound with 250 turns of wire carrying a current of 0.3 A. The relative permeability of the ring is 1500. Calculate the flux established in the ring. (13)

(OR)

b) i) Write a technical note on 'Transformer EMF and Motional EMF'. (6)

ii) Describe the relationship between field theory and circuit theory. (7)

15. a) i) The electric field intensity associated with a plane wave travelling in a perfect dielectric medium is given by $E_x(z, t) = 10\cos(2\pi \times 10^7 t - 0.1\pi z)$ V/m. What is the velocity of propagation ? (6)

ii) Derive the Poynting theorem and state its significance. (7)

(OR)

b) Write short notes on the following : (4+4+5)

i) Plane waves in lossless dielectrics.

ii) Plane waves in free space.

iii) Plane waves in good conductors.

PART – C (1×15=15 Marks)

16. a) Step by step, develop a condition between

i) Conductor and dielectric.

ii) Dielectric and dielectric. (15)

(OR)

b) From the basics, derive the expressions for Maxwell's equation in differential and integral form. (15)

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 80367

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Third Semester

Electrical and Electronics Engineering

EE 6302 — ELECTROMAGNETIC THEORY

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Determine the angle between $A = 2\vec{a}_x + 4\vec{a}_y$ and $B = 6\vec{a}_y - 4\vec{a}_z$.
2. State Stoke's Theorem.
3. Find the capacitance of an isolated spherical shell of radius a.
4. Find the magnitude of D for a dielectric material in which $E = 0.15$ MV/m and $\varepsilon_r = 5.25$.
5. State Ampere's Circuital Law.
6. A conductor 4 m long lies along the y-axis with the current of 10 A in a_y direction, if the field is $B = 0.05\,a_x$ Tesla calculate the force on the conductor.
7. Moist soil has conductivity of 10^{-3} S/m and $\varepsilon_r = 2.5$, determine the displacement current density if $E = 6.0 \times 10^{-6}\ sin\ 9.0 \times 10^9$ t (V/m).
8. State Faraday's Law.
9. Define standing wave ratio.
10. State the properties of uniform plane wave.

PART B — (5 × 13 = 65 marks)

11. (a) (i) State and Prove Divergence theorem (8)

(ii) Transform $4\vec{a}_x - 2\vec{a}_y - 4\vec{a}_z$ at (2, 3, 5) to cylindrical coordinates. (5)

Or

(b) (i) Derive the expression for electric field intensity due to uniformly charged circular disc of σ c/m². (8)

(ii) Find the force on a charge Q_1 of 20 μC at (0, 1, 2)m due to Q_2 of 300 μC at (2, 0, 0)m. (5)

12. (a) (i) Find the potential at $r_A = 5$ m with respect to $r_B = 15$ m due to point charge $Q = 500$ pC at the origin and zero reference at infinity. (6)

(ii) Find the capacitance of a parallel plate capacitor with dielectric $\varepsilon_{r1} = 1.5$ and $\varepsilon_{r2} = 3.5$ each occupy one half of the space between the plates of area 2 m² and $d = 10^{-3}$ m. (7)

Or

(b) (i) In spherical coordinates $V = -25$ V on a conductor at $r = 2$ cm and $V = 150$ V at $r = 35$ cm. The Space between the conductor is a dielectric of $\varepsilon_r = 3.12$. Find the surface charge densities on the conductor. (10)

(ii) Define Laplace and Poisson's equation. (3)

13. (a) Derive the expression for magnetic field intensity due to infinitely long straight conductor carrying a current of I amps along Z-axis. (13)

Or

(b) (i) Determine H for a solid cylindrical conductor of radius a, where the current I is uniformly distributed over the cross section. (5)

(ii) Calculate the inductance of a ring shaped coil of mean diameter 20 cm, wound on a wooden core of 2 cm diameter containing 200 turns. (8)

14. (a) Derive Maxwell's equation in both point and integral form for conducting medium and free Space. (13)

Or

(b) (i) Explain the concept of emf induction in static and time varying magnetic field. (8)

(ii) In a material for which $\sigma = 5.0$ S/m and $\varepsilon_r = 1$ with $E = 250 \sin 10^{10}$ t (V/m). Find J_c and J_D and also the frequency at which they equal magnitudes. (5)

15. (a) Derive the expression for electromagnetic wave equation for conducting and perfect dielectric medium. (13)

Or

(b) A 6580 MHz uniform plane wave is propagating in a material medium of $\varepsilon_r = 2.25$. If the amplitude of the electric field intensity of lossless medium is 500 V/m. Calculate the phase constant, propagation constant, velocity, wavelength and intrinsic impedance. (13)

PART C — (1 × 15 = 15 marks)

16. (a) A plane wave travelling in $+z$ direction in free space ($z < 0$) is normally incident at $z = 0$ on a conductor ($z > 0$) for which $\sigma = 61.7$ MS/m, $\mu_r = 1$. The free space E wave has a frequency f = 1.5 MHZ and an amplitude of 1.0 V/m at the interface it is given by $E(0, t) = 1.0 \sin 2\pi ft\, a_y$ (V/m). Analyse the wave and predict magnetic wave $H(z, t)$ at $z > 0$. (15)

Or

(b) Given that $A = 30e^{-r}\bar{a}_r - 2z\bar{a}_z$ in cylindrical coordinates, evaluate both sides of divergence theorem for the volume enclosed by $r = 2$, $z = 0$ and $z = 5$. (15)